The Strong CP Problem and Axions

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The Strong CP Problem and Axions

- The $U(1)_A$ Problem of QCD
- The QCD Vacuum and the Strong CP Problem
- Approaches to the Strong CP Problem
- $U(1)_{PQ}$ and Axions
- Axion Dynamics
- Invisible Axion Models
- Concluding Remarks

The $U(1)_A$ Problem of QCD

- In the 1970's the strong interactions had a puzzling problem, which became particularly clear with the development of QCD.
- The QCD Lagrangian for N flavors

$$L_{QCD} = -1/4 F_a^{\mu\nu} F_{a\mu\nu} - \Sigma_f \, \bar{q}_f (-i\gamma^\mu D_\mu + m_f) q_f$$

in the limit $m_f \to 0$ has a large global symmetry: $U(N)_V \times U(N)_A$

$$q_f \to [e^{i\alpha_a T_a/2}]_{ff'} q_{f'} \quad ; \quad q_f \to [e^{i\alpha_a T_a \gamma_5/2}]_{ff'} q_{f'}$$

Vector   Axial

- Since m_u, $m_d << \Lambda_{QCD}$, for these quarks m_f -> 0 limit is sensible. Thus expect strong interactions to be approximately $U(2)_V \times U(2)_A$ invariant.
- Indeed, experimentally know that

  $U(2)_V = SU(2)_V \times U(1)_V \equiv$ Isospin x Baryon #

  is a good approximate symmetry of nature

  $\Rightarrow$ (p, n) and ($\pi^{\pm}$, π°) multiplets in spectrum
- For axial symmetries, however, things are different. Dynamically, quark condensates

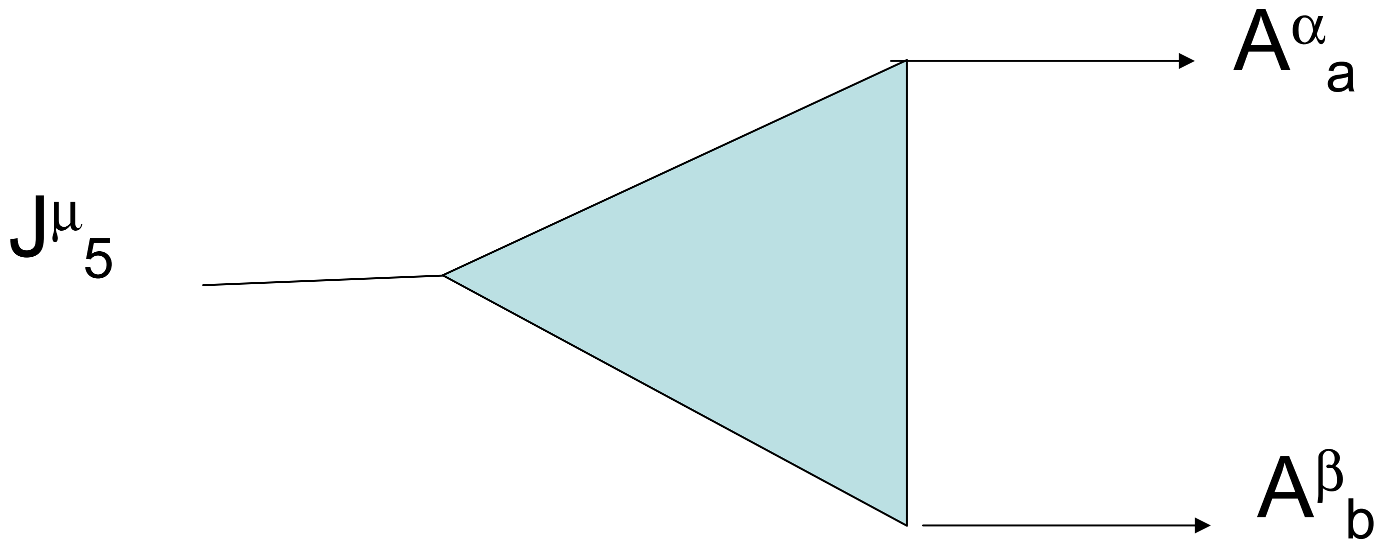
  $< \bar{u}u >=< \bar{d}d > \neq 0$ break $SU(2)_A$ down spontaneously and $\therefore$ no mixed parity multiplets

- However, because $U(2)_A$ is spontaneously broken symmetry, expect appearance in the spectrum of approximate Nambu-Goldstone bosons, with $m \approx 0$ [$m \to 0$ as m_u, $m_d \to 0$]
- For $U(2)_A$ would expect 4 such bosons (π, η). Although pions are light, $m_\pi \approx 0$, see no sign of another light state in the hadronic spectrum, since $m^2_\eta >> m^2_\pi$.
- Weinberg dubbed this the $U(1)_A$ problem and suggested that, somehow, there was no $U(1)_A$ symmetry in the strong interactions

The QCD Vacuum and the Strong CP Problem

- The resolution of the $U(1)_A$ problem, came through the realization that the QCD vacuum is more complicated [‘t Hooft].
- This complexity, in effect, is what makes $U(1)_A$ not a symmetry of QCD, even though it is an apparent symmetry of L_{QCD} in the limit $m_f \to 0$
- However, this more complicated vacuum gives rise to the strong CP problem. In essence, as we shall see, the question becomes why in QCD is CP not very badly broken?

- A possible resolution of the $U(1)_A$ problem seems to be provided by the chiral anomaly for axial currents [Adler Bell Jackiw]
- The divergence of axial currents, get quantum corrections from the triangle graph



with fermions going around the loop

- This anomaly gives a non-zero divergence

$$\partial_\mu J_5^\mu = \frac{g^2 N}{32\pi^2} F_a^{\mu\nu} \tilde{F}_{a\mu\nu}$$

where $\tilde{F}_a^{\mu\nu} = 1/2\varepsilon^{\mu\nu\alpha\beta} F_{\alpha\beta}$, even in symmetry limit

- Hence, in the m_f -> 0 limit, although formally QCD is invariant under a $U(1)_A$ transformation

$$q_f \to e^{i\alpha/2\gamma 5} q_f$$

the chiral anomaly affects the action

$$\delta W = \alpha \int d^4x \partial_\mu J_5^\mu = \alpha \frac{g^2 N}{32\pi^2} \int d^4x F_a^{\mu\nu} \tilde{F}_{a\mu\nu}$$

- However, matters are not that simple!

- This is because the pseudoscalar density entering in the anomaly is, in fact, a total divergence [Bardeen]:

$$F_a^{\mu\nu} \tilde{F}_{a\mu\nu} = \partial_\mu K^\mu$$

where

$$K^\mu = \varepsilon^{\mu\alpha\beta\gamma} A_{a\alpha} [F_{a\beta\gamma} - g/3\, f_{abc} A_{b\beta} A_{c\gamma}]$$

- This makes δW a pure surface integral

$$\delta W = \alpha\, g^2 N / 32\pi^2 \int d\sigma_\mu K^\mu$$

Hence, using the naïve boundary condition

$$A_a^{\ \mu} = 0 \ \text{ at } \infty \ \Rightarrow \int d\sigma_\mu K^\mu = 0$$

$U(1)_A$ appears to be a symmetry again!

- What 't Hooft showed, however, is that the correct boundary condition to use is that

$$A_a^{\mu} \text{ be a pure gauge at } \infty$$

i.e. either $A_a^{\mu}=0$ or gauge transformation of 0

- It turns out that, with these B. C., there are gauge configurations for which

$$\int d\sigma_{\mu} K^{\mu} \neq 0$$

and thus $U(1)_A$ is not a symmetry of QCD

- This is most easily understood for SU(2) QCD and in $A^o_a=0$ gauge [Callan Dashen Gross]. In this case one has only spatial gauge fields A^i_a

- Under a gauge transformation the A^i_a gauge fields transform as:

  $$\tfrac{1}{2}\tau_a A^i_a \equiv A^i \rightarrow \Omega A^i \Omega^{-1} + i/g\ \Omega\nabla^i \Omega^{-1}$$

  Thus vacuum configurations are either 0 or have the form $i/g\ \ \Omega\nabla^i \Omega^{-1}$
- In the $A^o_a=0$ gauge can further classify vacuum configurations by how Ω goes to unity as $r\rightarrow \infty$

  $$\Omega_n \ \rightarrow e^{\,i2\pi n} \text{ as } r \rightarrow \infty \quad [n=0, \pm1, \pm2,\ldots]$$

- The winding number n is related to the Jacobian of an $S_3\rightarrow S_3$ map and is given by

$$n = \frac{ig^3}{24\pi^2}\int d^3r \mathrm{Tr}\, \varepsilon_{ijk} A^i_n A^j_n A^k_n$$

- This expression is closely related to the Bardeen current K^μ. Indeed, in the $A^o{}_a=0$ gauge only $K^0\neq 0$ and one finds for pure gauge fields:

$$K^0=-g/3\varepsilon_{ijk}\varepsilon^{abc}A^i{}_a A^j{}_b A^k{}_c = 4/3ig\ \varepsilon_{ijk}\mathrm{Tr}\ A^i A^j A^k$$

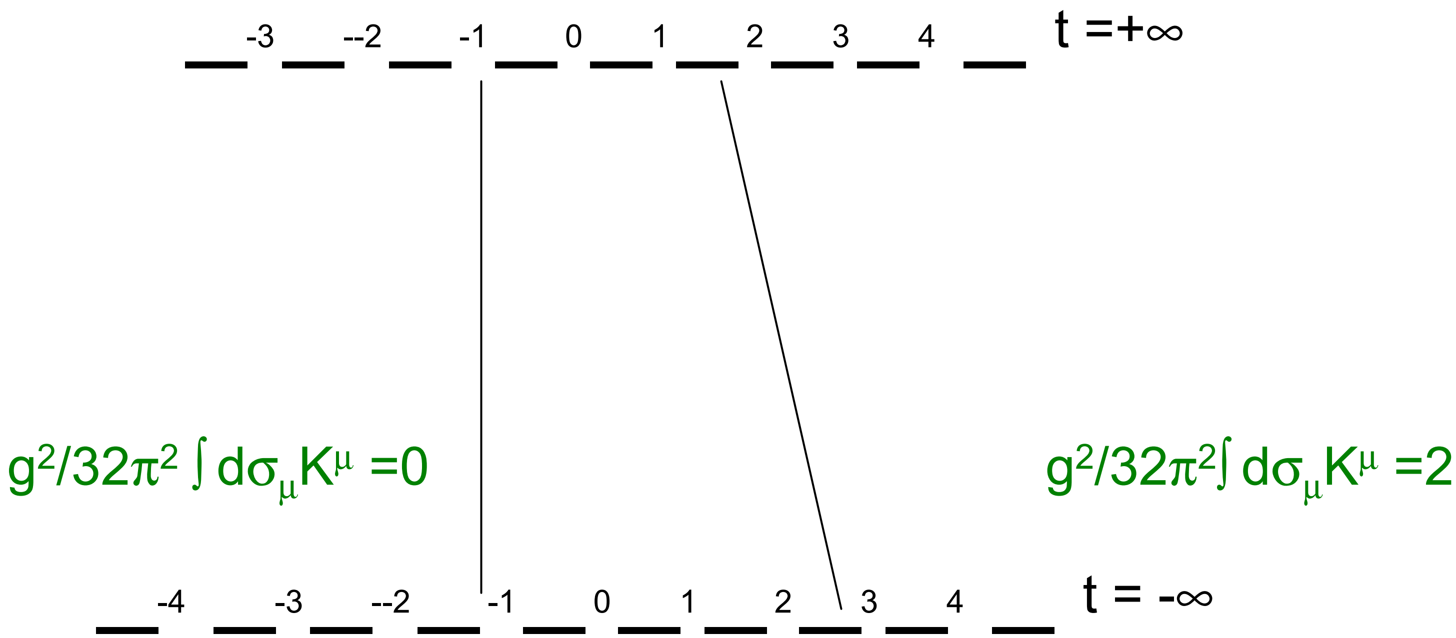
- True vacuum is superposition of these, so-called, n-vacua and is called the θ-vacuum:

$$|\theta> = \Sigma\, e^{-in\theta}\, |n>$$

- Easy to see that in vacuum to vacuum transitions there are transitions with $\int d\sigma_\mu K^\mu \neq 0$

$$n|_{t=+\infty} - n|_{t=-\infty} = g^2/32\pi^2 \int d\sigma_\mu K^\mu \,|^{t=+\infty}{}_{t=-\infty}$$

- Pictorially, one has



- In detail one can write for the vacuum to vacuum transition amplitude

$$ {}_+\langle\theta|\theta\rangle_- = \Sigma e^{im\theta}\, e^{-in\theta}\, {}_+\langle m|n\rangle_- = \Sigma_\nu\, e^{i\nu\theta}\, \Sigma_n\, {}_+\langle n+\nu|n\rangle_- $$

- Here the difference in winding numbers ν is given by

$$\nu = \frac{g^2}{32\pi^2}\int d\sigma_\mu K^\mu \Big|_{t=-\infty}^{t=+\infty} = \frac{g^2}{32\pi^2}\int d^4x F_a^{\mu\nu}\tilde{F}_{a\mu\nu}$$

- Using the usual path integral representation for $_+\langle\theta|\theta\rangle_-$ one sees that

$$_+\langle\theta|\theta\rangle_- = \sum_\nu \int \delta A e^{iS_{eff}[A]}\delta(\nu - \frac{g^2}{32\pi^2}\int d^4x F_a^{\mu\nu}\tilde{F}_{a\mu\nu})$$

which allows to re-interpret θ term as addition to usual QCD action

$$S_{eff} = S_{QCD} + \theta\frac{g^2}{32\pi^2}\int d^4x F_a^{\mu\nu}\tilde{F}_{a\mu\nu}$$

- Resolution of $U(1)_A$ problem, by recognizing complicated nature of QCD's vacuum, effectively adds and extra term to L_{QCD}

$$L_\theta = \theta \frac{g^2}{32\pi^2} F_a^{\mu\nu} \tilde{F}_{a\mu\nu}$$

- This term violates P and T, but conserves C. Strong bound on the neutron electric dipole moment $d_n < 1.1 \times 10^{-26}$ ecm requires the angle θ to be very small [$d_n \approx e\,\theta m_q/M_N^2 \Rightarrow \theta < 10^{-9} - 10^{-10}$]
- Why should this be so is the strong CP problem Problem actually worse if one considers the effect of chiral transformations on θ-vacuum

- Chiral transformations, because of the anomaly, change the θ-vacuum [Jackiw Rebbi]:

$$e^{i\alpha Q_5} | \theta > = | \theta + \alpha > \quad \text{(see Appendix)}$$

- If one includes weak interactions, the quark mass matrix is in general complex

$$L_{Mass} = - \bar{q}_{iR} M_{ij} q_{jL} + h.\,c.$$

  To diagonalize it one must, among other things, perform a chiral transformation which changes θ into

$$\theta_{total} = \theta + \text{Arg det } M$$

- Strong CP Problem: Why is this angle, coming from the strong and weak interactions, so small?

Approaches to the Strong CP Problem

- There are three possible “solutions” to Strong CP Problem:

i. Unconventional dynamics

ii. Spontaneously broken CP

iii. An additional chiral symmetry

However, in my opinion, only iii. is viable solution

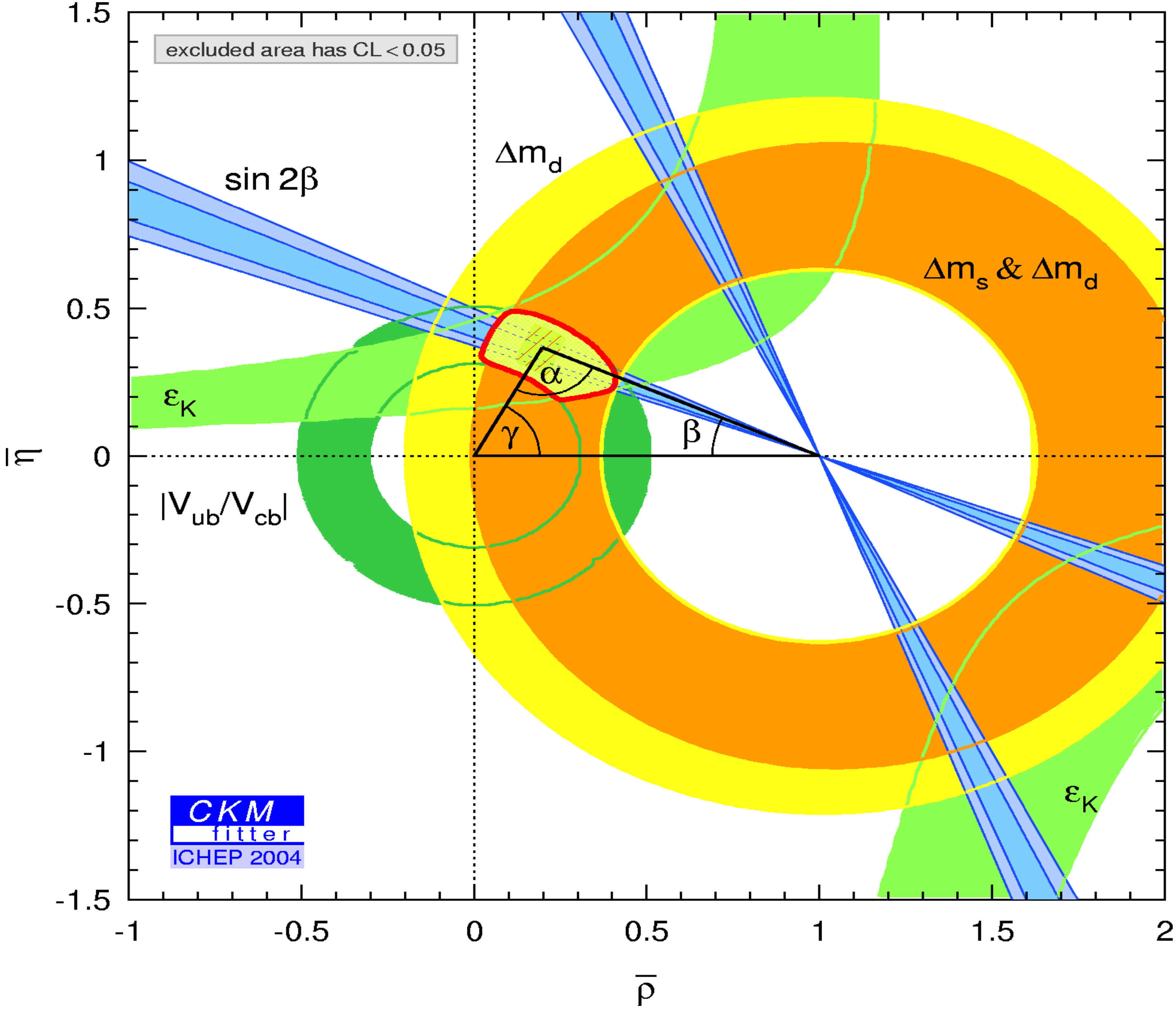
- It is, of course, also possible that, as a result of some anthropic reasons

$$\theta_{total} = \theta + \text{Arg det } M$$

just turns out to be of $O(10^{-10})$, but I doubt it!

- Approaches to i. unconventional dynamics are also not very believable:
- They either suggest that B.C. which gave rise to θ-vacuum is an artifact [but then, what is solution to $U(1)_A$ problem?], or use periodicity of vacuum energy $E(\theta) \sim \cos\theta$ to deduce that θ vanishes [but, why $\partial E/\partial\theta=0$?]
- The second possibility, ii Spontaneously broken CP, is more interesting
- If CP is a symmetry of nature, which is spontaneously broken, then can set $\theta=0$ at the Lagrangian level

- However, θ gets induced back at the loop-level, and to get $\theta < 10^{-9}$ one needs, in general, also to insure that $\theta_{1\text{-loop}}=0$
- Although models exist where this is so, theories with spontaneously broken CP need complex Higgs VEVs, leading to FCNC and domain walls, and introduce recondite physics [Barr Nelson] to avoid these problems
- In my view, however, the biggest drawback for this "solution" to the strong CP problem is that experimental data is in excellent agreement with the CKM Model– a model where CP is explicitly not spontaneously broken



excluded area has CL < 0.05
Δm_d
sin 2β
Δm_s & Δm_d
ε_K
α
γ
β
$|V_{ub}/V_{cb}|$
ε_K
CKM fitter
ICHEP 2004
$\bar{\eta}$
$\bar{\rho}$
1.5
1
0.5
0
-0.5
-1
-1.5
-1
-0.5
0
0.5
1
1.5
2

- Introducing iii an additional chiral symmetry is a very natural solution to strong CP problem since it, effectively, rotates θ -vacua away

$$e^{-i\,\theta Q_5} |\,\theta > = |\,0 >$$

- Two suggestions for this chiral symmetry:

i. The u-quark has no mass, $m_u = 0$

ii. SM has an additional global U(1) chiral symmetry [Peccei Quinn]

- $m_u = 0$ is disfavored by current algebra analysis [Leutwyler]. Further, it is difficult to understand why

$$\text{Arg det } M = 0$$

What is the origin of this chiral symmetry?

$U(1)_{PQ}$ and Axions

- Introducing a global $U(1)_{PQ}$ symmetry, which is necessarily spontaneously broken, replaces:

  $\theta_{total} = \theta + \text{Arg det } M \quad \Rightarrow \quad a(x) / f_a$

  Static CP viol. Angle　　　Dynamical CP cons.

  　　　　　　　　　　　　　Axion field

- The axion is the Goldstone boson of the broken [Weinberg Wilczek] $U(1)_{PQ}$ symmetry and f_a is scale of the breaking. Hence under $U(1)_{PQ}$

$$a(x) \rightarrow a(x) + \alpha f_a$$

- Formally, for $U(1)_{PQ}$ invariance the Lagrangian of SM is augmented by axion interactions:

$$L = L_{SM} + \theta_{total} \frac{g^2}{32\pi^2} F_a^{\mu\nu} \tilde{F}_{a\mu\nu} - \tfrac{1}{2} \partial^\mu a \partial_\mu a$$
$$+ L_{int}[\partial^\mu a / \mathrm{f}_a ; \psi] + \frac{a}{\mathrm{f}_a} \xi \frac{g^2}{32\pi^2} F_a^{\mu\nu} \tilde{F}_{a\mu\nu}$$

- Last term needed to give chiral anomaly of J^μ_{PQ}

$$\partial_\mu J^\mu_{PQ} = \xi \frac{g^2}{32\pi^2} F_a^{\mu\nu} \tilde{F}_{a\mu\nu}$$

and acts as an effective potential for axion field

- Minimum of potential occurs at $\langle a \rangle = -f_a/\xi \, \theta_{total}$

$$\left\langle \frac{\partial V_{eff}}{\partial a} \right\rangle = -\frac{\xi}{\mathrm{f}_a} \frac{g^2}{32\pi^2} \left\langle F_a^{\mu\nu} \tilde{F}_{a\mu\nu} \right\rangle_{\langle a \rangle} = 0$$

- Easy to understand the physics of PQ solution. If one neglects the effects of QCD then $U(1)_{PQ}$ symmetry allows any value for $\langle a \rangle$:

$$0 \leq \langle a \rangle \leq 2\pi$$

- Including the effects of the QCD anomaly generates a potential for the axion field which is periodic in the effective vacuum angle

$$V_{eff} \sim \cos[\theta_{total} + \xi \langle a \rangle / f_a]$$

- Minimizing this potential with respect to $\langle a \rangle$ gives the PQ solution

$$\langle a \rangle = -f_a/\xi \, \theta_{total}$$

- Hence theory written in terms of

$a_{phys} = a - \langle a \rangle$

has no longer a θ-term [this is the PQ solution]

- Furthermore, expanding V_{eff} at minimum gives the axion a mass [anomaly gives NG a mass]

$$m_a^2 = \left\langle \frac{\partial^2 V_{\text{eff}}}{\partial a^2} \right\rangle = -\frac{\xi}{\mathrm{f_a}} \frac{g^2}{32\pi^2} \frac{\partial}{\partial a} \left\langle F_a^{\mu\nu} \tilde{F}_{a\mu\nu} \right\rangle_{\langle a \rangle}$$

- Calculation of axion mass first done explicitly by current algebra techniques [Bardeen Tye]
- Here will give an effective Lagrangian derivation [Bardeen Peccei Yanagida], as it also gives readily axion couplings

Axion Dynamics

- In the original Peccei Quinn model, the $U(1)_{PQ}$ symmetry breakdown coincided with that of electroweak breaking $f_a = v_F$, with $v_F \approx 250$ GeV.
- However, this is not necessary. If $f_a >> v_F$ then axion is very light, very weakly coupled and very long lived [invisible axion models]
- Useful to derive first properties of weak-scale axions and then generalize the discussion to invisible axions
- To make SM $U(1)_{PQ}$ invariant must introduce 2 Higgs fields to absorb independent chiral transformations of u- and d-quarks (and leptons)

- Yukawa interactions in SM involve Higgs

$$L_{Yukawa} = \Gamma_{ij}^{u}\overline{Q}_{Li}\Phi_{1}u_{Rj} + \Gamma_{ij}^{d}\overline{Q}_{Li}\Phi_{2}d_{Rj} + \Gamma_{ij}^{\ell}\overline{L}_{Li}\Phi_{2}\ell_{Rj} + h.c.$$

- Defining $x=v_2/v_1$ and $v_F= \sqrt{(v_1^2 + v_2^2)}$, the axion is the common phase field in Φ_1 and Φ_2 which is orthogonal to the weak hypercharge

$$\Phi_1 = \frac{1}{\sqrt{2}} v_1 e^{iax/v_F}\begin{bmatrix}1\\0\end{bmatrix} \quad ; \quad \Phi_2 = \frac{1}{\sqrt{2}} v_2 e^{ia/xv_F}\begin{bmatrix}0\\1\end{bmatrix}$$

- See that L_{Yukawa} is invariant under the $U(1)_{PQ}$ transformation

$a \rightarrow a + \alpha\, v_F$; $u_{Ri} \rightarrow e^{-i\alpha x}\, u_{Ri}$; $d_{Ri}\, l_{Ri} \rightarrow e^{-i\alpha/x}\, d_{Ri}\, l_{Ri}$

- Let us focus on the quark pieces. The current $J^{\mu}_{PQ}=-v_F \partial^{\mu}a + x \Sigma_i \bar{u}_{iR} \gamma^{\mu} u_{iR} + 1/x \Sigma_i \bar{d}_{iR} \gamma^{\mu} d_{iR}$ identifies the strong anomaly coefficient ξ as:

$$\xi = N/2(x + 1/x) = N_g(x + 1/x)$$

- To compute the axion mass and mixings from an effective chiral Lagrangian we need to separate out light u- and d-quarks from rest.
- For these purposes one introduces a 2x2 matrix of NG fields

$$\Sigma = \exp[\, i(\tau.\pi + \eta)/f_{\pi}\,]$$

and the $U(2)_V x U(2)_A$ invariant eff. Lagrangian

$$L_{chiral} = -f_{\pi}^2/4 \; Tr\, \partial_{\mu}\Sigma^{\dagger} \partial^{\mu}\Sigma$$

- To L_{chiral} must add $U(2)_V x U(2)_A$ breaking terms which mimic the $U(1)_{PQ}$ invariant Yukawa interactions of the u- and d-quarks.
- This is accomplished by adding

$$L_{mass} = \tfrac{1}{2}(f_\pi m^o_\pi)^2 \, Tr[\Sigma A M + (\Sigma A M)^\dagger]$$

where

$$A = \begin{bmatrix} e^{-i x a / v_F} & 0 \\ 0 & e^{-i a / x v_F} \end{bmatrix} \quad ; \quad M = \begin{bmatrix} \frac{m_u}{m_u + m_d} & 0 \\ 0 & \frac{m_d}{m_u + m_d} \end{bmatrix}$$

and under PQ-transformations

$$a \rightarrow a + \alpha v_F \quad ; \quad \Sigma \rightarrow \Sigma \begin{bmatrix} e^{i \alpha x} & 0 \\ 0 & e^{i \alpha / x} \end{bmatrix}$$

- However, L_{mass} only gives part of the physics. Indeed, the quadratic terms in L_{mass} involving neutral fields

$$L^2_{mass} = -\tfrac{1}{2} m^{o}_{\pi}{}^{2}\{m_u/(m_u+m_d)[\pi+\eta-xf_\pi/v_F a]^2 + m_d/(m_u+m_d)[\eta-\pi-f_\pi/xv_F a]^2\}$$

give the wrong ratio for m_η^2/ m_π^2

$$m_\eta^2/ m_\pi^2 = m_d/m_u \approx 1.6 \text{ [the } U(1)_A \text{ problem!]}$$

and the axion is still massless

- To account for the effect of the anomaly in both $U(1)_A$ and $U(1)_{PQ}$ one must add a further effective mass term which gives the η the right mass and produces a mass for the axion

- It is easy to see that such a term has the form [Bardeen Peccei Yanagida]

$L_{anomaly} = -\frac{1}{2} m_o^2 \, [\eta + \{[f_\pi/v_F]\, [(N_g-1)(x+1/x)/2]\} a\,]^2$

where $m_o^2 \approx m_\eta^2 >> m_\pi^2$

- Coefficient in front of a in $L_{anomaly}$ details the relative strength of the couplings of η and a to $F_a^{\mu\nu}\tilde{F}_{a\mu\nu}$ .Naively, one would imagine

$$\{\} = f_\pi/v_F \, \xi/2 = f_\pi/v_F \, N_g/2(x+1/x)$$

However, only the contribution of heavy quarks to the PQ anomaly should be included (hence $N_g \rightarrow (N_g-1)$) since light quark interactions of axions are included already in L_{mass}

- Diagonalization of the quadratic terms in L_{mass} and $L_{anomaly}$ gives both the axion mass and the parameters for a-π and a –η mixing for the PQ model.
- Convenient to define

$$m_a^{st} = m_\pi f_\pi / v_F [\sqrt{m_u m_d}/(m_u + m_d)] \approx 25 \text{ KeV}$$

- Then can characterize all axion models by 4 parameters $\{ \lambda_m; \lambda_3 ; \lambda_0 ; K_{a\gamma\gamma} \}$ of O(1). To wit:

$$m_a = \lambda_m \, m_a^{st} [v_F / f_a]$$

$$\xi_{a\pi} = \lambda_3 [f_\pi / f_a] \, ; \, \xi_{a\eta} = \lambda_0 [f_\pi / f_a]$$

$$L_{a\gamma\gamma} = \frac{\alpha}{4\pi} K_{a\gamma\gamma} \frac{a_{\text{phys}}}{\text{f}_\text{a}} F^{\mu\nu} \tilde{F}_{\mu\nu}$$

- A simple calculation for weak-scale axions, where $f_a = v_F$, gives:

$$\lambda_m = N_g(x + 1/x)$$

$$\lambda_3 = ½[(x - 1/x) - N_g(x + 1/x)(m_d - m_u)/(m_u + m_d)]$$

$$\lambda_0 = ½(1 - N_g)(x + 1/x)$$

- To compute the coupling $K_{a\gamma\gamma}$ one must consider the em anomaly of the PQ current

$$\partial_\mu J^\mu_{PQ} = \xi_\gamma \frac{\alpha}{4\pi} F^{\mu\nu} \tilde{F}_{\mu\nu}$$

Leptons also contribute to ξ_γ and one finds

$$\xi_\gamma = N_g\{[3(2/3)^2]x + [3(-1/3)^2 + (-1)^2]1/x\}$$

$$= 4/3 N_g(x + 1/x)$$

- As before, we must separate out the light quark contributions in the anomaly, since they are counted by the coupling of the π° and η to 2γ.
- Adding the lepton and heavy quark contributions of the axion coupling to the em anomaly

$$\xi_\gamma^{eff}=4/3N_g(x+1/x)-4/3x-1/3x$$

to that coming from the π and η mixing

$$\lambda_3+5/3\,\lambda_0$$

gives, finally,

$$K_{a\gamma\gamma}=N_g(x+1/x)[m_u/(m_u+m_d)]$$

Invisible Axion Models

- Original PQ model, where $f_a = v_F$, was long ago ruled out by experiment.
- For example, one can estimate the branching ratio [Bardeen Peccei Yanagida]

$$BR(K^+ \to \pi^+ + a) \approx 3 \times 10^{-5} \lambda_0^{\,2}$$
$$\approx 3 \times 10^{-5} (x+1/x)^2$$

which is well above the KEK bound

$$BR(K^+ \to \pi^+ + \text{nothing}) < 3.8 \times 10^{-8}$$

- However, invisible axion models, where $f_a >> v_F$, are still viable

- These invisible axion models introduce fields which carry PQ charge but are SU(2)XU(1) singlets
- Two types of models have been proposed

i) KSVZ [Kim; Shifman Vainshtein Zakharov]

Only a scalar field σ with $f_a = \langle\sigma\rangle \gg v_F$ and a superheavy quark Q with $M_Q \sim f_a$ carry PQ charge

ii) DFSZ [Dine Fischler Srednicki; Zhitnisky]

Adds to PQ model a scalar field σ which carries PQ charge and $f_a = \langle\sigma\rangle \gg v_F$

- For these models, one can repeat the calculations we just did to get the axion mass and couplings
- Will do this for the KSVZ model because it is simple and illustrates well what we just did
- The KSVZ axion does not interact with leptons and only interacts with light quarks as the result of the strong and em anomalies
- The superheavy quark Q induces the following couplings [e_Q is the em charge of Q]

$$L_{axion} = \frac{a}{f_a}\left[\frac{g^2}{32\pi^2} F_a^{\mu\nu}\widetilde{F}_{a\mu\nu} + 3e_Q^2 \frac{\alpha}{4\pi} F^{\mu\nu}F_{\mu\nu}\right]$$

- Since in the KSVZ model the ordinary Higgs do not carry PQ charge, the only interactions of the axion come from the effective anomaly mass term which here is given by

$$L_{anomaly} = -½\, m_o^2\, [\eta + \{[f_\pi/f_a]\ [1/2]\}a\ ]^2$$

- To the above one must add the standard quadratic term coming from the light quarks

$$L^2 = -½\, m^o{}_\pi{}^2 \{m_u/(m_u+m_d)[\pi+\eta]^2 + m_d/(m_u+m_d)[\eta-\pi]^2\}$$

- Diagonalizing $L_{anomaly}$ plus L^2 gives the axion parameters:

$$\lambda_m = 1\ ;\ \lambda_3 = -½(m_d-m_u)/(m_u+m_d)\ ;\ \lambda_0 = -½$$

- Note that since in the KSVZ model $\lambda_m=1$ the axion mass is given by the formula:

$$m_a = m_a^{st}\,[v_F / f_a] \approx 6.3\,[10^6 \text{ GeV} / f_a] \text{ eV}$$

- The calculation of $K_{a\gamma\gamma}$ in this model is equally straightforward. To the contribution of the superheavy quark in the em anomaly [$3e_Q^2$], one must add that coming from the mixing of the axion with the $\pi°$ and η [$\lambda_3 + 5/3\,\lambda_0$]
- This gives, finally,

$$K_{a\gamma\gamma} = 3e_Q^2 - (4m_d + m_u)/3(m_u + m_d)$$

- I will not go through the analogous calculation for the DFSZ model, but just quote the results
- It proves convenient to define

$$X_1=2v_2^2/v_F^2\ ,\ X_2=2v_1^2/v_F^2\ ,$$

where $v_F=\sqrt{(v_1^2+v_2^2)}$ and v_1 and v_2 are Higgs VEVs, and to rescale $f_a \to f_a/2N_g$ to make $\lambda_m \equiv 1$, so that also in the DFSZ model

$$m_a = m_a^{st}\,[v_F / f_a] \approx 6.3\ [10^6\ \mathrm{GeV} / f_a]\ \mathrm{eV}$$

- One then finds

$$\lambda_3 = \frac{1}{2}\left[\frac{X_1 - X_2}{2N_g} - \frac{m_d - m_u}{m_d + m_u}\right] \quad ; \quad \lambda_0 = (1-N_g)/\ 2N_g$$

$$K_{a\gamma\gamma} = \frac{4}{3} - \frac{4m_d + m_u}{3(m_d + m_u)}$$

- Although KSVZ and DFSZ axions are very light, very weakly coupled and very long-lived, they are not totally invisible
- Astrophysics gives bounds on m_a since axion emission, through $\gamma e \to ae$ and Primakoff processes causes energy loss ~1/ f_a affecting stellar evolution.
- Other upper bounds on m_a come from SN1987a, since axion emission through $NN \to NNa$ in core collapse affects neutrino spectrum.
- Typically bounds allow axions lighter than

$$m_a \leq 1\text{-}10^{-3}\ \mathrm{eV}$$

- Remarkably, cosmology gives a lower bound on axion mass (upper bound on f_a) [Preskill Wise Wilczek; Abbott Sikivie; Dine Fischler]
- Physics is simple to understand. When Universe goes through PQ phase transition at $T \sim f_a >> \Lambda_{QCD}$ anomaly ineffective and $\langle a_{phys} \rangle$ is arbitrary. Eventually, when Universe cools to $T \sim \Lambda_{QCD}$ the axion gets a mass and $\langle a_{phys} \rangle \rightarrow 0$.
- Coherent $\mathbf{p_a}=0$ axion oscillations towards minimum contribute to Universe's energy density and act as cold dark matter. WMAP data provides bound on CDM and $\therefore$ axions:

$$\Omega_a h^2 \leq 0.12$$

- Quote result of a recent calculation of axion contribution to Universe's energy density by Fox Pierce Thomas:

$$\Omega_a h^2 = 0.5[(f_a/\xi)/10^{12}\ \text{GeV}]^{7/6}[\theta_i^2 + \sigma_\theta^2]\gamma$$

  Here ξ is coefficient of PQ anomaly, θ_I is initial misallignment angle and σ_θ its mean square fluctuation and γ is a possible dilution factor

- For $\gamma=1$, and using for θ_i an average angle $\theta_i^2 = \langle\theta^2\rangle = \pi^2/3$ and neglecting fluctuations, WMAP data gives the following cosmological bound for the PQ scale:

$$f_a/\xi < 3 \times 10^{11}\ \text{GeV} \quad \text{or} \quad 2.1 \times 10^{-5}\ \text{eV} < m_a$$

Concluding Remarks

- After more than 25 years, preferred solution to strong CP problem remains having a $U(1)_{PQ}$ symmetry in the theory and its concomitant axions
- Although Fermi scale axions have been ruled out, invisible axions models are still viable and axion oscillations could account for the dark matter in the Universe
- No totally compelling invisible axion models exist, but it is encouraging that experimentalists are actively searching for axions

Appendix: θ-vacua and chirality

- The gauge matrices Ω_n can be obtained by compounding: $\Omega_n = [\Omega_1]^n$. It follows thus that on an n-vacuum state

$$\Omega_1 |n> = |n+1>$$

- Hence n-vacua are not gauge invariant, but the θ-vacuum is;

$$\Omega_1 |\theta> = \Sigma\, e^{-in\theta}\, \Omega_1\, |n> = \Sigma\, e^{-in\theta}\, |n+1> = e^{i\theta}\, |\,\theta>$$

- In a theory with N massless quarks there is a conserved but gauge variant chiral current

$$J_{c5}^{\mu} = J_5^{\mu} - g^2 N/32\pi^2\, K^{\mu}$$

- The associated time independent chiral charge $Q_{c5} = \int d^3x \, J_{c5}{}^{o}$, as a result, shifts under gauge transformations which change the n-vacua

$$\Omega_1 \, Q_{c5} \, \Omega_1{}^{-1} = Q_{c5} + N$$

- Consider

$$\Omega_1 e^{\, i\alpha/N \, Q_{c5}} |\theta> \; = \Omega_1 e^{\, i\alpha/N \, Q_{c5}} \, \Omega_1{}^{-1} \, \Omega_1 \, |\theta>$$

$$= e^{\, i(\alpha + \theta)} \, e^{\, i\alpha/N \, Q_{c5}} |\theta>$$

which shows that a chiral rotation changes the θ-vacuum [Jackiw Rebbi]

$$e^{\, i\alpha/N \, Q_{c5}} |\theta> \; = |\theta + \alpha >$$