

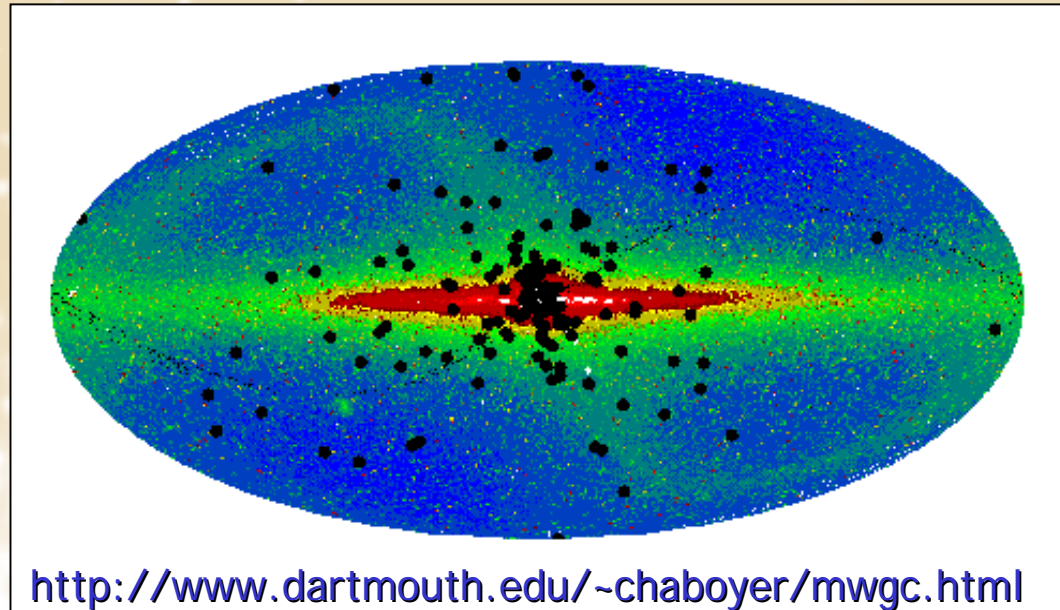
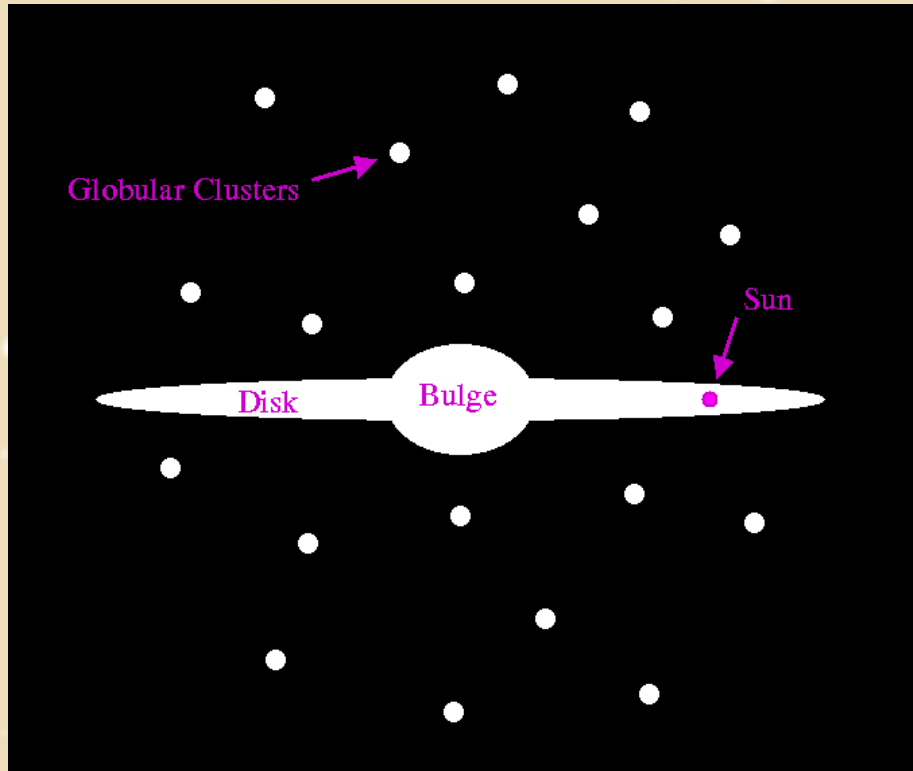
ILIAS Axion Training, 30 Nov-2 Dec 2005, CERN, Geneva

# Astrophysical Axion Bounds

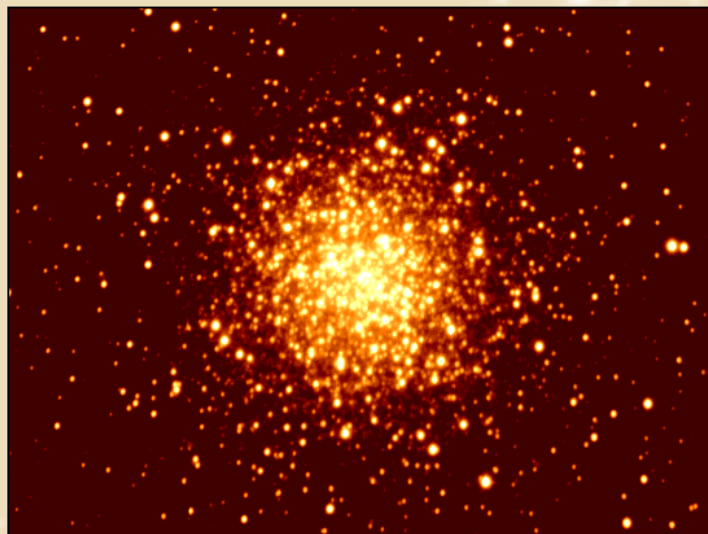
Georg G. Raffelt

Max-Planck-Institut für Physik, München, Germany

# Globular Clusters of the Milky Way

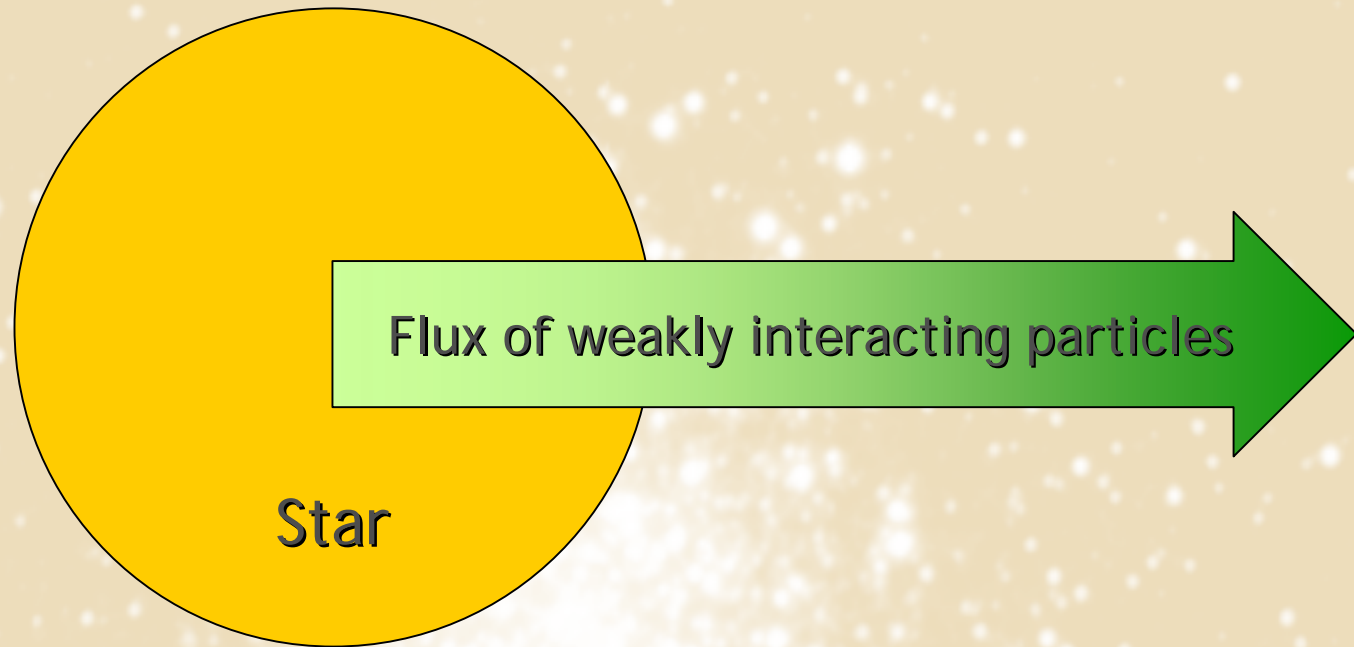


Globular clusters on top of the  
FIRAS 2.2 micron map of the Galaxy



The galactic globular cluster M3

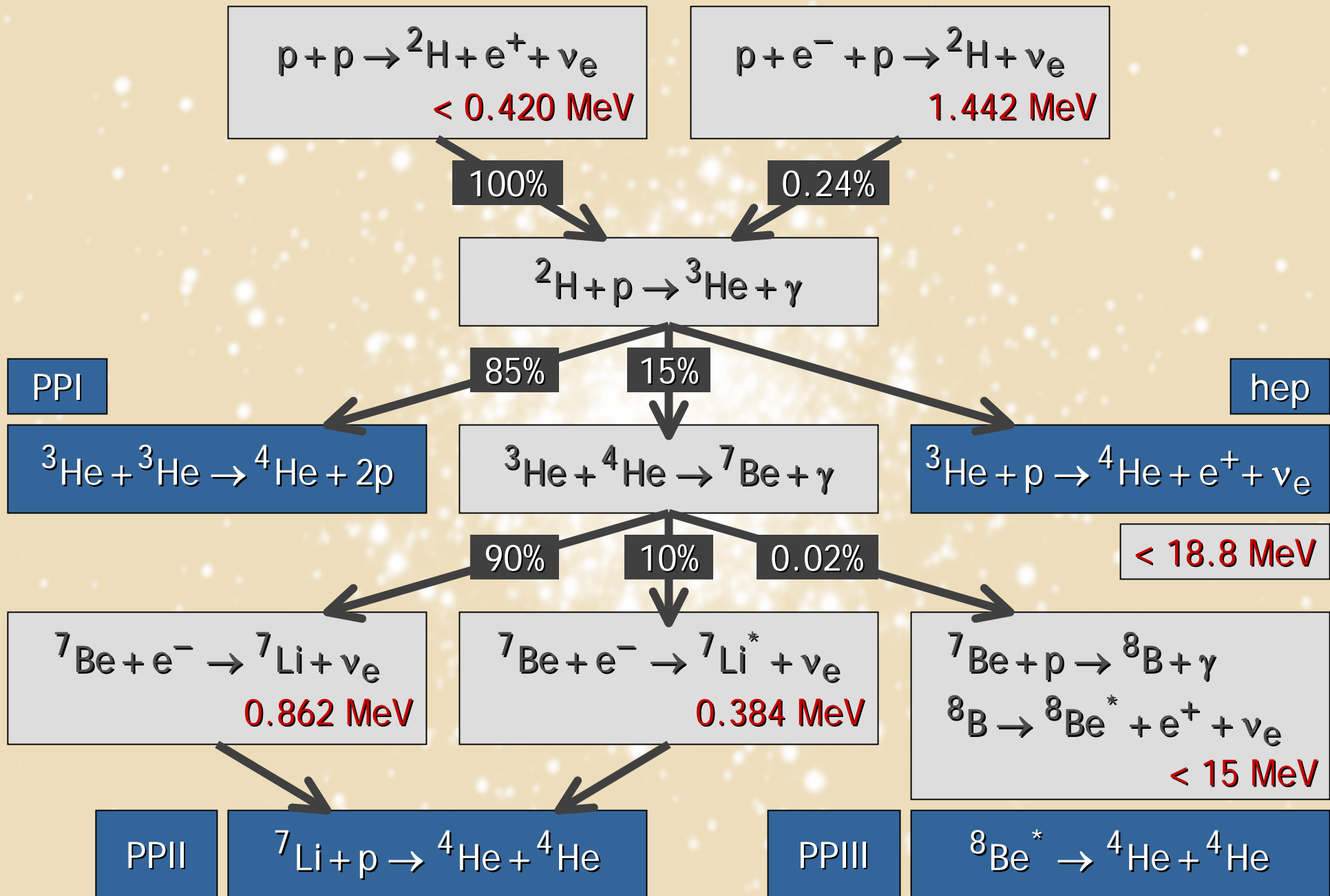
# Basic Argument



- Invisible axions have very small mass
- Emission from stellar plasma not suppressed by threshold effects (analogous to neutrinos)
- New energy-loss channel
- Back-reaction on stellar properties and evolution

- What are the emission processes?
- What are the observable consequences?

# Hydrogen burning: Proton-Proton Chains



# Neutrinos from Thermal Plasma Processes

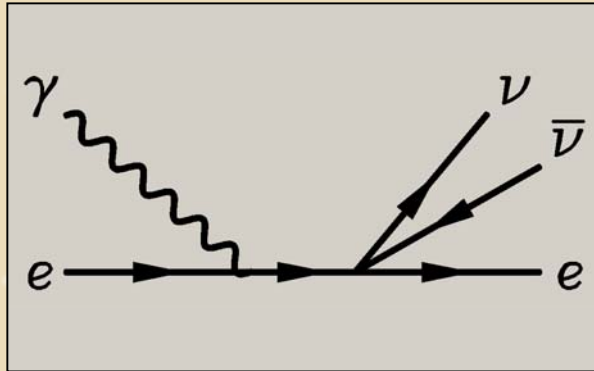
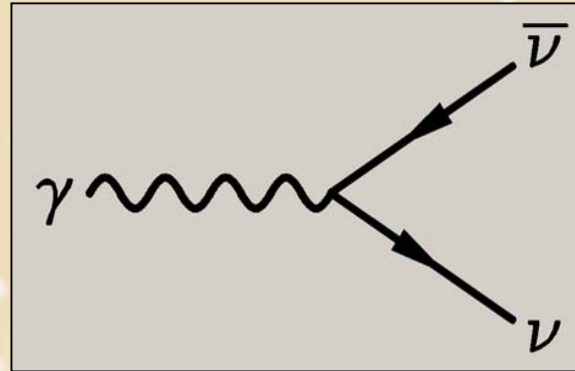
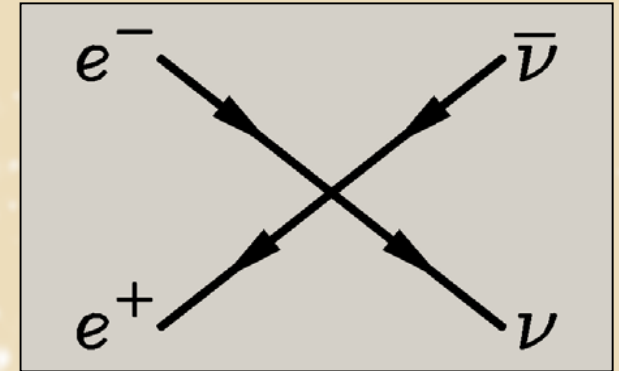


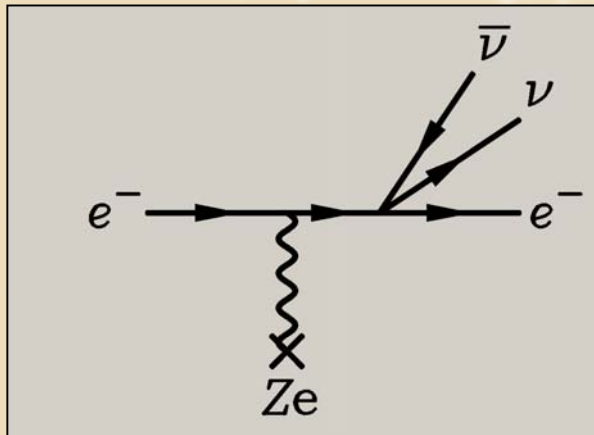
Photo (Compton)



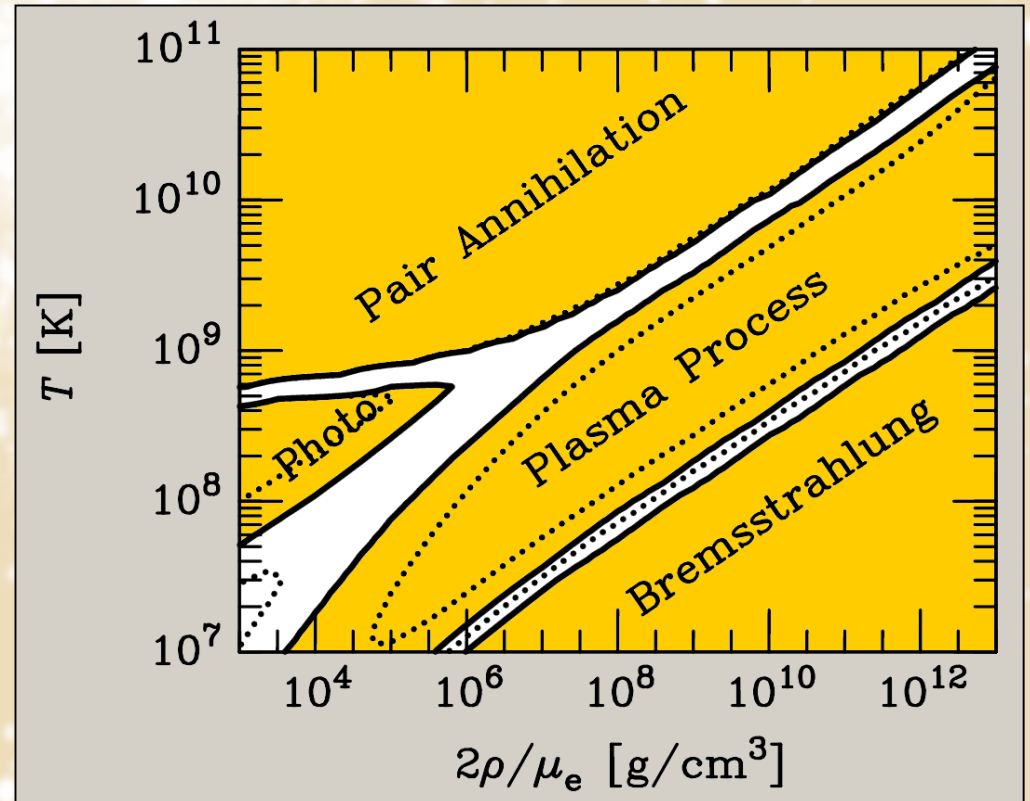
Plasmon decay



Pair annihilation



Bremsstrahlung

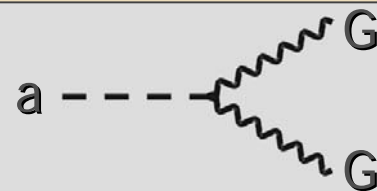


These processes first discussed in 1961-63 after V-A theory

# Axion Properties

Gluon coupling  
(Generic property)

$$L_{aG} = \frac{\alpha_s}{8\pi f_a} G\tilde{G}a$$



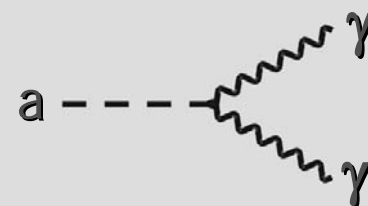
Mass

$$m_a = \frac{0.6 \text{ eV}}{f_a / 10^7 \text{ GeV}} \approx \frac{m_\pi f_\pi}{f_a}$$

Photon coupling

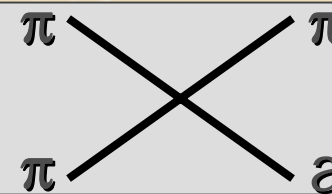
$$L_{a\gamma} = -\frac{g_{a\gamma}}{4} F\tilde{F}a = g_{a\gamma} \vec{E} \cdot \vec{B}a$$

$$g_{a\gamma} = \frac{\alpha}{2\pi f_a} \left( \frac{E}{N} - 1.92 \right)$$



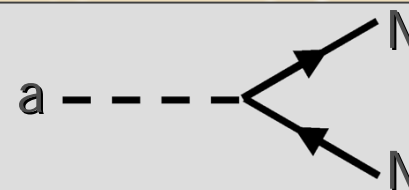
Pion coupling

$$L_{a\pi} = \frac{C_{a\pi}}{f_a f_\pi} (\pi^0 \pi^+ \partial_\mu \pi^- + \dots) \partial^\mu a$$



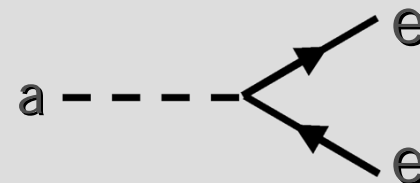
Nucleon coupling  
(axial vector)

$$L_{aN} = \frac{C_N}{2f_a} \bar{\Psi}_N \gamma^\mu \gamma_5 \Psi_N \partial_\mu a$$



Electron coupling  
(optional)

$$L_{ae} = \frac{C_e}{2f_a} \bar{\Psi}_e \gamma^\mu \gamma_5 \Psi_e \partial_\mu a$$





# Axion or Graviton Emission Processes in Stars

Nucleons	$\frac{C_N}{2f_a} \bar{\Psi}_N \gamma_\mu \gamma_5 \Psi_N \partial^\mu a$	Nucleon Bremsstrahlung	
Photons	$C_\gamma \frac{\alpha}{2\pi f_a} \vec{E} \cdot \vec{B} a$	Primakoff	
Electrons	$\frac{C_e}{2f_a} \bar{\Psi}_e \gamma_\mu \gamma_5 \Psi_e \partial^\mu a$	Compton	
		Pair Annihilation	
		Electromagnetic Bremsstrahlung	

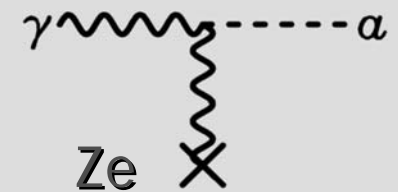
# Primakoff Process in the Sun

Interaction  
Lagrangian

$$\mathcal{L}_{a\gamma} = -\frac{1}{4} g_{a\gamma} F_{\mu\nu} \tilde{F}^{\mu\nu} a = g_{a\gamma} \vec{E} \cdot \vec{B} a$$

Primakoff  
cross section

$$\frac{d\sigma_{\gamma \rightarrow a}}{d\Omega} = \frac{g_{a\gamma}^2 Z^2 \alpha}{8\pi} \frac{|\vec{k}_a \times \vec{k}_\gamma|^2}{|\vec{k}_a - \vec{k}_\gamma|^4}$$



Conversion rate  
(screening effects,  
no nuclear recoil)

$$\Gamma_{\gamma \rightarrow a} = \frac{g_{a\gamma}^2 T k_S^2}{32\pi} \left[ \left( 1 + \frac{k_S^2}{4E^2} \right) \ln \left( 1 + \frac{4E^2}{k_S^2} \right) - 1 \right]$$

Screening scale  
(non-relativistic  
non-degenerate)

$$\kappa_S^2 = \frac{k_S^2}{4T^2} = \frac{\pi\alpha}{T^3} n_B \left( Y_e + \sum_j Z_j^2 Y_j \right)$$

Sun  $\kappa_S^2 \approx 12$   
HB Star  $\kappa_S^2 \approx 2.5$

- G. Raffelt, "Astrophysical axion bounds diminished by screening effects", Phys. Rev. D 33 (1986) 897 (Part of GR's Ph.D. Thesis)
- Consistent with results from FTD methods, see Altherr, Petitgirard & del Rio Gaztelurrutia, Astropart. Phys. 2 (1994) 175



# Energy-Loss Rate of the Sun

Conversion rate

$$\Gamma_{\gamma \rightarrow a} = \frac{g_{a\gamma}^2 T \kappa_S^2}{32\pi} \left[ \left( 1 + \frac{\kappa_S^2}{4E^2} \right) \ln \left( 1 + \frac{4E^2}{\kappa_S^2} \right) - 1 \right]$$

$\approx g_{10}^2 10^{-15} \text{s}^{-1}$  for few keV-energy photons (Sun)

$$g_{10} = \frac{g_{a\gamma}}{10^{-10} \text{GeV}^{-1}}$$

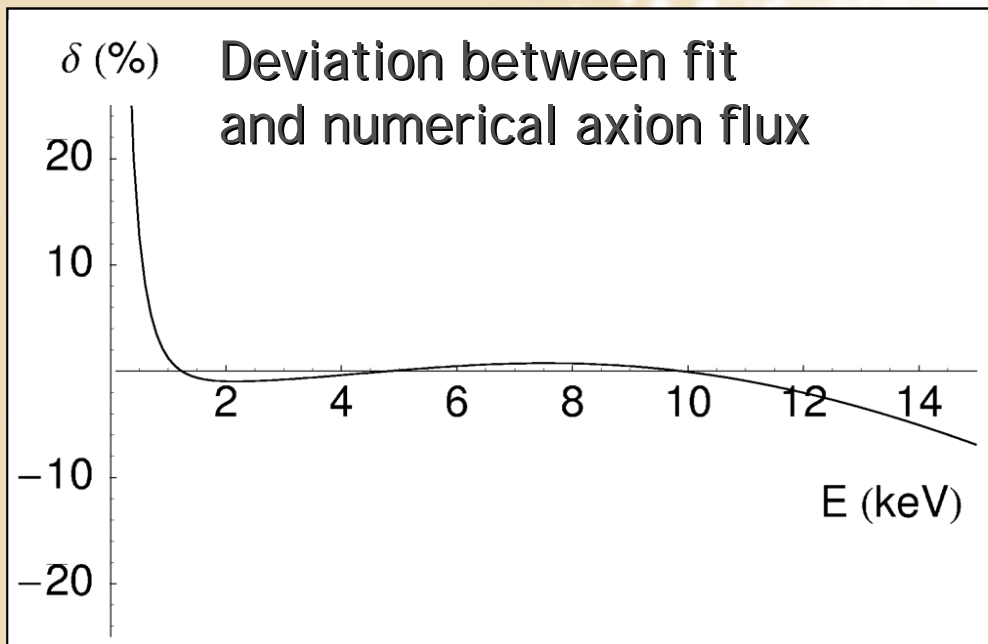
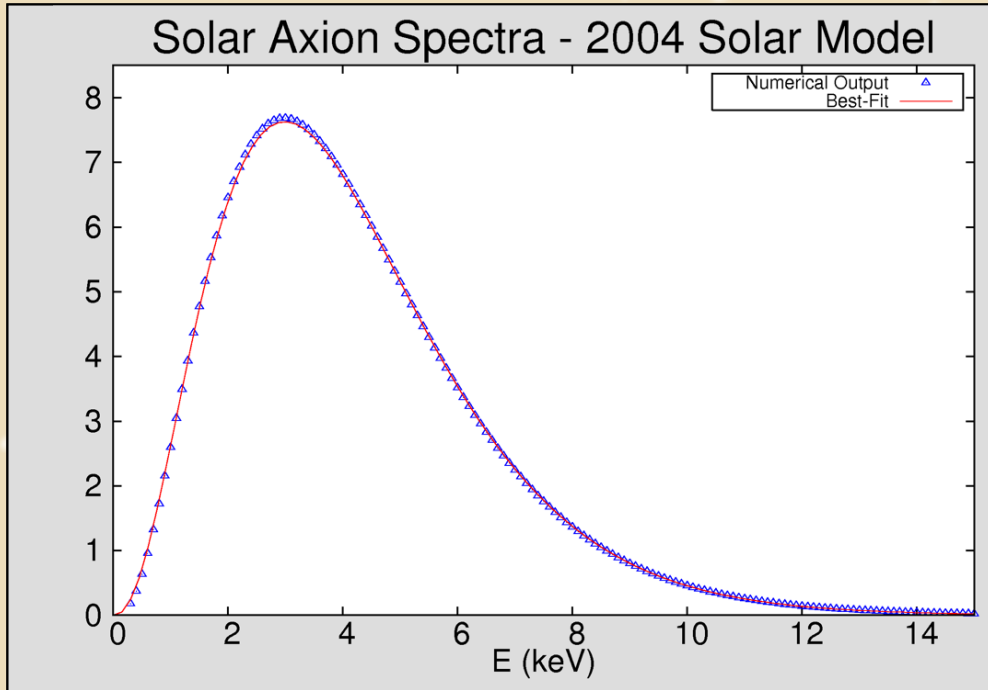
Energy-Loss Rate

$$Q = \int \frac{2d^3\vec{k}_\gamma}{(2\pi)^3} \frac{\Gamma_{a \rightarrow \gamma} E}{e^{E/T} - 1} = \frac{g_{a\gamma}^2 T^7}{4\pi} F(\kappa_S^2)$$
$$F(\kappa_S^2) = \frac{\kappa_S^2}{2\pi^2} \int_0^\infty dx \left[ (x^2 + \kappa_S^2) \ln \left( 1 + \frac{x^2}{\kappa_S^2} \right) - x^2 \right] \frac{x}{e^x - 1}$$

Solar Axion  
Luminosity

$$L_a = g_{10}^2 1.85 \times 10^{-3} L_{\text{sun}}$$

# Solar Axion Spectrum



“Power-law” fit

$$\frac{d\Phi_a}{dE} = A \left( \frac{E}{\bar{E}} \right)^\alpha e^{-(\alpha+1) E/\bar{E}}$$

Average energy

$$\langle E \rangle = \bar{E}$$

For  $\alpha = 2$  the fit is identical with a Maxwell-Boltzmann distribution

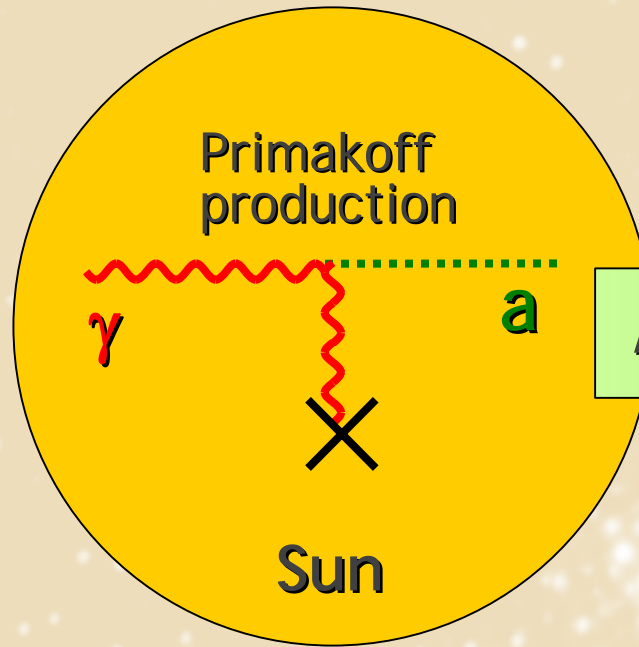
Determine  $A$ ,  $\alpha$ , and  $\bar{E}$  such that the total axion flux,  $\langle E \rangle$  and  $\langle E^2 \rangle$  are exactly reproduced

With 2004 solar model

$$\bar{E} = 4.196 \text{ keV}$$

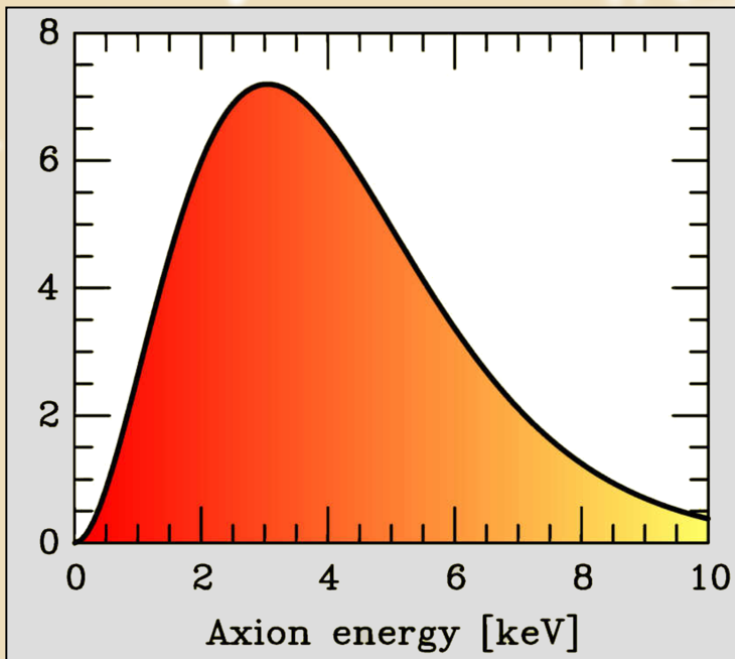
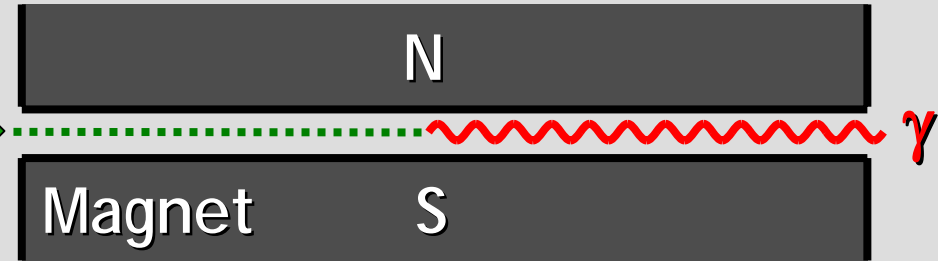
$$\alpha = 2.481$$

# Search for Solar Axions



## Axion Helioscope (Sikivie 1983)

### Axion-Photon-Oscillation



- Tokyo Axion Helioscope (Results since 1998)
- CERN Axion Solar Telescope (CAST) (Results since 2003)

### Alternative technique:

#### Bragg conversion in crystal

Experimental limits on solar axion flux from dark-matter experiments (SOLAX, COSME, DAMA, ...)

# Basics of Stellar Evolution

# Equations of Stellar Structure

Assume spherical symmetry and static structure (neglect kinetic energy)  
 Excludes: Rotation, convection, magnetic fields, supernova-dynamics, ...

Hydrostatic equilibrium

$$\frac{dP}{dr} = -\frac{G_N M_r \rho}{r^2}$$

Energy conservation

$$\frac{dL_r}{dr} = 4\pi r^2 \varepsilon \rho$$

Energy transfer

$$L_r = \frac{4\pi r^2}{3\kappa\rho} \frac{d(aT^4)}{dr}$$

$r$  Radius from center  
 $P$  Pressure  
 $G_N$  Newton's constant  
 $\rho$  Mass density  
 $M_r$  Integrated mass up to  $r$   
 $L_r$  Luminosity (energy flux)  
 $\varepsilon$  Local rate of energy generation [erg/g/s]  
 $\varepsilon = \varepsilon_{\text{nuc}} + \varepsilon_{\text{grav}} - \varepsilon_{\nu}$

$\kappa$  Opacity  
 $\kappa^{-1} = \kappa_{\gamma}^{-1} + \kappa_C^{-1}$

$\kappa_{\gamma}$  Radiative opacity  
 $\kappa_{\gamma}\rho = \langle \lambda_{\gamma} \rangle_{\text{Rosseland}}^{-1}$

$\kappa_C$  Electron conduction

## Literature

- Clayton: Principles of stellar evolution and nucleosynthesis (Univ. Chicago Press 1968)
- Kippenhahn & Weigert: Stellar structure and evolution (Springer 1990)

# Virial Theorem and Hydrostatic Equilibrium

Hydrostatic equilibrium

$$\frac{dP}{dr} = -\frac{G_N M_r \rho}{r^2}$$

Integrate both sides

$$\int_0^R dr 4\pi r^3 P' = -\int_0^R dr 4\pi r^3 \frac{G_N M_r \rho}{r^2}$$

L.h.s. partial integration  
with  $P = 0$  at surface  $R$

$$-3 \int_0^R dr 4\pi r^2 P = E_{\text{grav}}^{\text{tot}}$$

Classical monatomic gas:  $P = \frac{2}{3}U$   
( $U$  density of internal energy)

$$U^{\text{tot}} = -\frac{1}{2}E_{\text{grav}}^{\text{tot}}$$

Average energy of single  
"atoms" of the gas

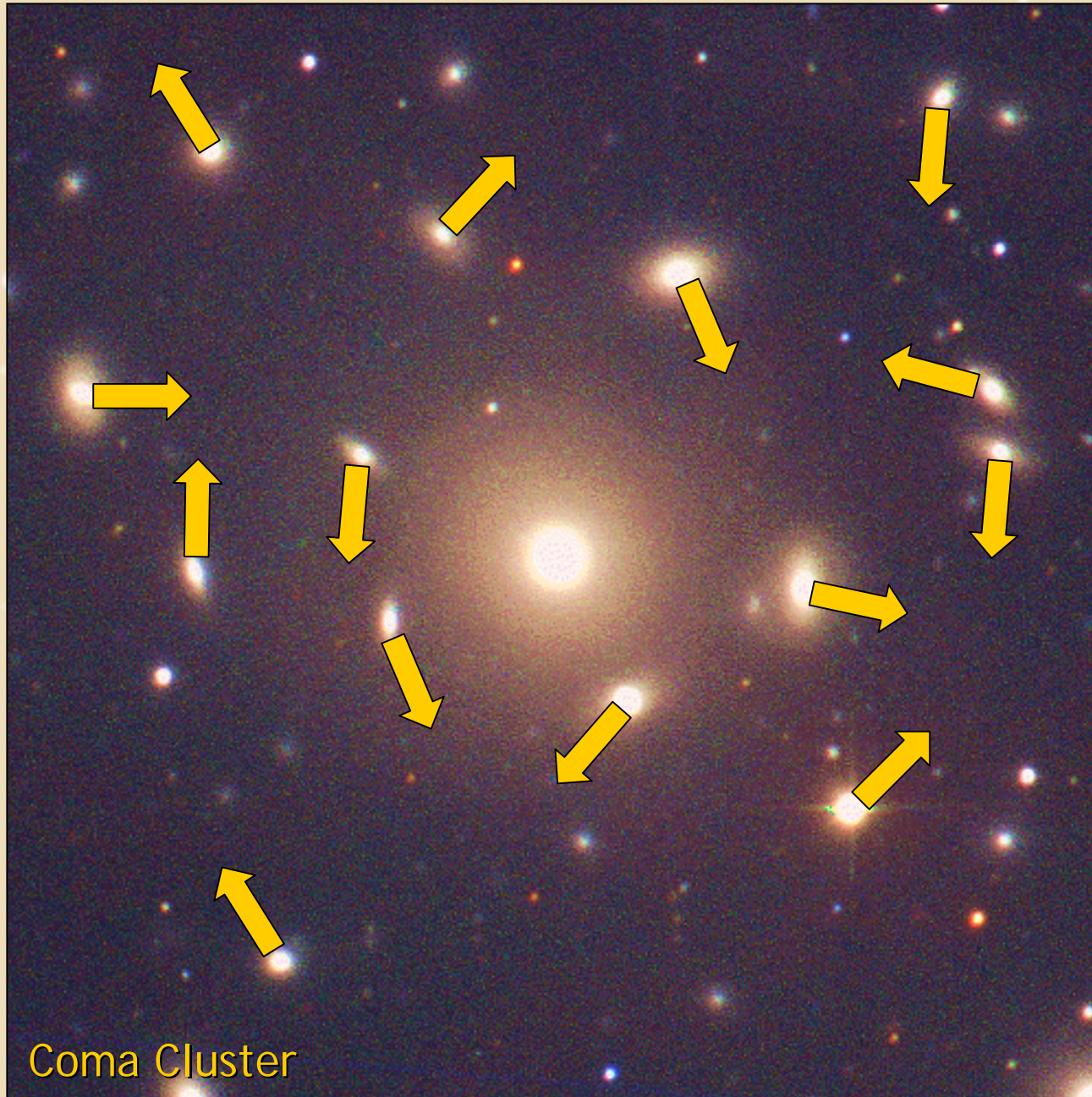
$$\langle E_{\text{kin}} \rangle = -\frac{1}{2} \langle E_{\text{grav}} \rangle$$

Virial Theorem

Most important tool to understand  
self-gravitating systems



# Dark Matter in Galaxy Clusters



A gravitationally bound system of many particles obeys the virial theorem

$$2\langle E_{\text{kin}} \rangle = -\langle E_{\text{grav}} \rangle$$

$$2\left\langle \frac{mv^2}{2} \right\rangle = \left\langle \frac{G_N M_r m}{r} \right\rangle$$

$$\langle v^2 \rangle \approx G_N M_r \langle r^{-1} \rangle$$

Velocity dispersion  
from Doppler shifts  
and geometric size



**Total Mass**

# Virial Theorem Applied to the Sun

$$\langle E_{\text{kin}} \rangle = -\frac{1}{2} \langle E_{\text{grav}} \rangle$$

Virial Theorem

Approximate Sun as a homogeneous sphere with

$$\text{Mass } M_{\text{sun}} = 1.99 \times 10^{33} \text{ g}$$

$$\text{Radius } R_{\text{sun}} = 6.96 \times 10^{10} \text{ cm}$$

Gravitational potential energy of a proton near center of the sphere

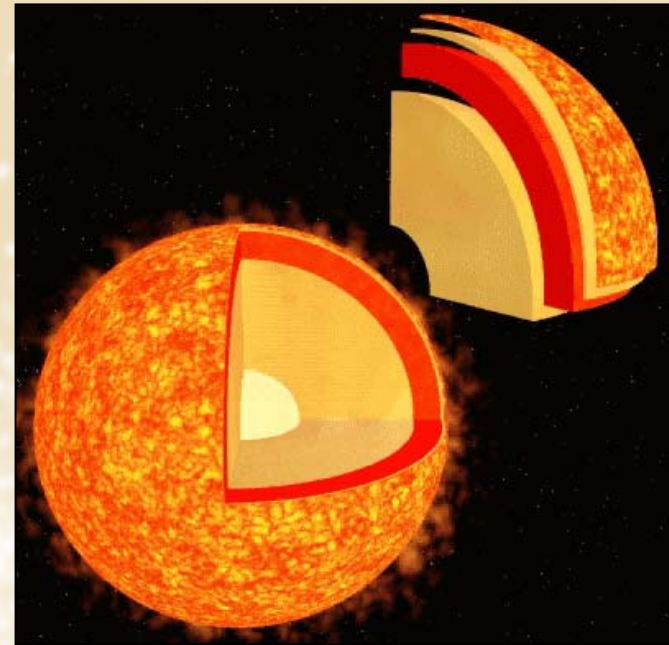
$$\langle E_{\text{grav}} \rangle = -\frac{3}{2} \frac{G_{\text{N}} M_{\text{sun}} m_{\text{p}}}{R_{\text{sun}}} = -3.2 \text{ keV}$$

Thermal velocity distribution

$$\langle E_{\text{kin}} \rangle = \frac{3}{2} k_{\text{B}} T = -\frac{1}{2} \langle E_{\text{grav}} \rangle$$

Estimated temperature

$$T = 1.1 \text{ keV}$$



Central temperature from standard solar models

$$T_{\text{c}} = 1.56 \times 10^7 \text{ K} \\ = 1.34 \text{ keV}$$



# Thermonuclear Reactions and Gamow Peak

Coulomb repulsion prevents nuclear reactions, except for Gamow tunneling

Tunneling probability

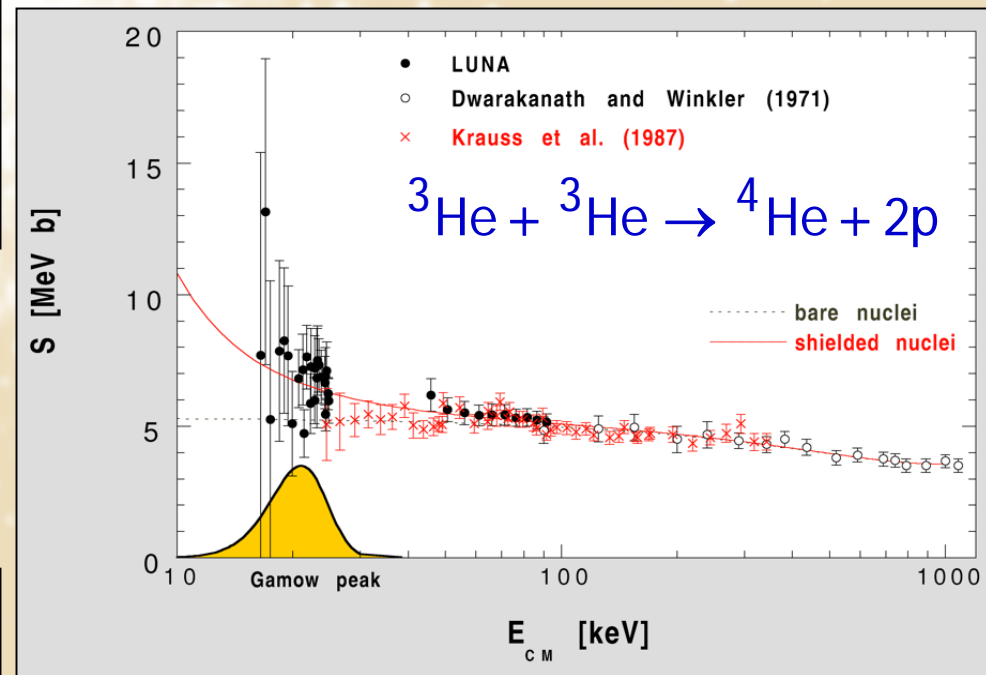
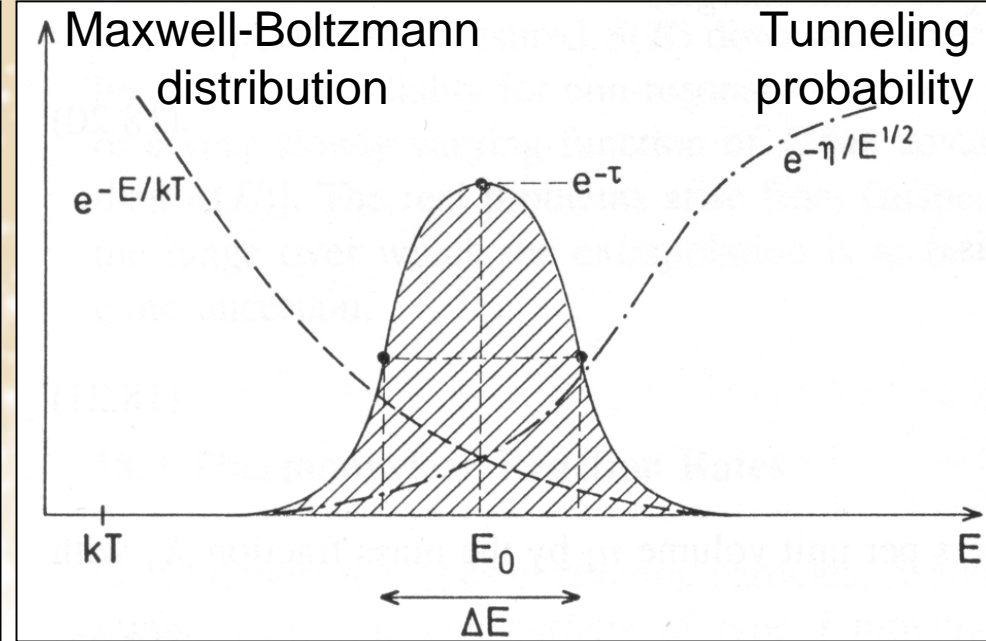
$$p \propto E^{-1/2} e^{-2\pi\eta}$$

With Sommerfeld parameter

$$\eta = \left( \frac{m}{2E} \right)^{1/2} Z_1 Z_2 e^2$$

Parameterize cross section with astrophysical S-factor

$$S(E) = \sigma(E) E e^{2\pi\eta(E)}$$



LUNA Collaboration, nucl-ex/9902004

# Main Nuclear Burnings

**Hydrogen burning**  $4p + 2e^- \rightarrow {}^4\text{He} + 2\nu_e$

- Proceeds by pp chains and CNO cycle
- No higher elements are formed because no stable isotope with mass number 8
- Neutrinos from  $p \rightarrow n$  conversion
- Typical temperatures:  $10^7$  K ( $\sim 1$  keV)

**Helium burning**



“Triple alpha reaction” because  ${}^8\text{Be}$  unstable, builds up with concentration  $\sim 10^{-9}$



Typical temperatures:  $10^8$  K ( $\sim 10$  keV)

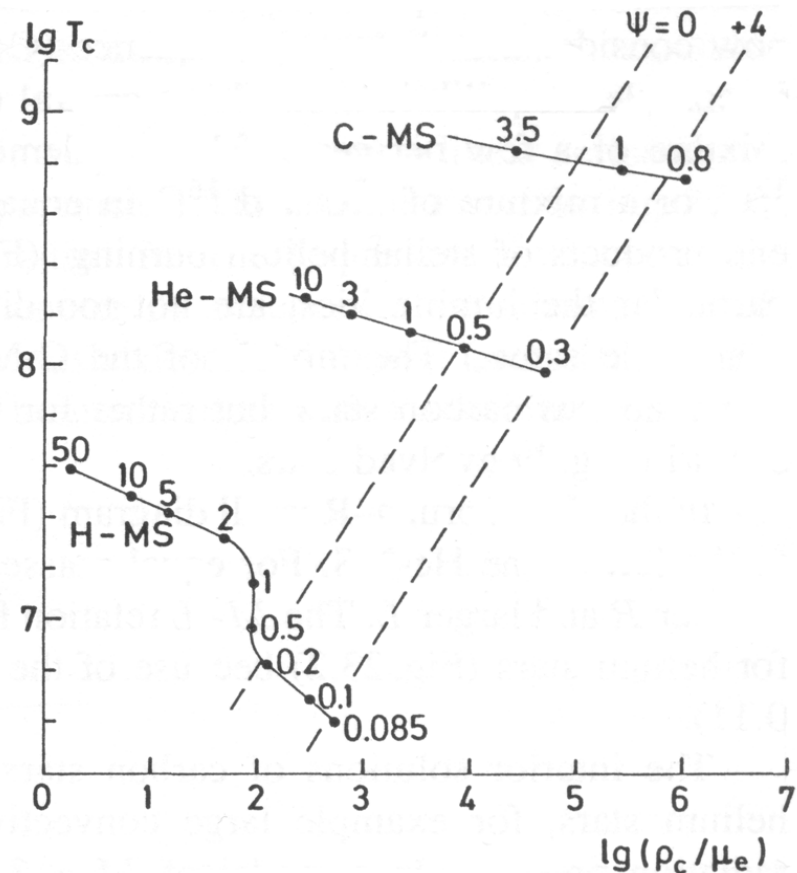
**Carbon burning**

Many reactions, for example



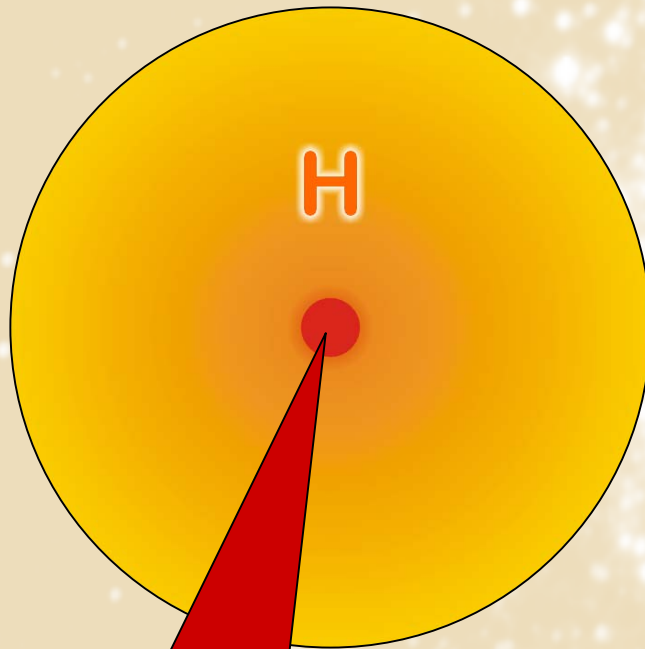
Typical temperatures:  $10^9$  K ( $\sim 100$  keV)

- Each type of burning occurs at a very different T but a broad range of densities
- Never co-exist in same location



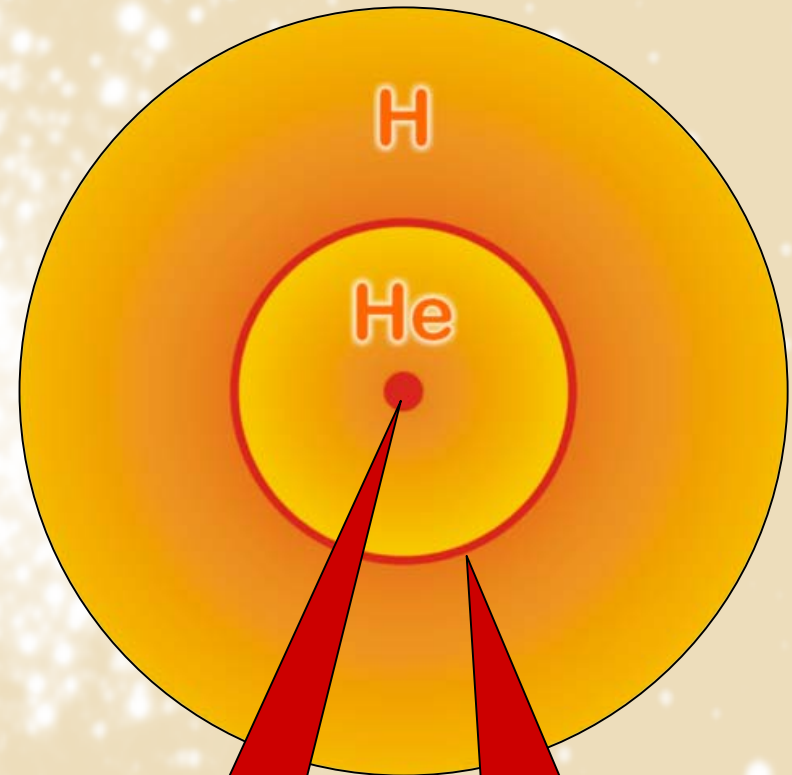
# Hydrogen Exhaustion

Main-sequence star



Hydrogen Burning


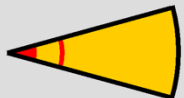
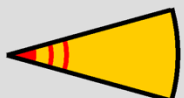

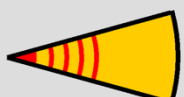
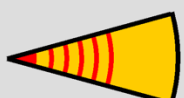
Helium-burning star



Helium  
Burning

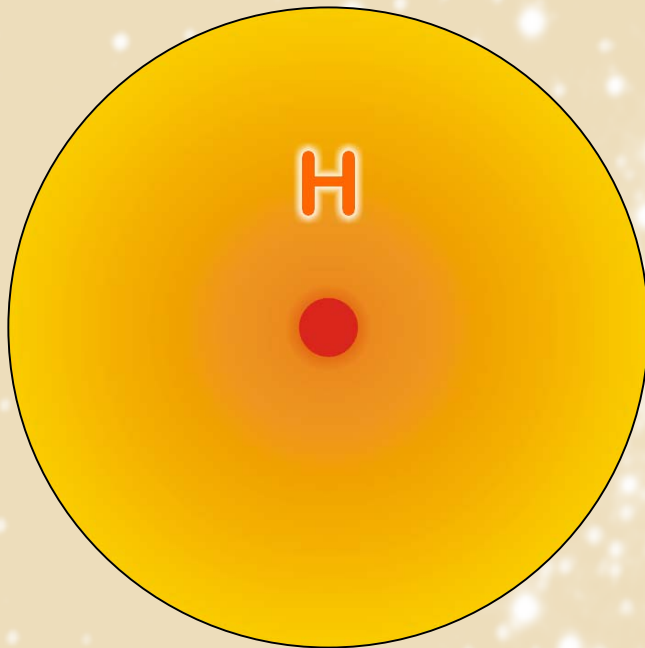
Hydrogen  
Burning

# Burning Phases of a 15 Solar-Mass Star

Burning Phase	Dominant Process	$T_C$ [keV]	$\rho_C$ [g/cm <sup>3</sup> ]	$L_\gamma$ [10 <sup>4</sup> L <sub>sun</sub> ]		Duration [years]
				$L_V/L_\gamma$		
 Hydrogen	H → He	3	5.9	2.1	—	1.2 × 10 <sup>7</sup>
 Helium	He → C, O	14	1.3 × 10 <sup>3</sup>	6.0	1.7 × 10 <sup>-5</sup>	1.3 × 10 <sup>6</sup>
 Carbon	C → Ne, Mg	53	1.7 × 10 <sup>5</sup>	8.6	1.0	6.3 × 10 <sup>3</sup>
 Neon	Ne → O, Mg	110	1.6 × 10 <sup>7</sup>	9.6	1.8 × 10 <sup>3</sup>	7.0
 Oxygen	O → Si	160	9.7 × 10 <sup>7</sup>	9.6	2.1 × 10 <sup>4</sup>	1.7
 Silicon	Si → Fe, Ni	270	2.3 × 10 <sup>8</sup>	9.6	9.2 × 10 <sup>5</sup>	6 days



# Self-Regulated Nuclear Burning



Main-Sequence Star

Virial Theorem  $\langle E_{\text{kin}} \rangle = -\frac{1}{2} \langle E_{\text{grav}} \rangle$

## Small Contraction

- Heating
- Increased nuclear burning
- Increased pressure
- Expansion

## Additional energy loss ("cooling")

- Loss of pressure
- Contraction
- Heating
- Increased nuclear burning

## Hydrogen burning at a nearly fixed T

- Gravitational potential nearly fixed:  
 $G_N M/R \sim \text{constant}$
- $R \propto M$  (More massive stars bigger)

# Modification of Stellar Properties by Axion Emission

Homologous  
changes of  
stellar structure

Assume that some small perturbation (e.g. axion emission) leads to “homologous” modification of stellar structure, i.e. every point is mapped to a new position  $r' = yr$   
Requires power-law relations for constitutive relations

- Nuclear burning rate  $\epsilon \propto \rho^n T^m$
- Mean opacity  $\kappa \propto \rho^s T^t$

Implies for other quantities:

- Density  $\rho'(r') = y^{-3} \rho(r)$
- Pressure  $p'(r') = y^{-4} p(r)$
- Temperature gradient  $dT'(r')/dr' = y^{-2} dT(r)/dr$

Impact of small  
exotic energy loss

Modified nuclear burning rate  $\epsilon \propto (1 - \delta_x) \epsilon_{\text{nuc}}$

Assume Kramers opacity law  $s = 1, \quad t = -3.5$

Hydrogen burning  $n = 1, \quad m = 4 - 6$

$$\frac{\delta R}{R} = \frac{-2\delta_x}{2m+5} \quad \frac{\delta L_\gamma}{L_\gamma} = \frac{\delta_x}{2m+5} \quad \frac{\delta T}{T} = \frac{\delta_x}{2m+5}$$

# Degenerate Stars ("White Dwarfs")

Assume T very small  
 → No thermal pressure  
 → Electron degeneracy is pressure source

Pressure ~ Momentum density x Velocity

- Electron density  $n_e = p_F^3 / (3\pi^2)$
- Momentum  $p_F$  (Fermi momentum)
- Velocity  $v \propto p_F / m_e$
- Pressure  $P \propto p_F^5 \propto \rho^{5/3} \propto M^{5/3} R^{-5}$
- Density  $\rho \propto MR^{-3}$   
(Stellar mass M and radius R)

Hydrostatic equilibrium

$$\frac{dP}{dr} = -\frac{G_N M_r \rho}{r^2}$$

With  $dP/dr \sim -P/R$  we have approximately

$$P \propto G_N M \rho R^{-1} \propto G_N M^2 R^{-4}$$

Inverse mass-radius relationship  
 for degenerate stars:  $R \propto M^{-1/3}$

$$R = 10,500 \text{ km} \left( \frac{0.6 M_{\text{sun}}}{M} \right)^{1/3} (2Y_e)^{5/3}$$

( $Y_e$  electrons per nucleon)

For sufficiently large mass,  
 electrons become relativistic

- Velocity = speed of light
- Pressure

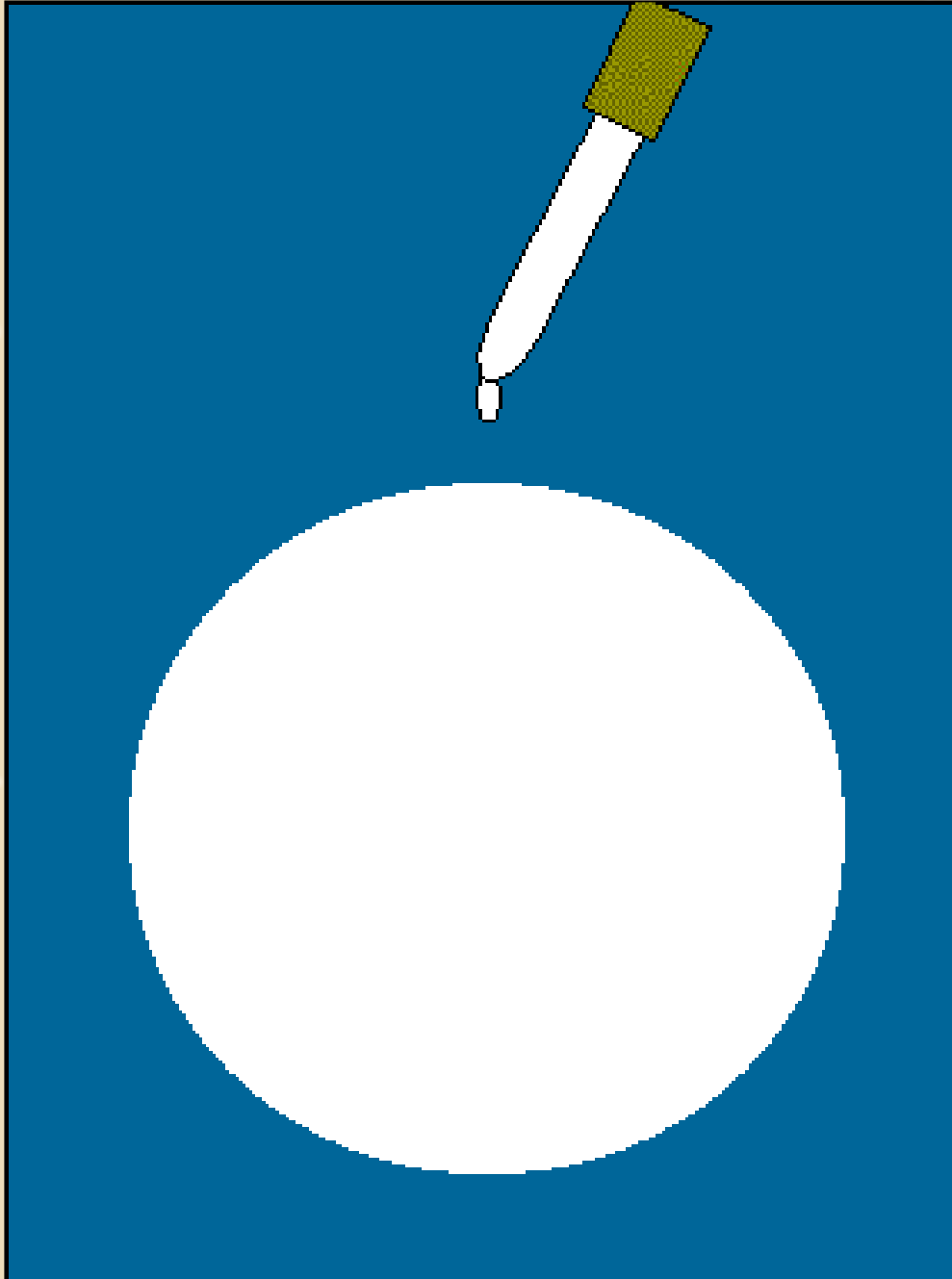
$$P \propto p_F^4 \propto \rho^{4/3} \propto M^{4/3} R^{-4}$$

No stable configuration

Chandrasekhar mass limit

$$M_{\text{Ch}} = 1.457 M_{\text{sun}} (2Y_e)^2$$

# Degenerate Stars



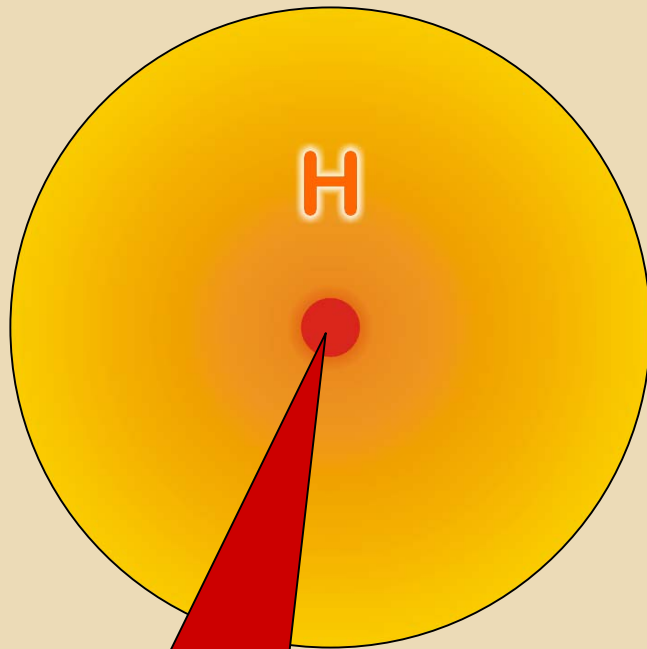
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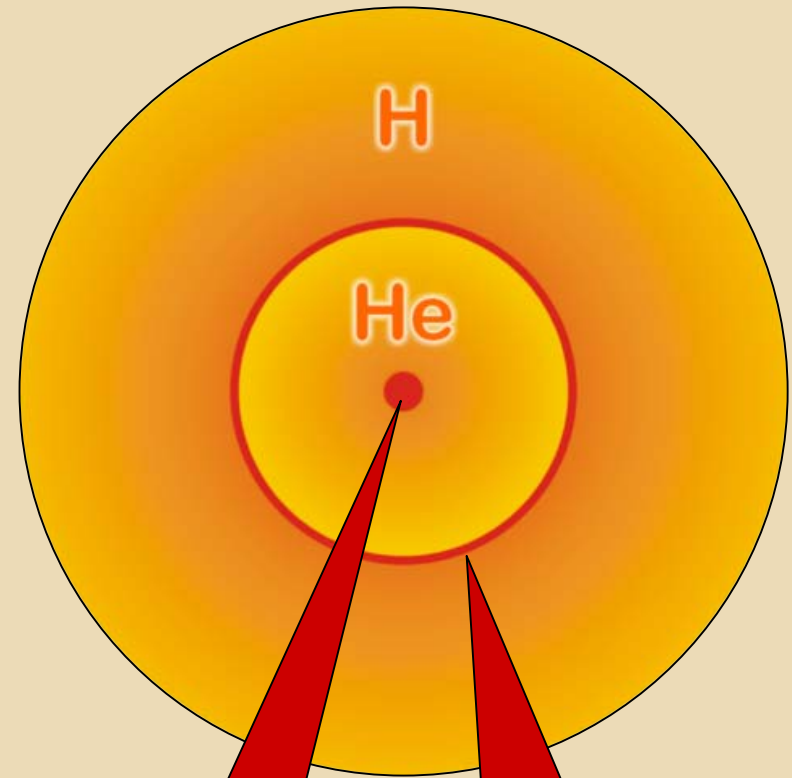
# Stellar Collapse

Main-sequence star



Hydrogen Burning

Helium-burning star



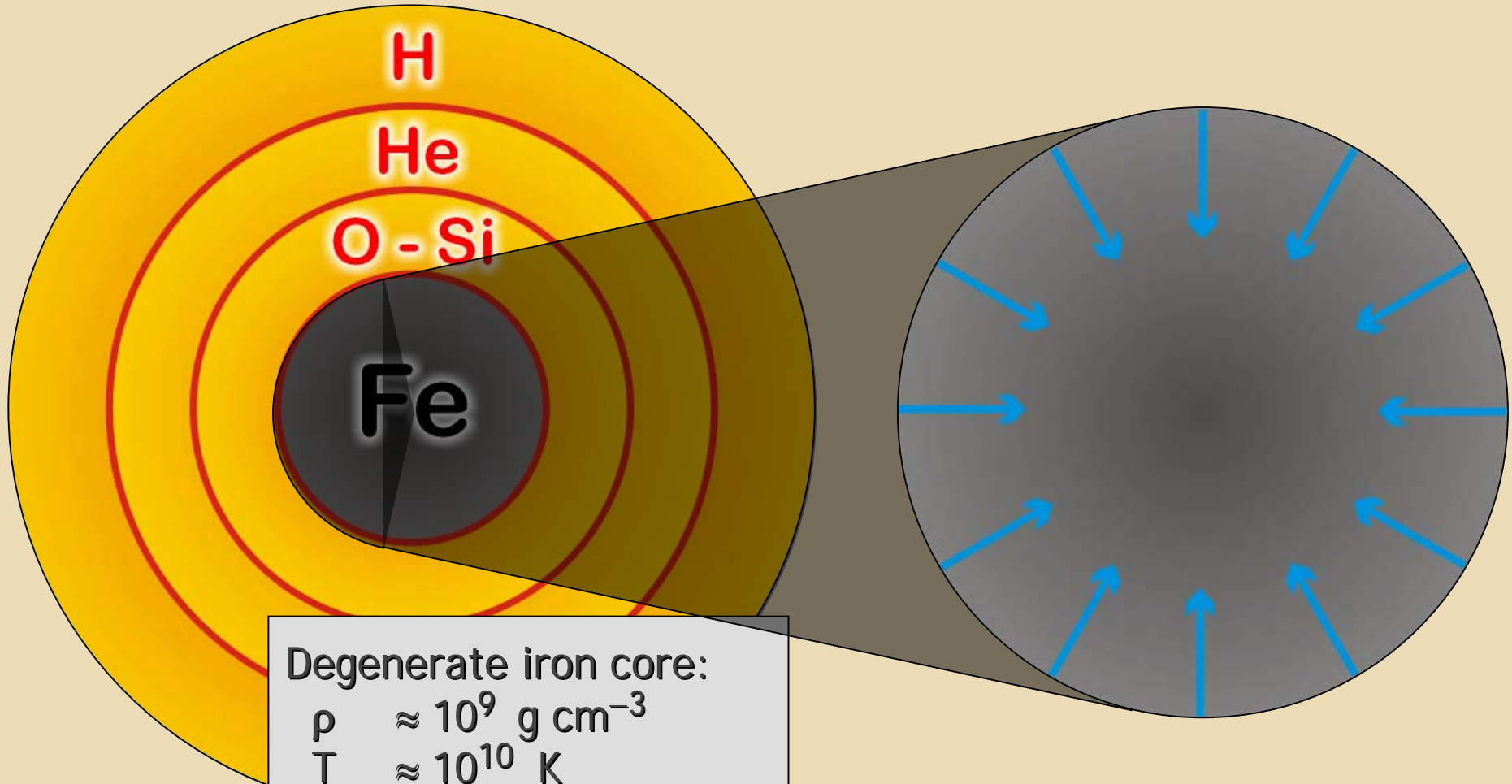
Helium  
Burning

Hydrogen  
Burning

# Stellar Collapse

Onion structure

Collapse (implosion)



Degenerate iron core:

$$\rho \approx 10^9 \text{ g cm}^{-3}$$

$$T \approx 10^{10} \text{ K}$$

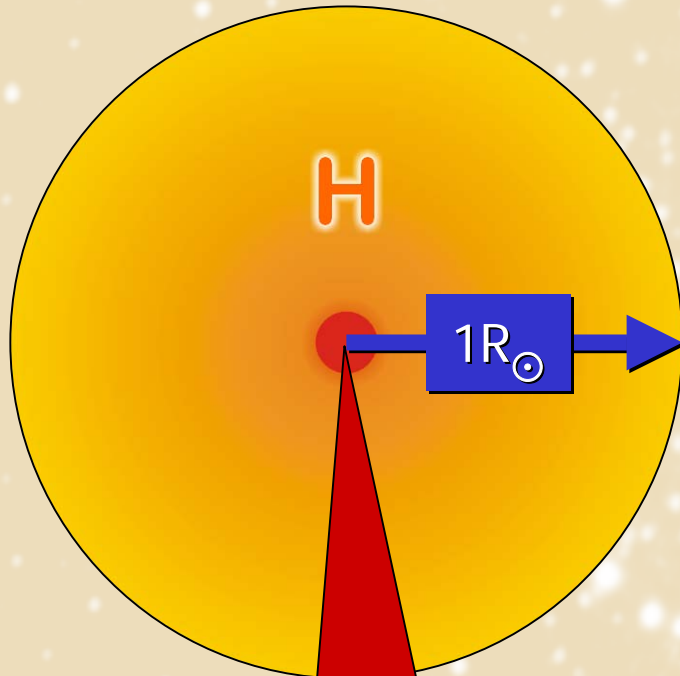
$$M_{\text{Fe}} \approx 1.5 M_{\text{sun}}$$

$$R_{\text{Fe}} \approx 8000 \text{ km}$$



# Giant Stars

Main-sequence star  $1M_{\odot}$   
(Hydrogen burning)



$\epsilon_{\text{nuc}}(\text{H})$  relates to  
 $T \propto \Phi_{\text{grav}} \propto M/R$   
of full star

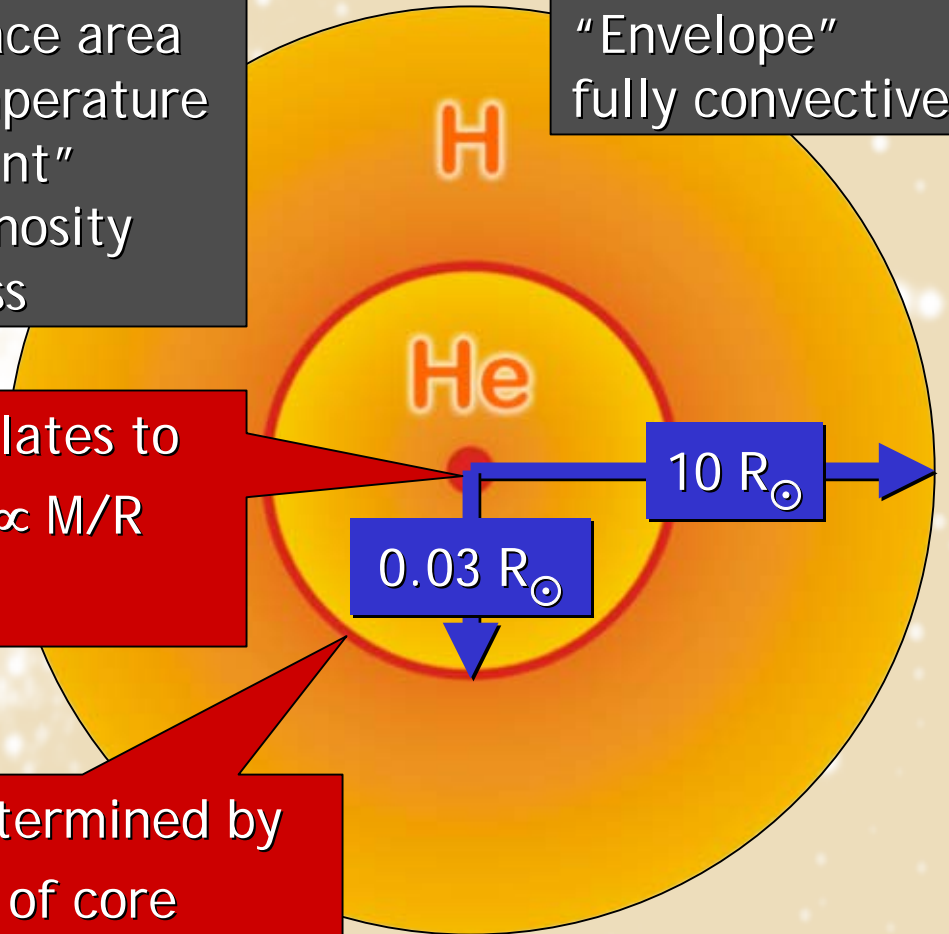
Helium-burning star  $1M_{\odot}$

Large surface area  
→ low temperature  
→ "red giant"  
Large luminosity  
→ mass loss

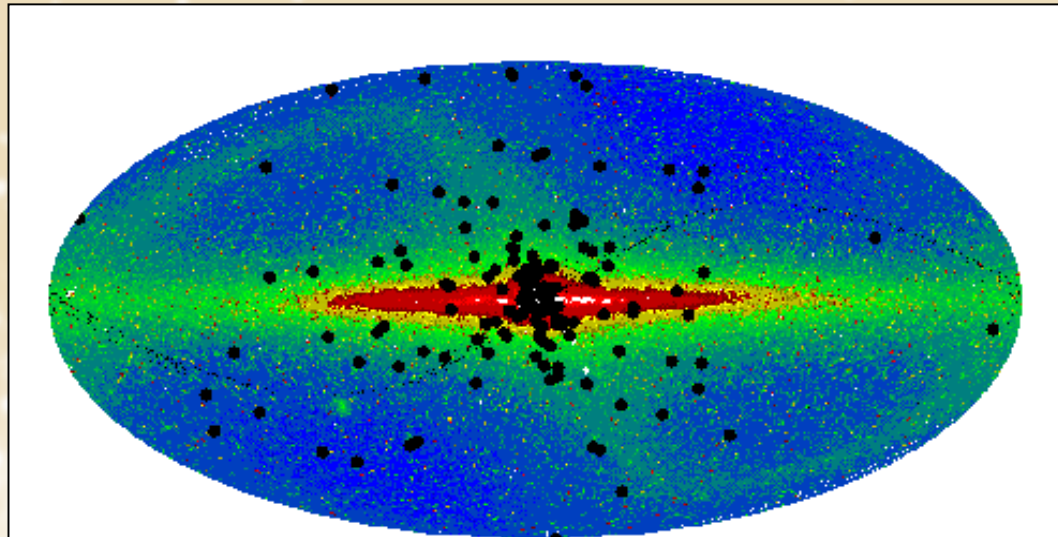
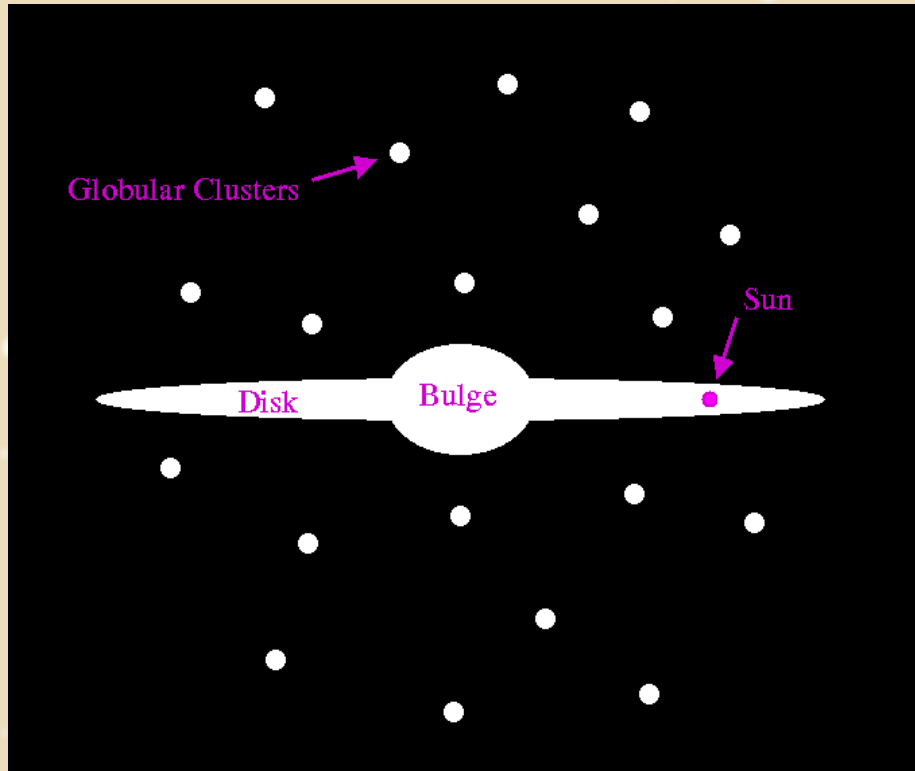
"Envelope"  
fully convective

$\epsilon_{\text{nuc}}(\text{He})$  relates to  
 $T \propto \Phi_{\text{grav}} \propto M/R$   
of core

$\epsilon_{\text{nuc}}(\text{H})$  determined by  
 $T \propto \Phi_{\text{grav}}$  of core  
→ huge  $L(\text{H})$

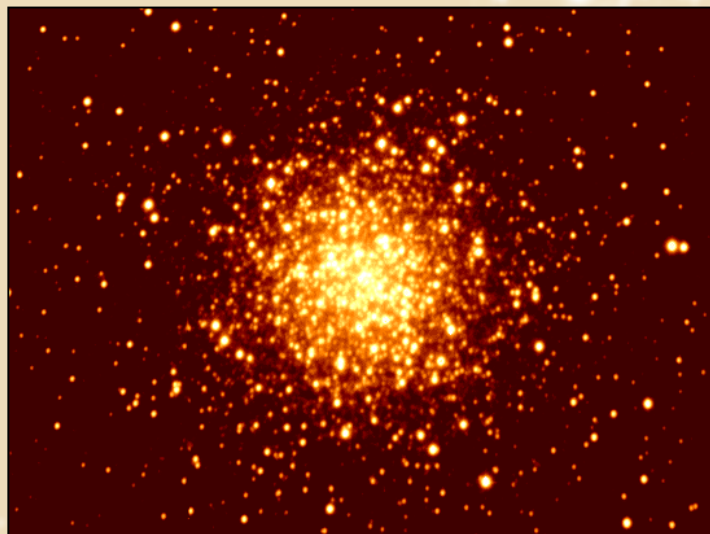


# Globular Clusters of the Milky Way



<http://www.dartmouth.edu/~chaboyer/mwgc.html>

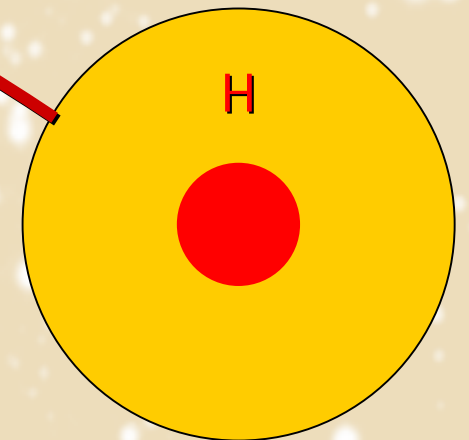
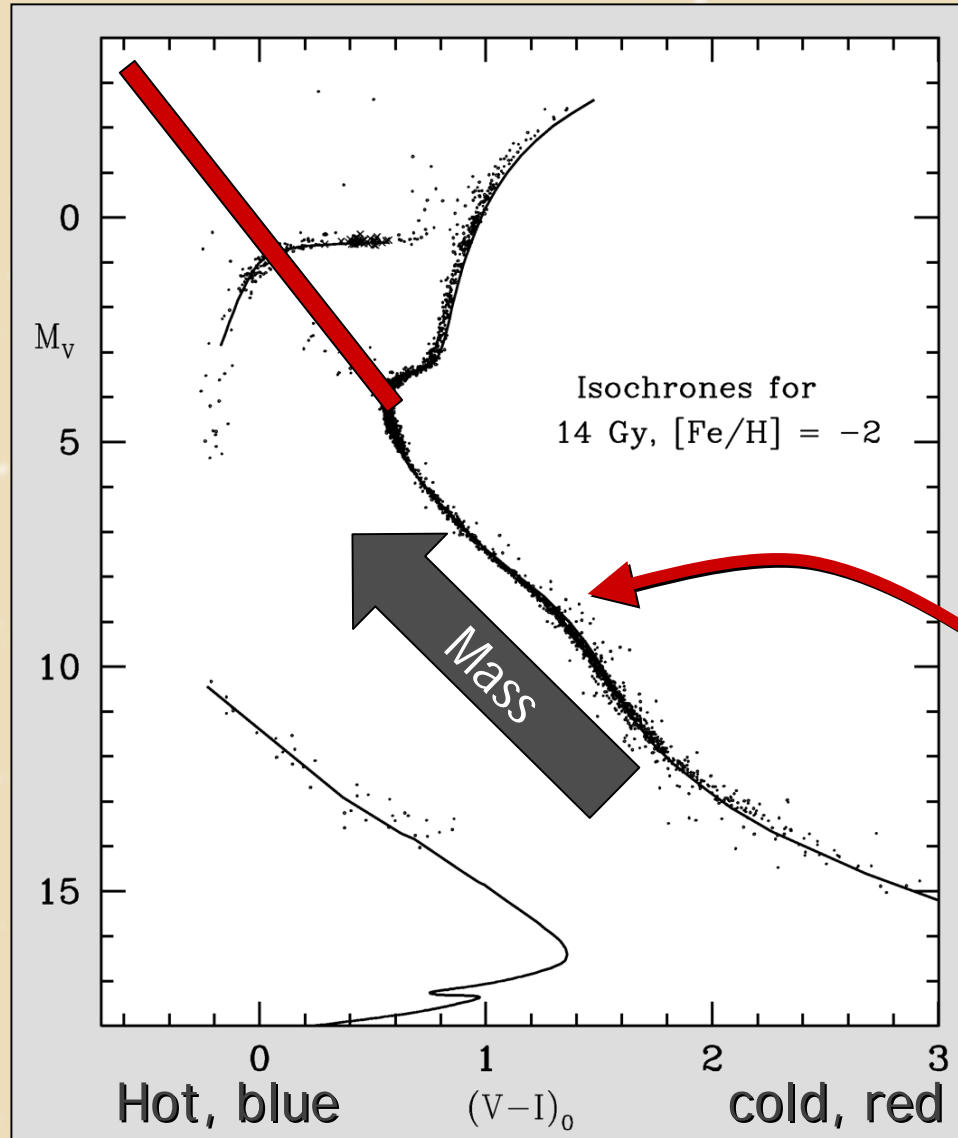
Globular clusters on top of the  
FIRAS 2.2 micron map of the Galaxy



The galactic globular cluster M3

# Color-Magnitude Diagram for Globular Clusters

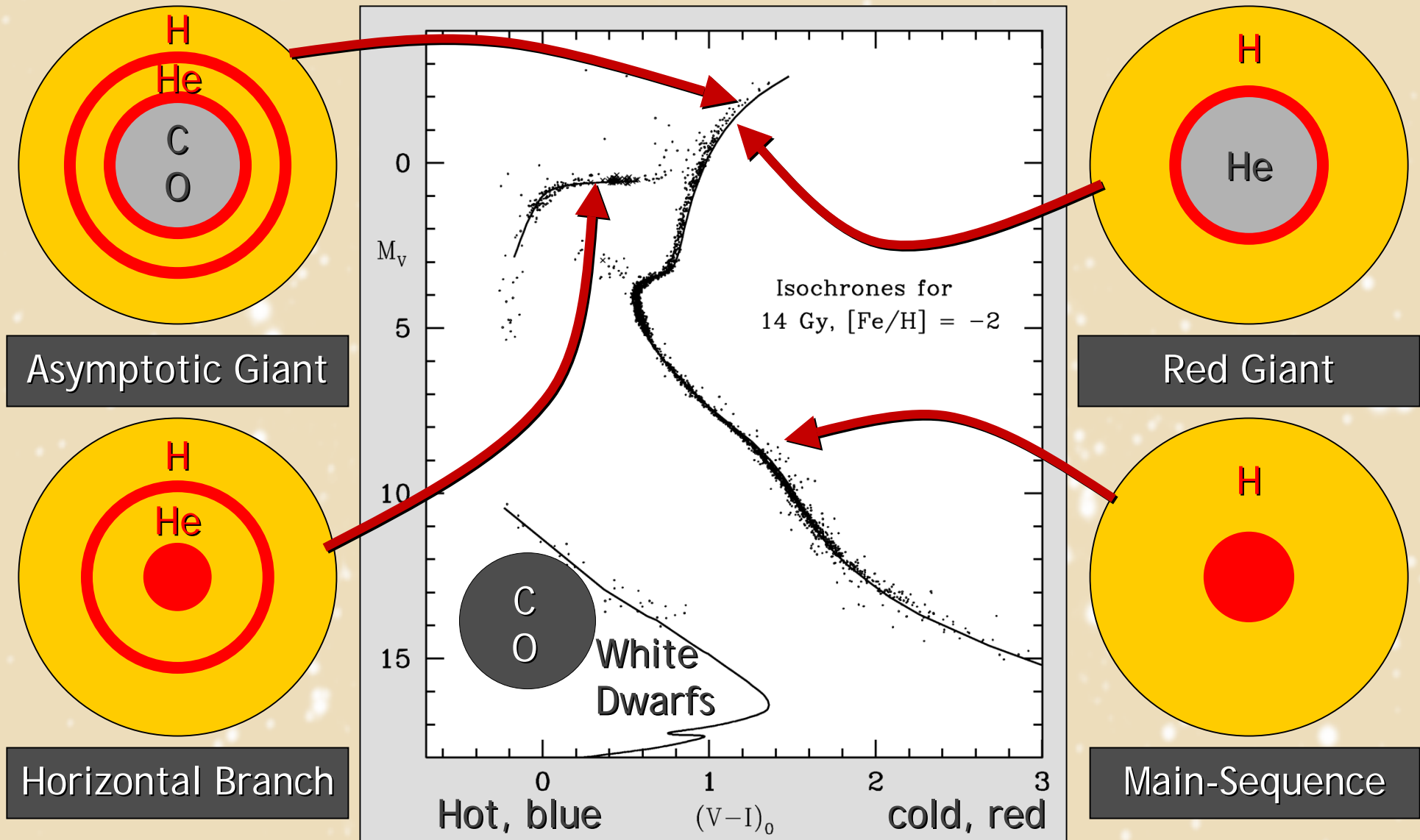
- Stars with  $M$  so large that they have burnt out in a Hubble time
- No new star formation in globular clusters



Main-Sequence

Color-magnitude diagram synthesized from several low-metallicity globular clusters and compared with theoretical isochrones (W.Harris, 2000)

# Color-Magnitude Diagram for Globular Clusters

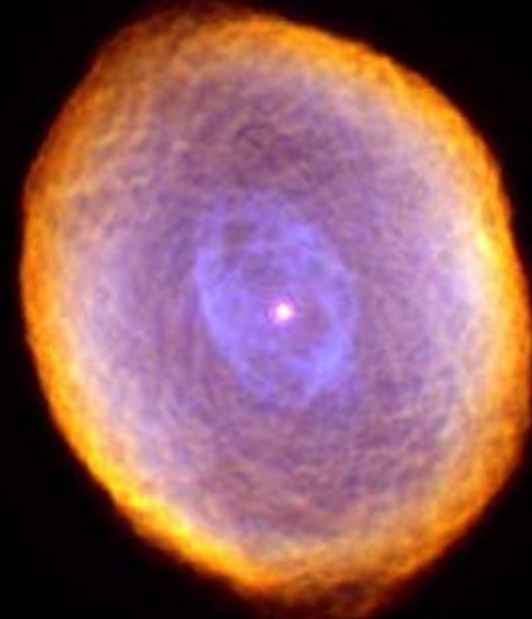


Color-magnitude diagram synthesized from several low-metallicity globular clusters and compared with theoretical isochrones (W.Harris, 2000)

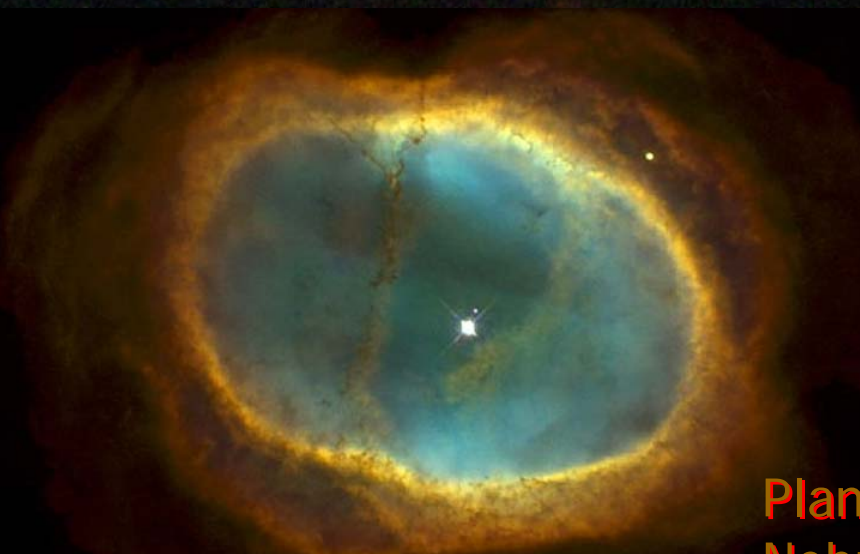


# Planetary Nebulae

Hour  
Glass  
Nebula



Planetary  
Nebula IC 418



Planetary  
Nebula NGC 3132

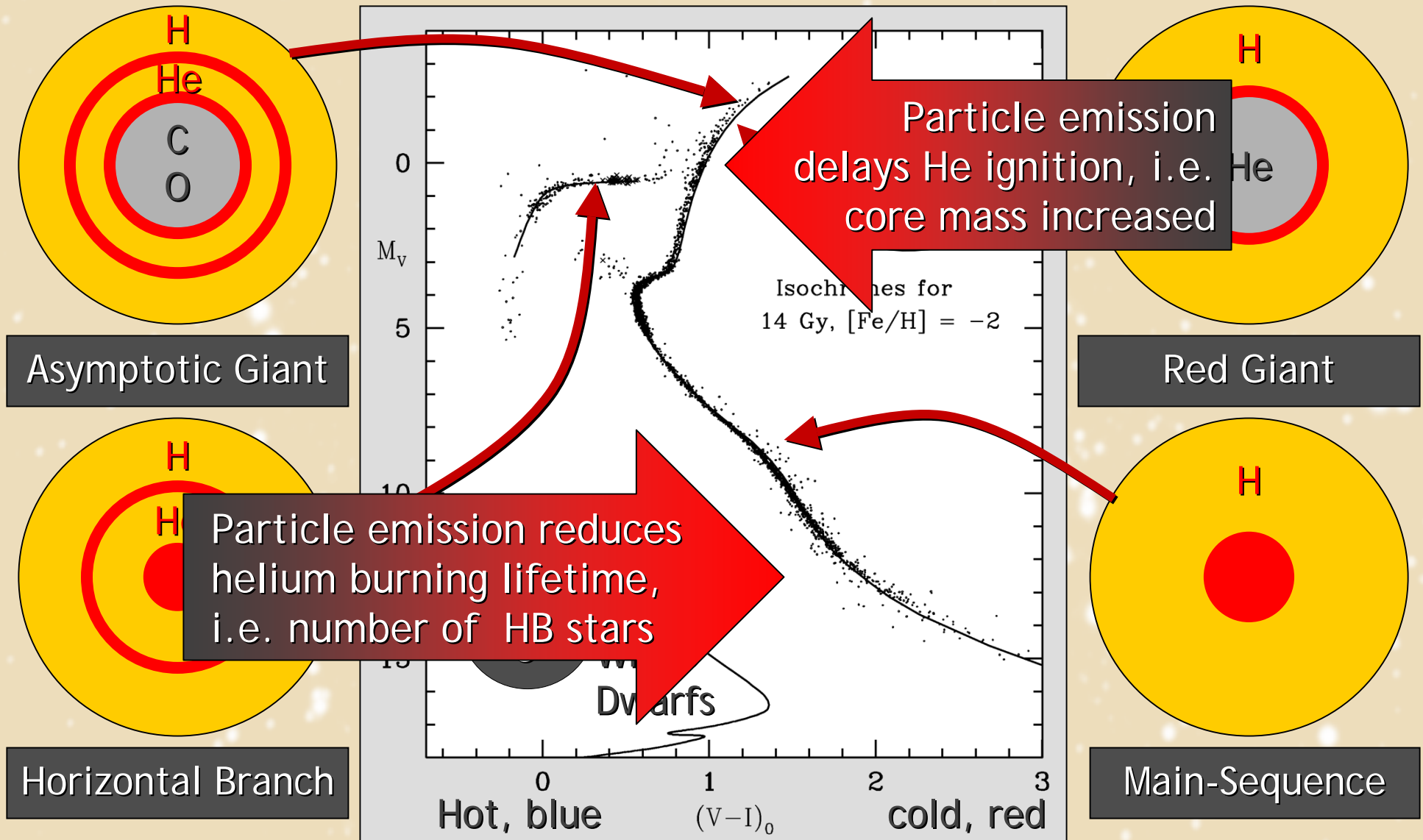
Eskimo  
Nebula



# Globular-Cluster Limit on Axion-Photon Coupling

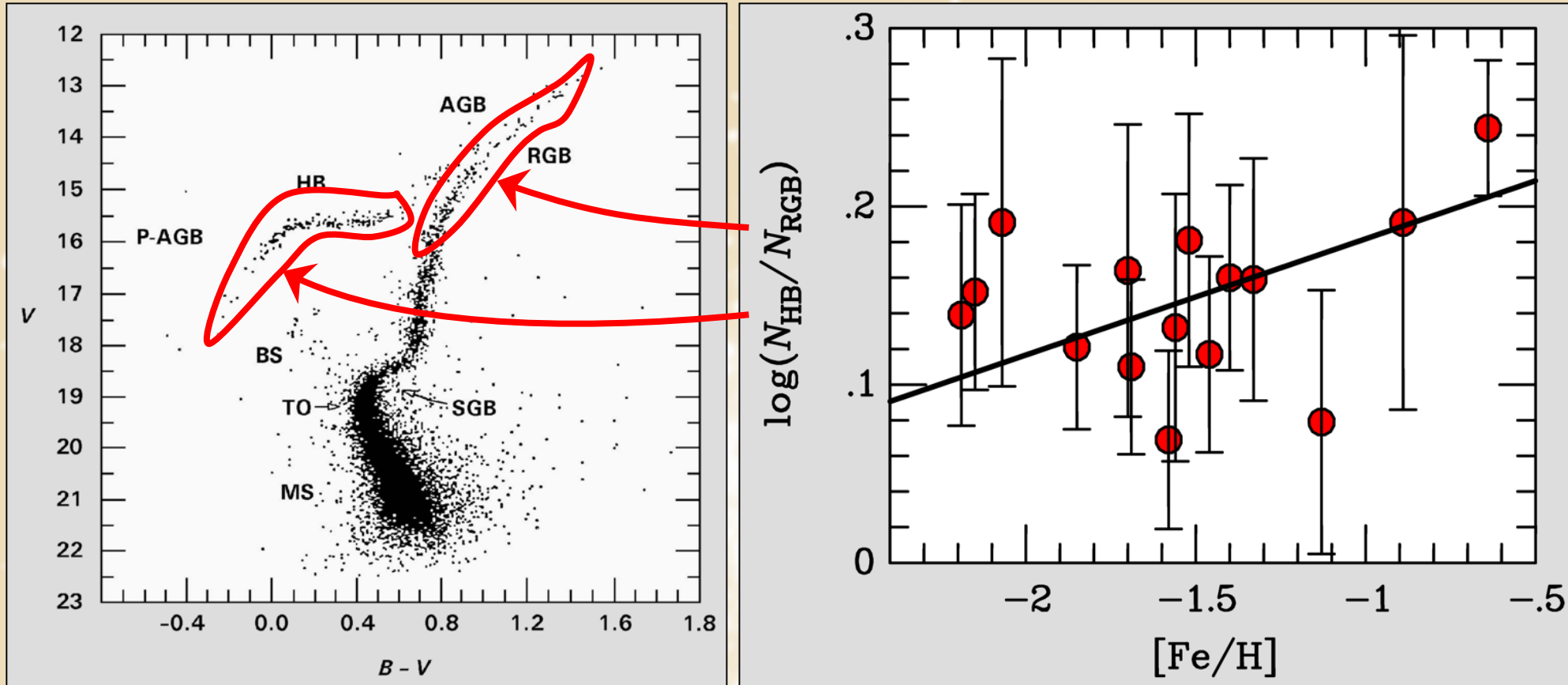


# Color-Magnitude Diagram for Globular Clusters



Color-magnitude diagram synthesized from several low-metallicity globular clusters and compared with theoretical isochrones (W.Harris, 2000)

# Helium-Burning Lifetime of Horizontal-Branch Stars

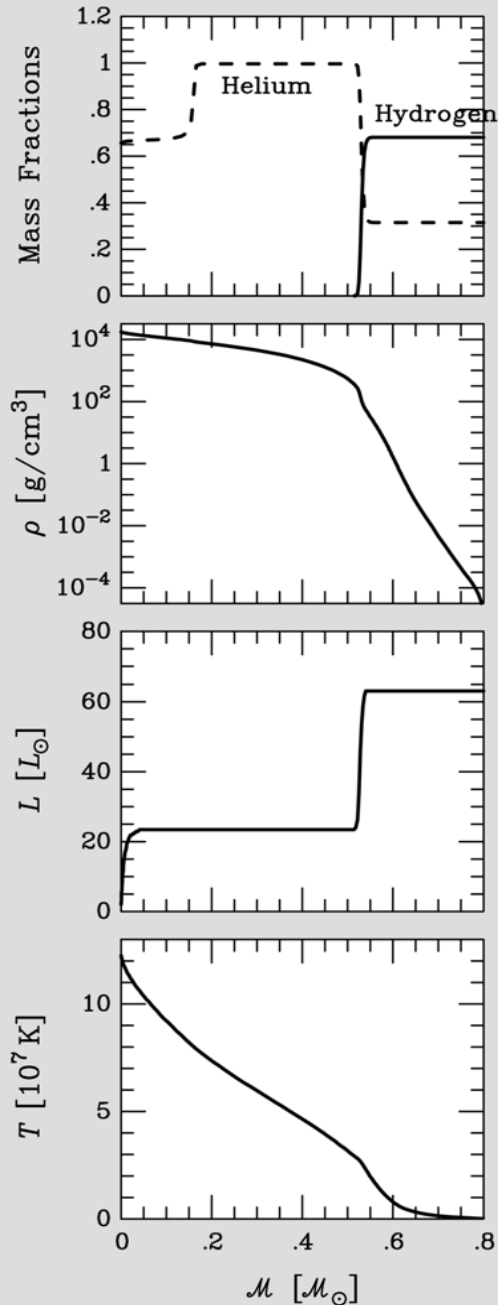
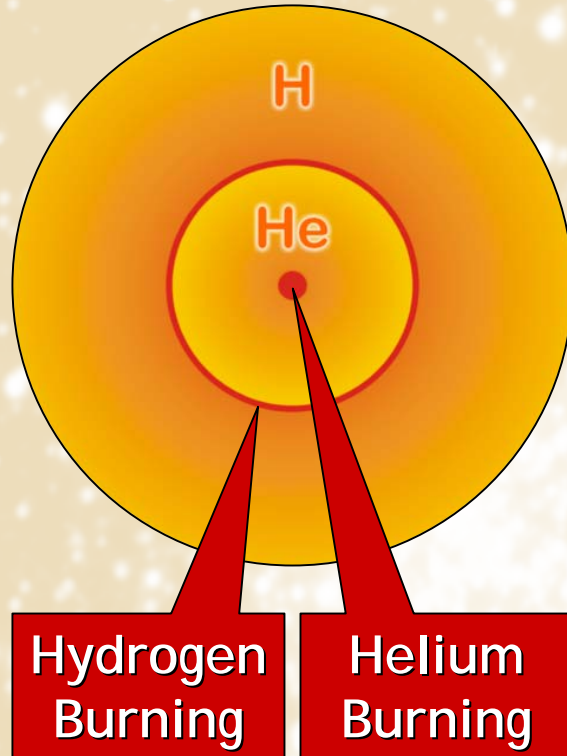


Number ratio of HB-Stars/Red Giants in 15 galactic globular clusters  
(Buzzoni et al. 1983)

Helium-burning lifetime established within  $\pm 10\%$

# Globular-Cluster Limit on Axion-Photon Coupling

## Helium-burning star

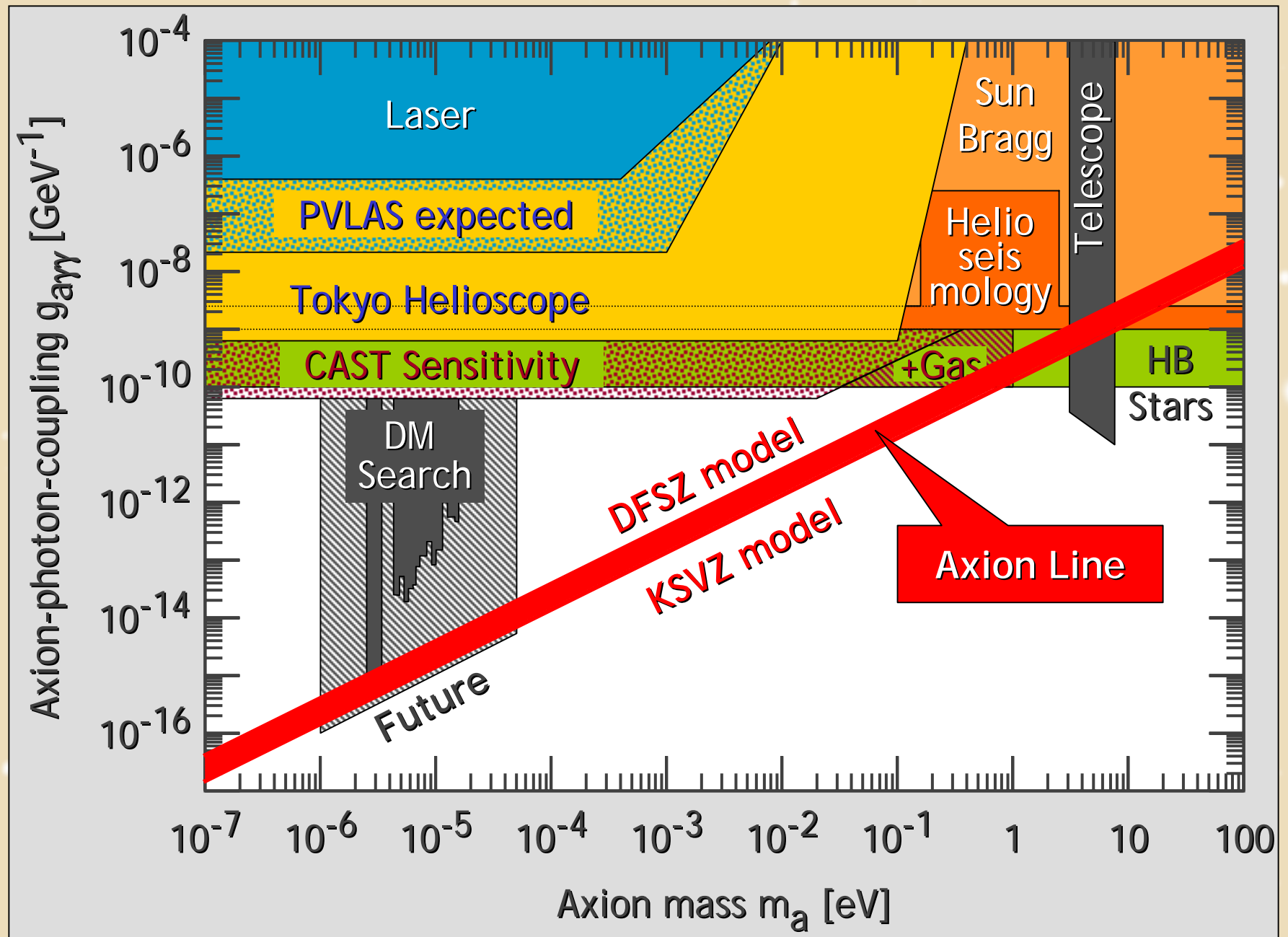


- Helium-burning luminosity
  - $L_{3\alpha} \approx 20 L_\odot$
  - $T \approx 10$  keV
  - $\rho \approx 10^4$  g cm<sup>-3</sup>
- Core-average nuclear energy generation rate
  - $\epsilon_{3\alpha} \approx 80$  erg g<sup>-1</sup> s<sup>-1</sup>
- Core-average Primakoff emission rate
  - $\epsilon_{\text{Primakoff}} \approx g_{10}^2 30$  erg g<sup>-1</sup> s<sup>-1</sup>
- Reduction of helium-burning lifetime

$$\frac{\tau}{\tau_0} \approx \frac{1}{1 + 0.4 g_{10}^2}$$

- Adopt nominal limit  $g_{10} < 1$  (More restrictive limit if using 10% precision for helium burning lifetime)

# Limits on Axion-Photon-Coupling



# Model-Dependence of Axion-Photon Coupling

Translating limits on the axion-photon coupling into limits on the Peccei-Quinn scale or axion mass depends on model uncertainties

Light quark mass ratio

$$z = \frac{m_u}{m_d} = 0.3 - 0.7$$

$$z = 0.56 \quad \text{"Canonical value"}$$

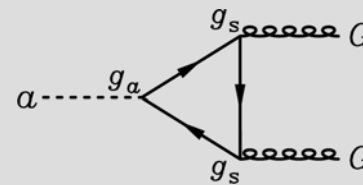
Mass

$$m_a = \frac{f_\pi m_\pi}{f_a} \frac{\sqrt{z}}{1+z} \quad \frac{\sqrt{z}}{1+z} = 0.42 - 0.49$$

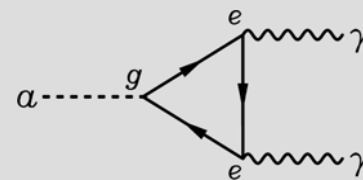
Axion-photon coupling

$$L_{a\gamma} = -\frac{g_{a\gamma}}{4} F\tilde{F}a = g_{a\gamma} \vec{E} \cdot \vec{B}a$$

$$g_{a\gamma} = \frac{\alpha}{2\pi f_a} \left( \frac{E}{N} - \underbrace{\frac{2}{3} \frac{4+z}{1+z}}_{1.8-2.2} \right)$$



Gluon anomaly coefficient N



Electromagnetic anomaly coefficient E

$E/N = 0$  (KSVZ),  $E/N = 8/3$  (DFSZ), or many other ...  
But requires fine-tuning to strongly suppress  $g_{a\gamma}$

# Astrophysical Axion Bounds



Experiments

Tele  
scope

Globular clusters  
( $a$ - $\gamma$ -coupling)

Axion dark matter possible  
(Late inflation scenario)

DM o.k.

(String scenario)

Too much DM

Direct  
search



# Free Streaming vs Trapping of New Particles

Free Streaming  
Mean Free Path  $\gg$  Stellar Radius

Trapping  
Mean Free Path  $\ll$  Stellar Radius

Energy conservation

$$\frac{dL_r}{dr} = 4\pi r^2 \varepsilon \rho$$

Hydrostatic Equilibrium

$$\frac{dP}{dr} = -\frac{G_N M_r \rho}{r^2}$$

Energy transfer

$$L_r = \frac{4\pi r^2}{3\kappa\rho} \frac{d(aT^4)}{dr}$$

Weakly interacting particles constitute a new **energy-loss channel** in addition to neutrinos and thus violate “energy conservation,” reducing the available nuclear energy

$$\varepsilon = \varepsilon_{\text{nuc}} - \varepsilon_{\nu} - \varepsilon_x$$

Strong effect on stellar evolution when

$$\varepsilon_x \text{ comparable to } \varepsilon_{\text{nuc}}$$

Weakly interacting particles achieve local thermal equilibrium and thus contribute an **energy-transfer channel** in addition to photons and conduction

$$\kappa^{-1} = \kappa_C^{-1} + \kappa_{\gamma}^{-1} + \kappa_x^{-1}$$

Relation to average mean free path

$$(\kappa_{\gamma}\rho)^{-1} = \langle \lambda_{\gamma} \rangle_{\text{Rosseland}}$$

Strong effect on stellar structure when

$$\lambda_x \gtrsim \lambda_{\gamma}$$

Strongest effect of new particles when mean free path  $\sim$  stellar radius



# Supernova 1987A Limits on the Axion-Nucleon Coupling



Sanduleak -69 202



Supernova 1987A

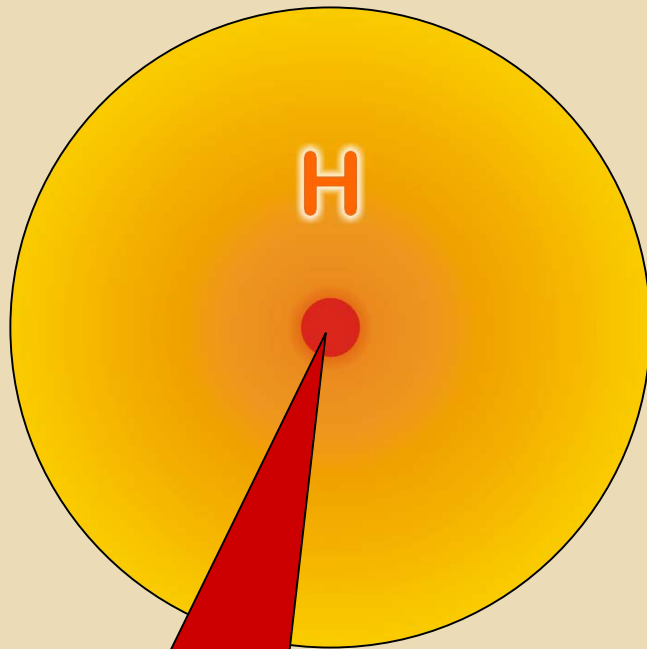
23 February 1987





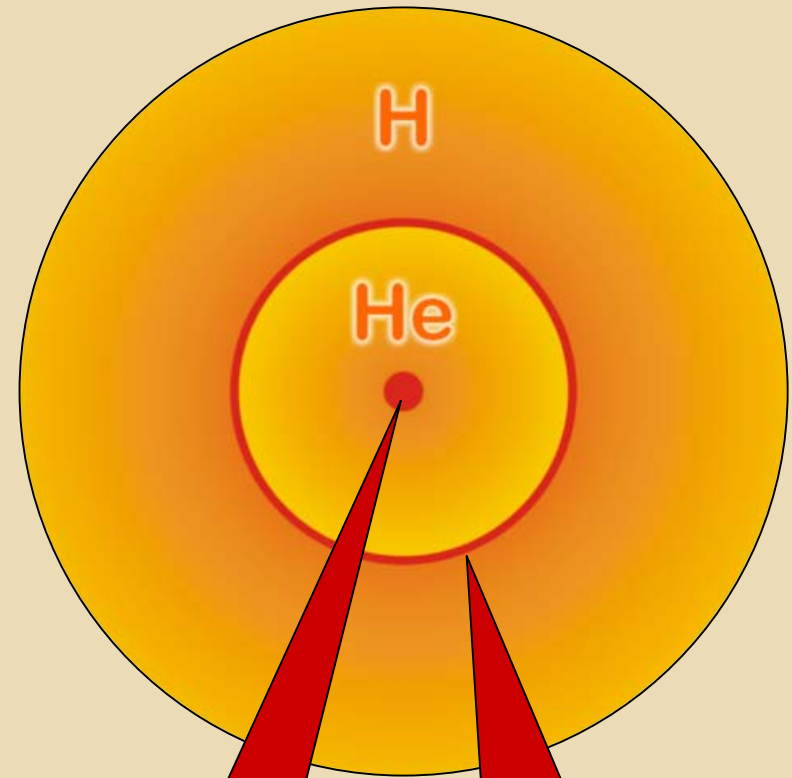
# Stellar Collapse

Main-sequence star



Hydrogen Burning

Helium-burning star



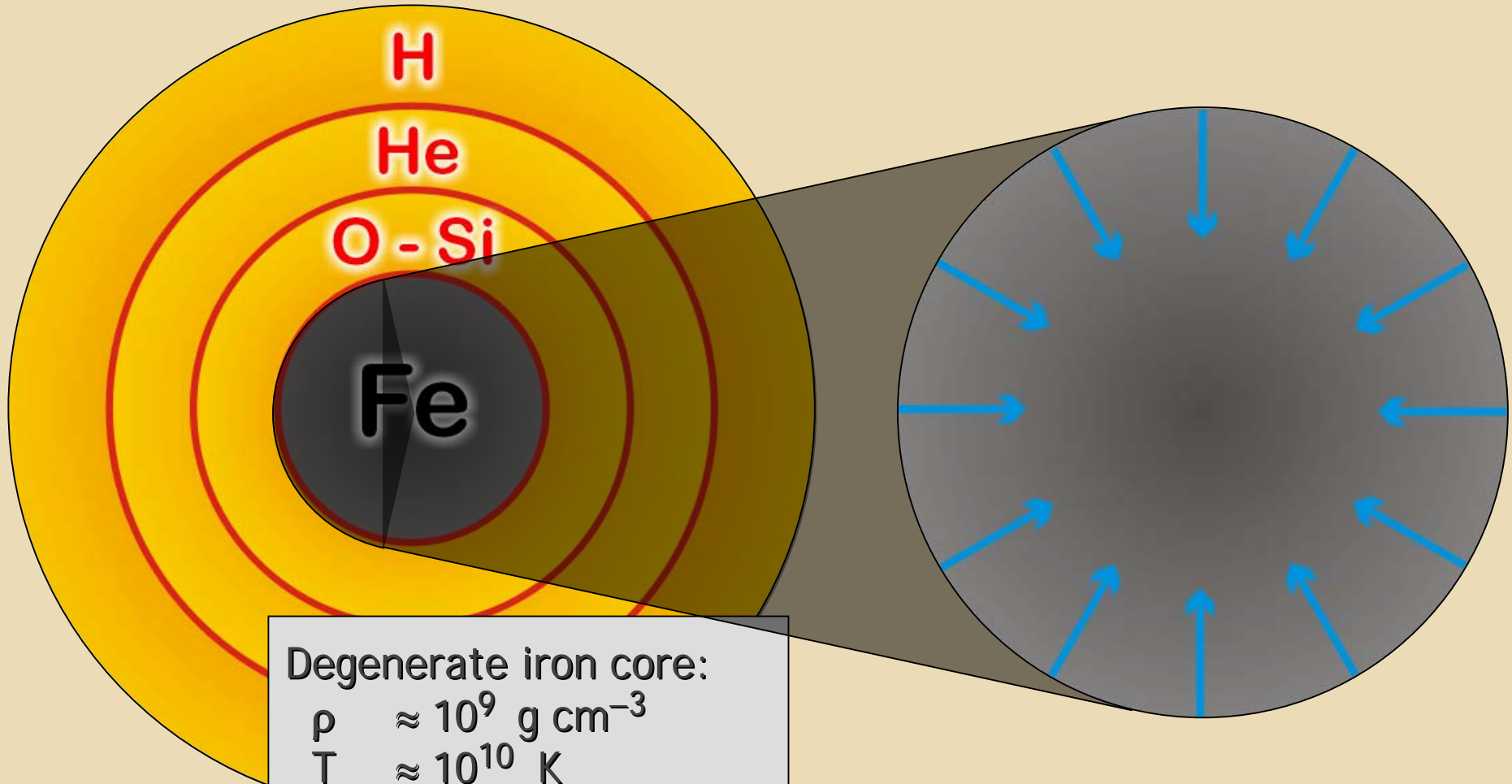
Helium  
Burning

Hydrogen  
Burning

# Stellar Collapse

Onion structure

Collapse (implosion)



Degenerate iron core:

$$\rho \approx 10^9 \text{ g cm}^{-3}$$

$$T \approx 10^{10} \text{ K}$$

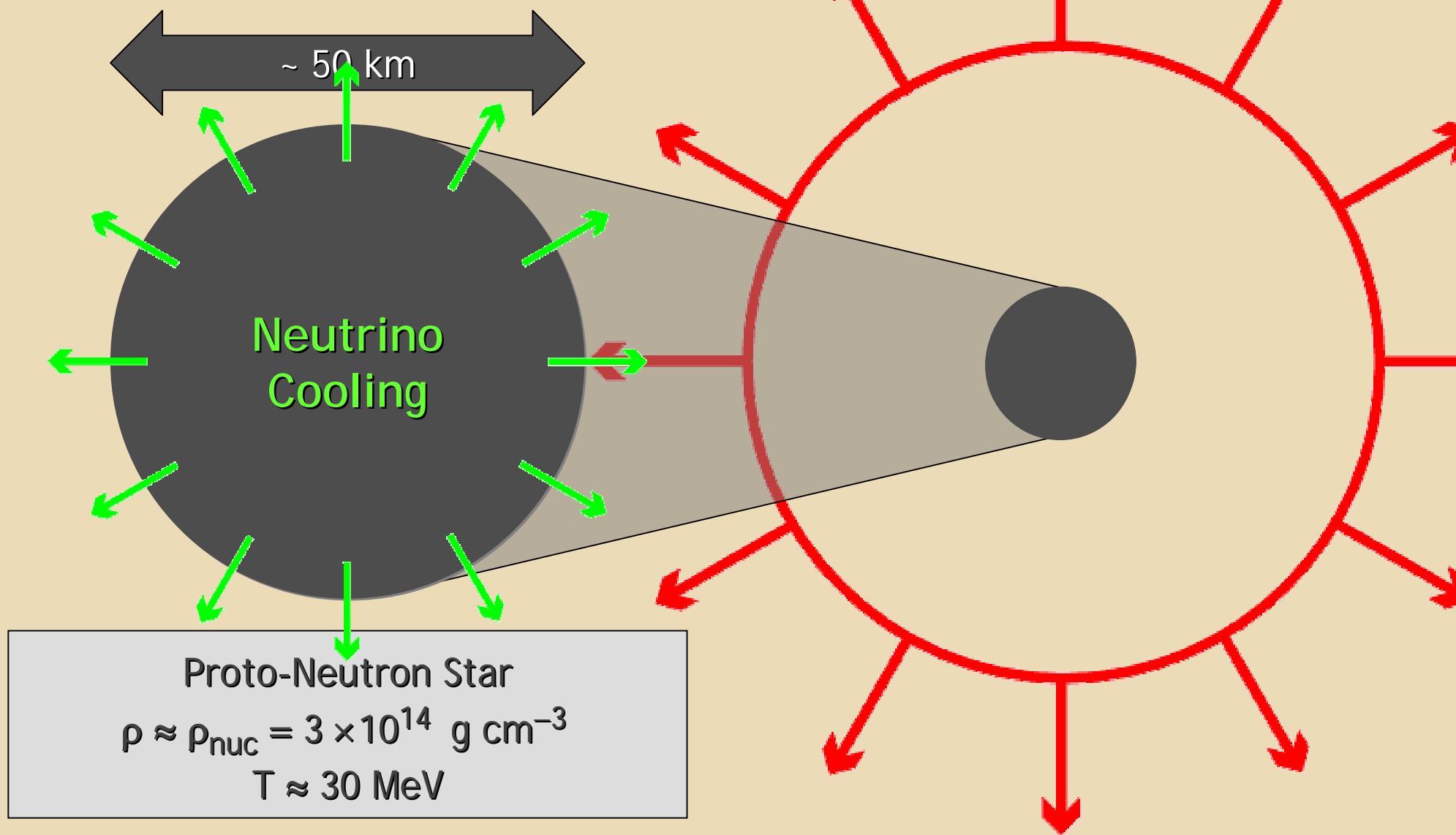
$$M_{\text{Fe}} \approx 1.5 M_{\text{sun}}$$

$$R_{\text{Fe}} \approx 8000 \text{ km}$$

# Stellar Collapse and Supernova Explosion

Newborn Neutron Star

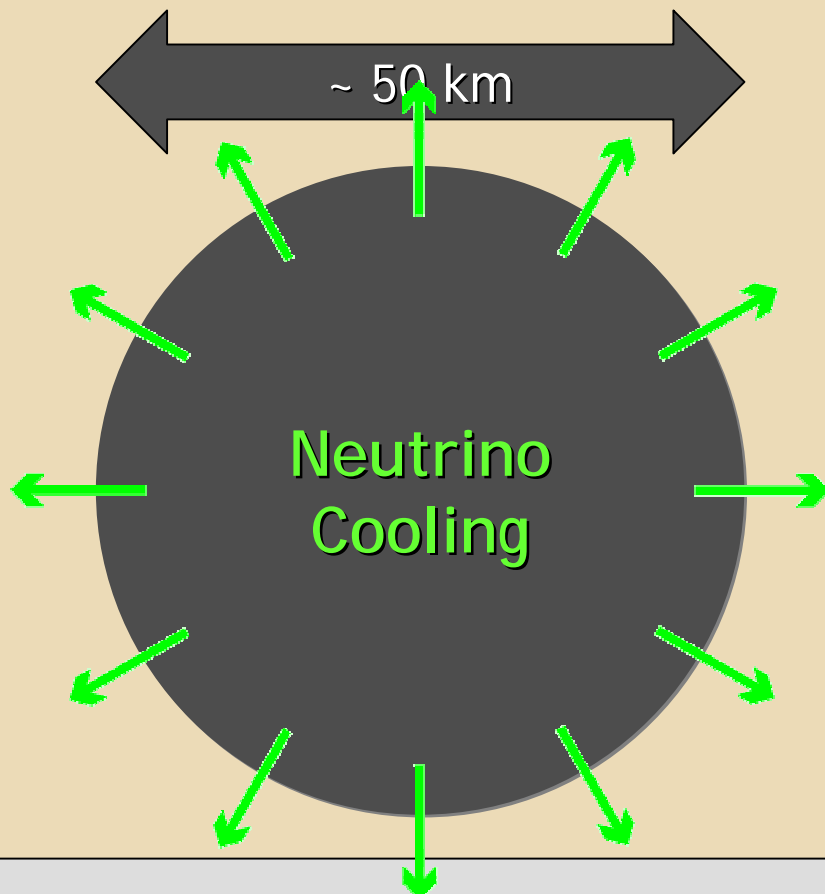
Explosion





# Stellar Collapse and Supernova Explosion

## Newborn Neutron Star



Proto-Neutron Star  
 $\rho \approx \rho_{\text{nuc}} = 3 \times 10^{14} \text{ g cm}^{-3}$   
 $T \approx 30 \text{ MeV}$

Gravitational binding energy

$$E_b \approx 3 \times 10^{53} \text{ erg} \approx 17\% M_{\text{SUN}} c^2$$

This shows up as

99% Neutrinos

1% Kinetic energy of explosion  
(1% of this into cosmic rays)

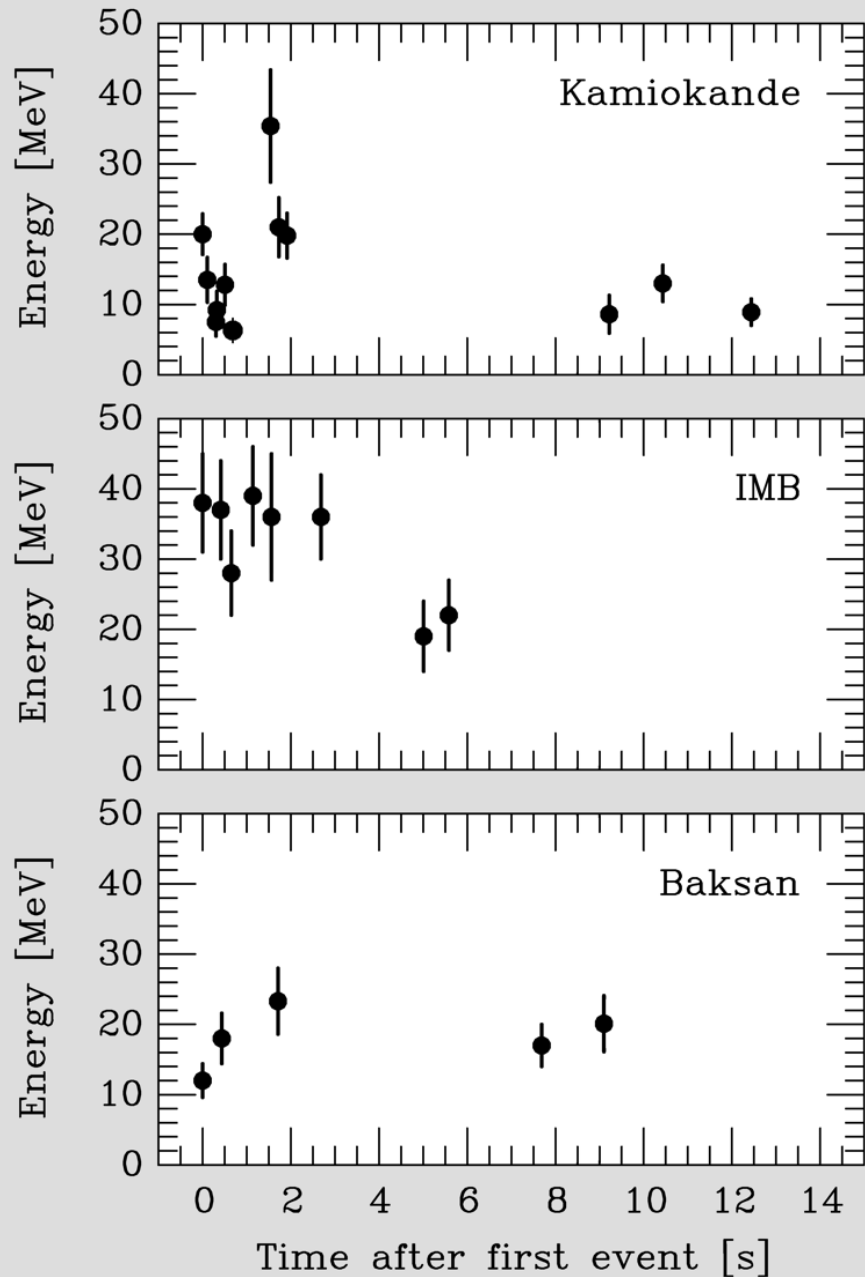
0.01% Photons, outshine host galaxy

Neutrino luminosity

$$L_\nu \approx 3 \times 10^{53} \text{ erg} / 3 \text{ sec}$$
$$\approx 3 \times 10^{19} L_{\text{SUN}}$$

While it lasts, outshines the entire visible universe

# Neutrino Signal of Supernova 1987A



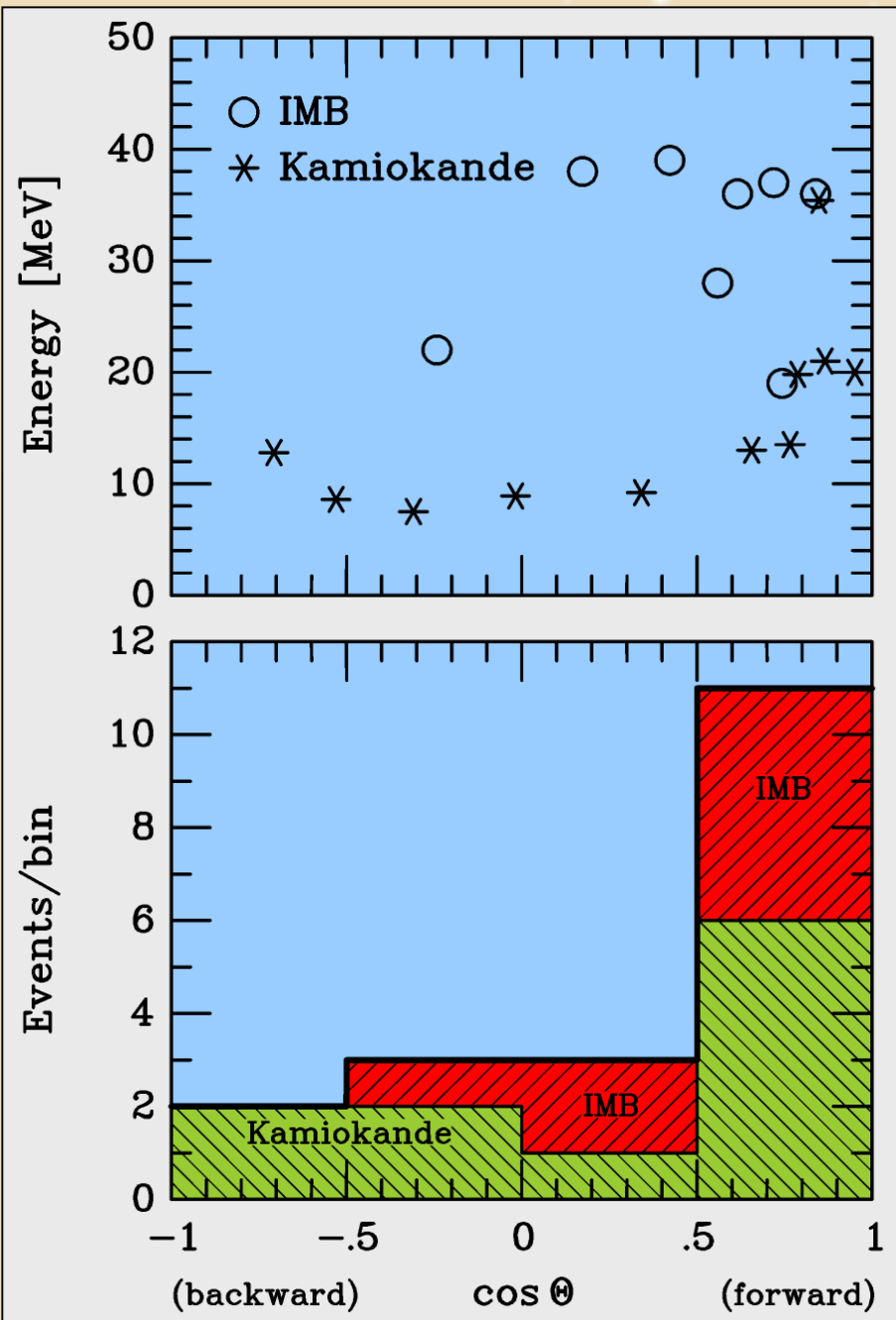
Kamiokande (Japan)  
Water Cherenkov detector  
Clock uncertainty  $\pm 1$  min

Irvine-Michigan-Brookhaven (US)  
Water Cherenkov detector  
Clock uncertainty  $\pm 50$  ms

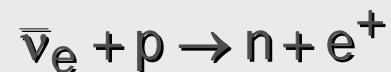
Baksan Scintillator Telescope  
(Soviet Union)  
Clock uncertainty  $+2/-54$  s

Within clock uncertainties,  
signals are contemporaneous

# Angular Distribution of SN 1987A Neutrinos



## Main detection reaction



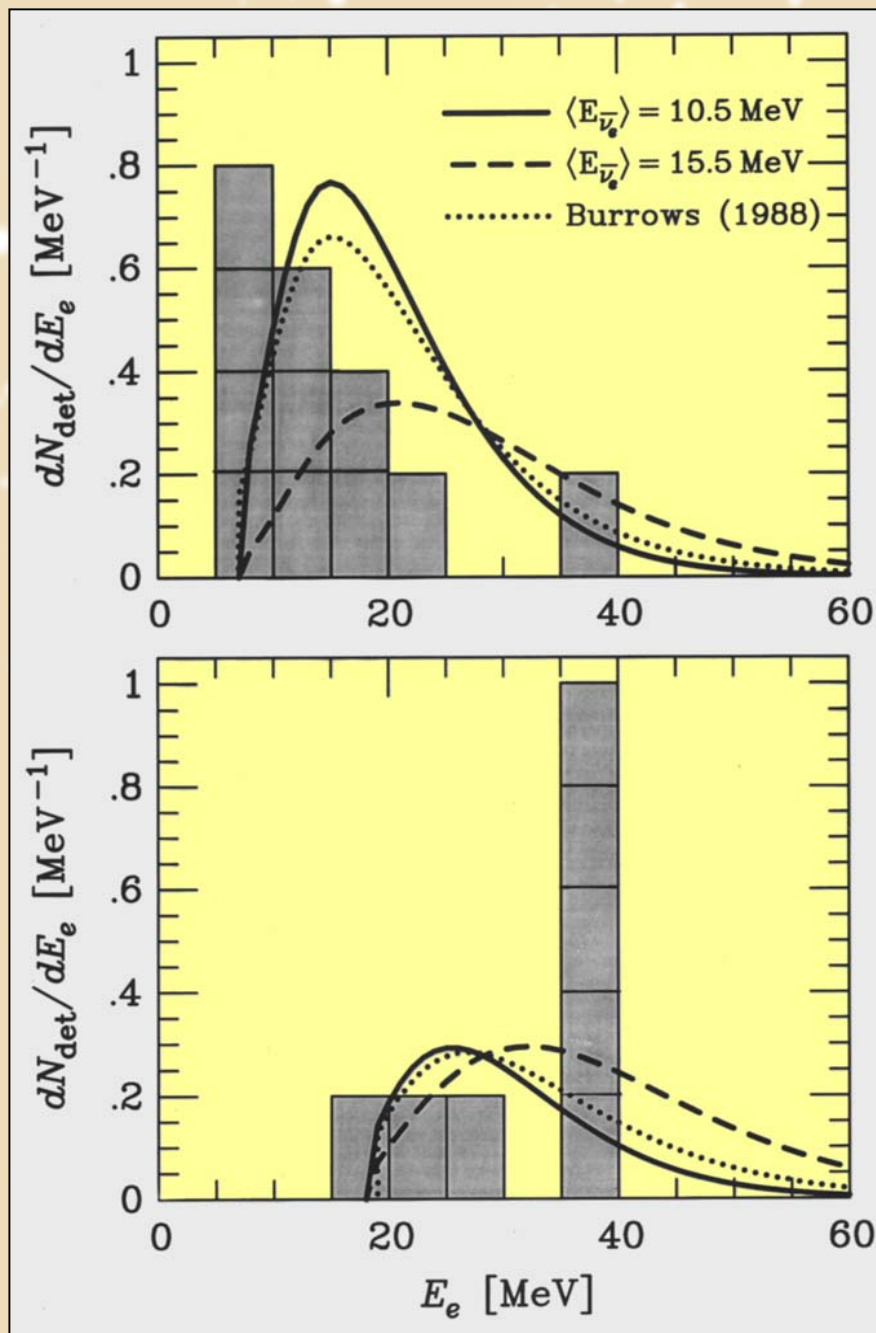
is essentially isotropic for the relevant energies.

Expect only a fraction of an event from forward-peaked reaction



Observed signal compatible with isotropy only at approx. 0.1% CL, but no alternative known

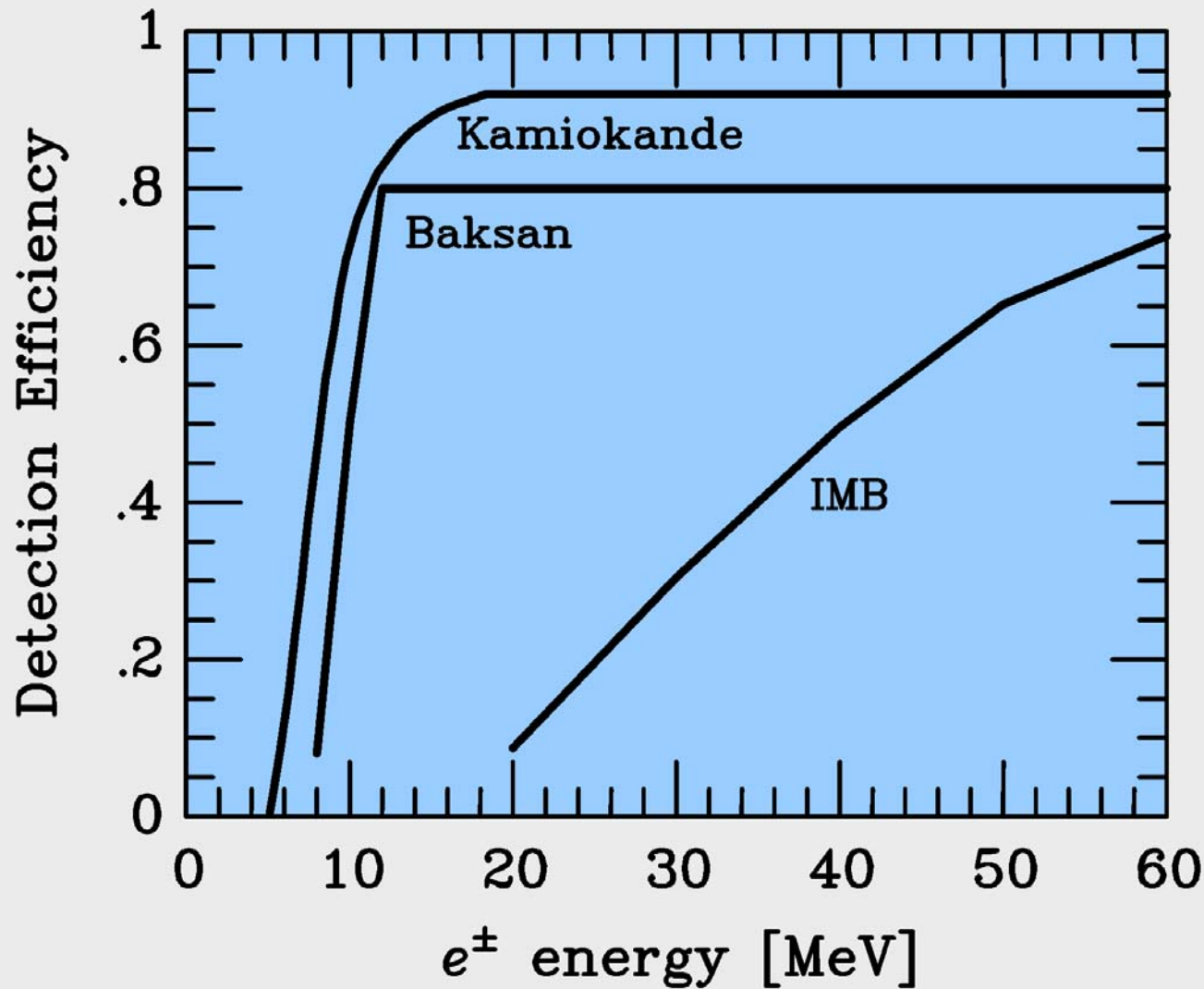
# Energy Distribution of SN 1987A Neutrinos



Kamiokande II

IMB

# Trigger Efficiencies at the Detectors



Fiducial volumes for  
SN 1987A detection

Kamiokande II  
2140 tons water  
( $1.43 \times 10^{32}$  protons)

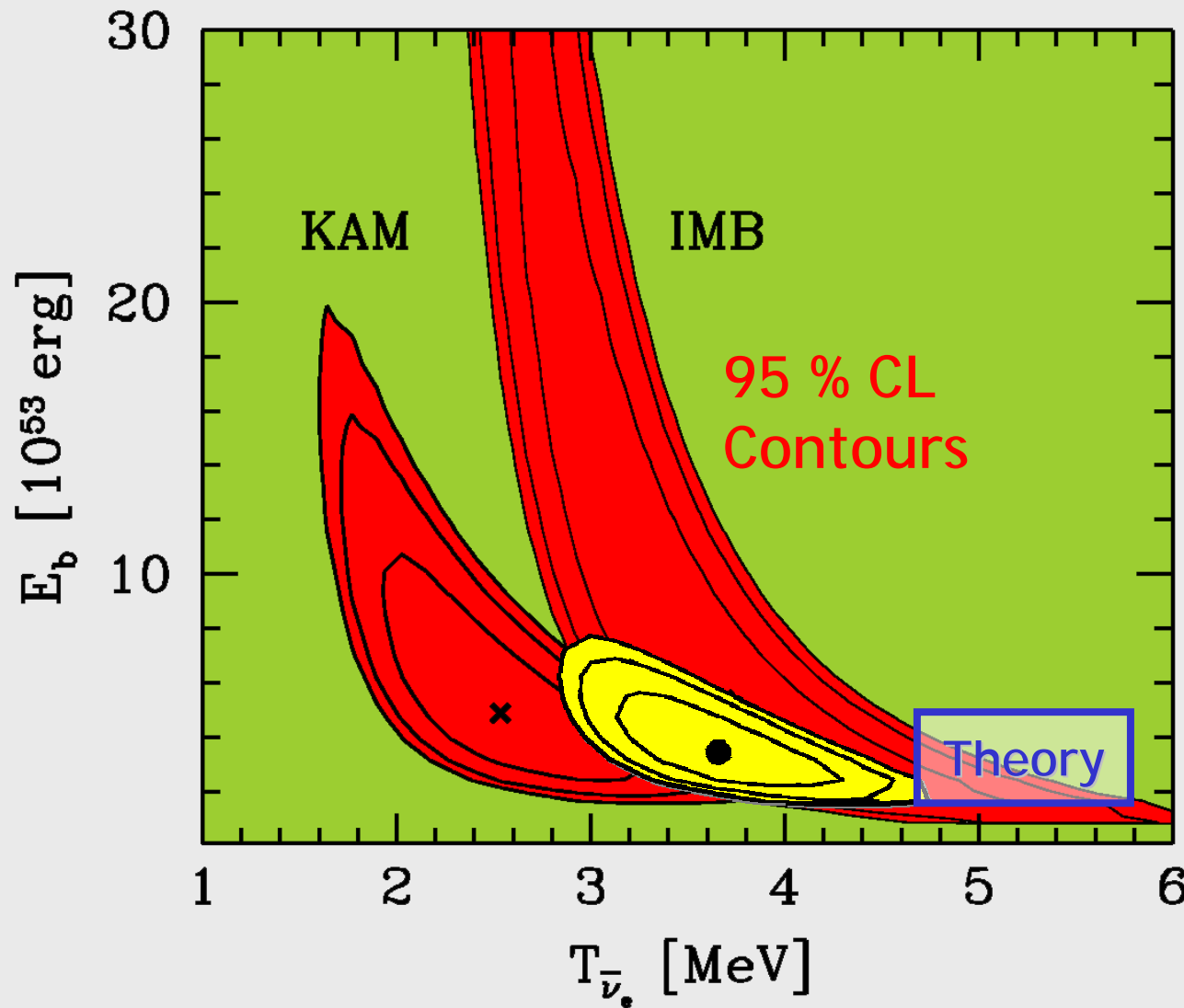
IMB  
6800 tons water  
( $4.6 \times 10^{32}$  protons)

BST  
200 tons scintillator  
( $1.88 \times 10^{31}$  protons)



# Interpreting SN 1987A Neutrinos

Total Binding Energy

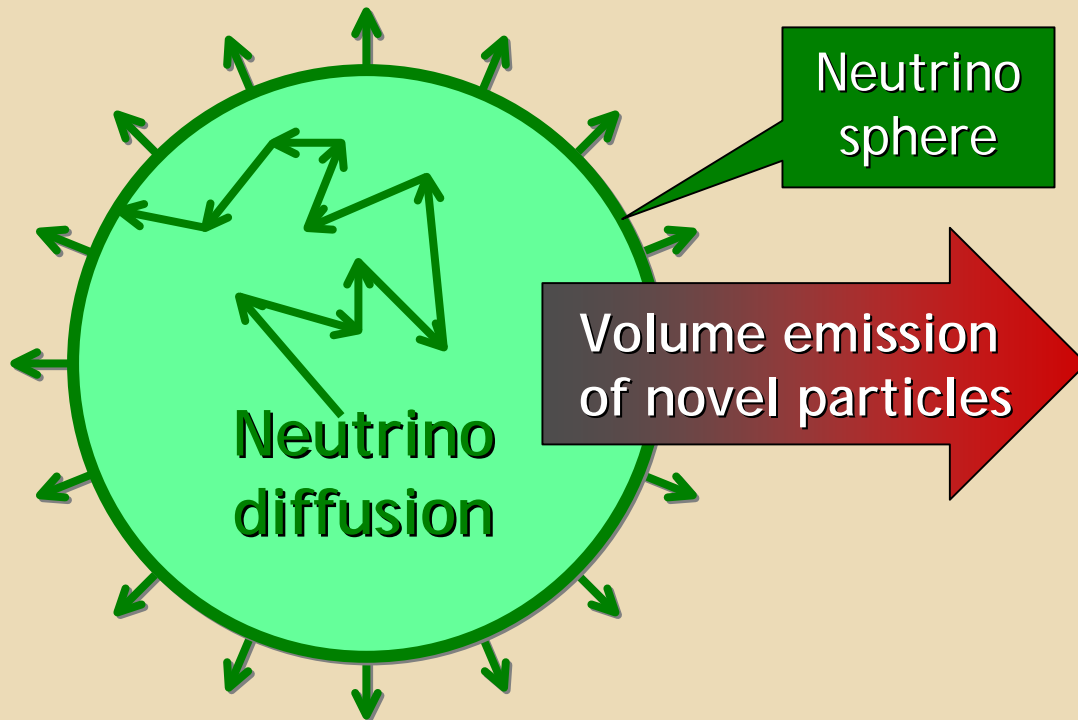


Jegerlehner,  
Neubig & Raffelt,  
PRD 54 (1996) 1194

Assume thermal  
spectra and  
equipartition of  
energy between  
the six degrees  
of freedom  
 $\nu_e, \nu_\mu, \nu_\tau$  and their  
antiparticles

Spectral  $\bar{\nu}_e$  Temperature

# The Energy-Loss Argument

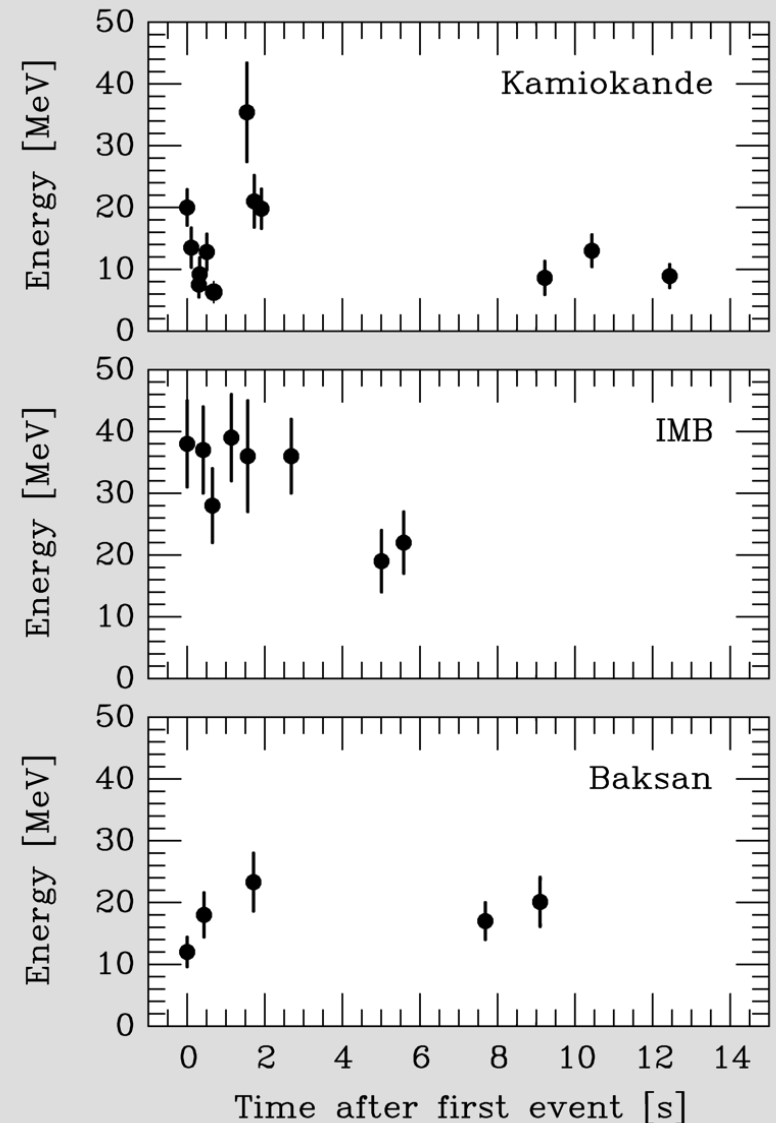


Assuming that the neutrino burst was not shortened by more than  $\sim 1/2$  leads to an approximate requirement on a novel energy-loss rate of

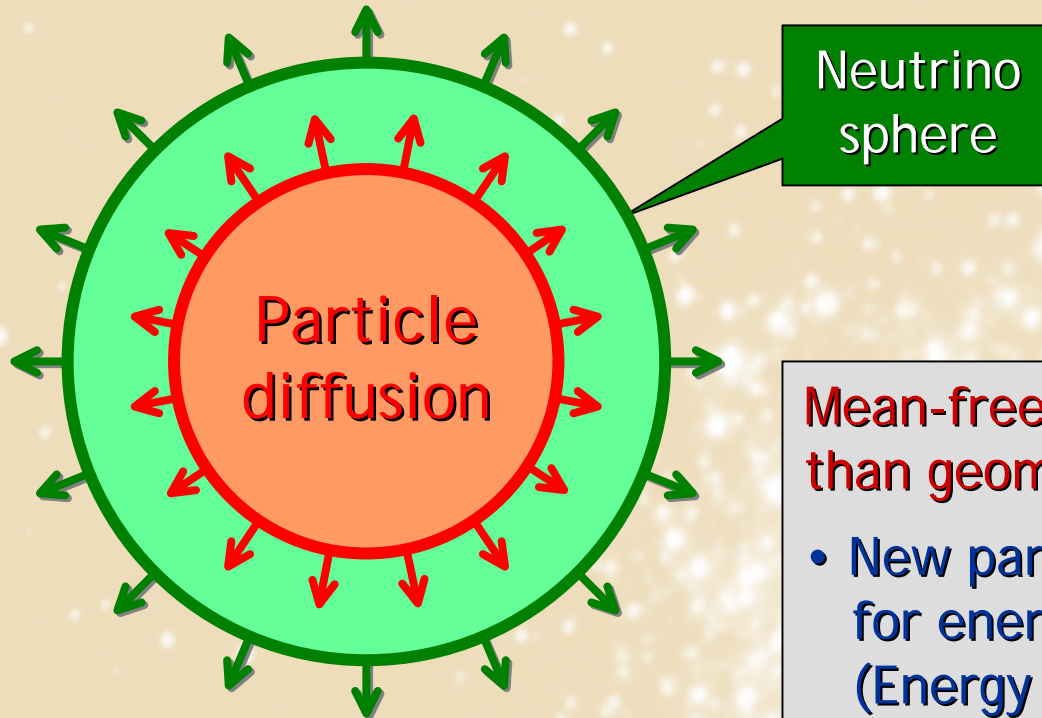
$$\epsilon_x < 10^{19} \text{ erg g}^{-1} \text{ s}^{-1}$$

for  $\rho \approx 3 \times 10^{14} \text{ g cm}^{-3}$  and  $T \approx 30 \text{ MeV}$

## SN 1987A neutrino signal



# The Energy-Loss Argument in the Trapping Limit



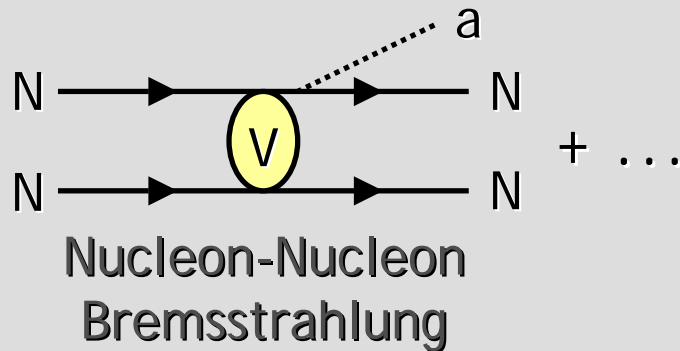
Mean-free-path of new particles less than geometric dimension of star

- New particles are more important for energy transfer than neutrinos (Energy transfer  $\propto$  mfp)
- Efficiency of energy transfer must be less than that of neutrinos or else speed up cooling of PNS, again shortening the observed SN 1987A signal

# Axion Emission from a Nuclear Medium

Axion-nucleon interaction is of current-current form:

$$L_{\text{int}} = \frac{C_N}{2f_a} \bar{\Psi}_N \gamma_\mu \gamma_5 \Psi_N \partial^\mu a = \frac{C_N}{2f_a} J_\mu^A \partial^\mu a$$



Energy loss rate (erg cm<sup>-3</sup> s<sup>-1</sup>)

$$Q = \int d\Gamma_a \int d\Gamma_{\text{Nucleons}} |M|^2 \omega$$

$$= \left( \frac{C_N}{2f_a} \right)^2 \frac{n_B}{4\pi^2} \int_0^\infty d\omega \omega^4 S(-\omega)$$

Axion energy

Dynamical structure function

Difficulties include:

- Realistic nucleon-nucleon interaction potential (even in vacuum)
- Many-body effects (effective mass, spin-spin correlations ...)
- Axion couplings in the nuclear medium
- Multiple-scattering effects:

Frequency of NN collisions exceeds typical axion energy

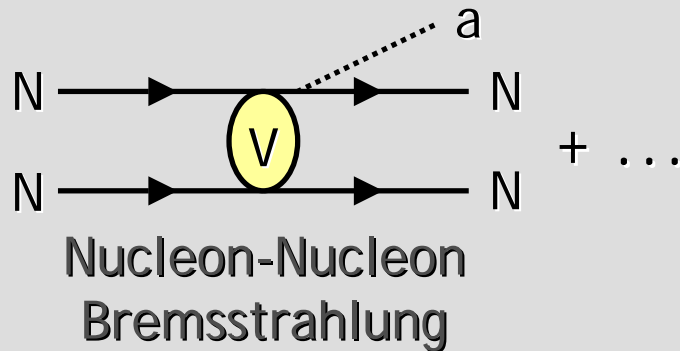
$$\tau_{\text{coll}} < \omega^{-1}$$

Expect LPM-type destructive interference effects

# Axion Emission from a Nuclear Medium

Axion-nucleon interaction is of current-current form:

$$L_{\text{int}} = \frac{C_N}{2f_a} \bar{\Psi}_N \gamma_\mu \gamma_5 \Psi_N \partial^\mu a = \frac{C_N}{2f_a} J_\mu^A \partial^\mu a$$



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Axion energy

Dynamical structure function

Diffic

• Rea

• Mar

• Axio

• Mul

Frec

$\tau_{\text{col}}$

Exp

- Fundamentally the dynamical structure function is a correlator of the nucleon axial current
- Non-relativistic nucleons: ~ nucleon spin density operator  $\sigma$

$$S(\omega, k) = \frac{4}{3n_B} \int_{-\infty}^{+\infty} dt e^{i\omega t} \langle \vec{\sigma}(t, k) \cdot \vec{\sigma}(0, -k) \rangle$$

- Example for the fluctuation and dissipation theorem of linear-response theory: Axion emission determined by spontaneous nucleon spin fluctuations



# Properties of the Dynamical Structure Function

Nucleon spin-density autocorrelation function

$$S(\omega, k) = \frac{4}{3n_B} \int_{-\infty}^{+\infty} dt e^{i\omega t} \langle \sigma(t, k) \cdot \sigma(0, -k) \rangle$$

Normalization, ignoring many-body correlations

$$\int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} S(\omega, k) = \frac{1}{n_B} \int \frac{2d^3p}{(2\pi)^3} f_p (1 - f_{p+k})$$

Detailed balancing

$$S(-\omega, k) = e^{-\omega/T} S(\omega, k)$$

consequence of non-commuting  $\sigma(t)$  at different times

Symmetric form

$$\bar{S}(\omega, k) = \frac{S(-\omega, k) + S(\omega, k)}{2} \rightarrow S(\omega, k) = \frac{2\bar{S}(\omega, k)}{1 + e^{-\omega/T}}$$

Long-wavelength limit ( $k \rightarrow 0$ )

$$\bar{S}(\omega) = \frac{4}{3} \int_{-\infty}^{+\infty} dt e^{i\omega t} \left\langle \frac{s(t) \cdot s(0) + s(0) \cdot s(t)}{2} \right\rangle$$

Is Fourier transform of single-nucleon spin correlation function

$$\bar{R}(t) = \frac{4}{3} \left\langle \frac{s(t) \cdot s(0) + s(0) \cdot s(t)}{2} \right\rangle$$

# Spin Relaxation Rate

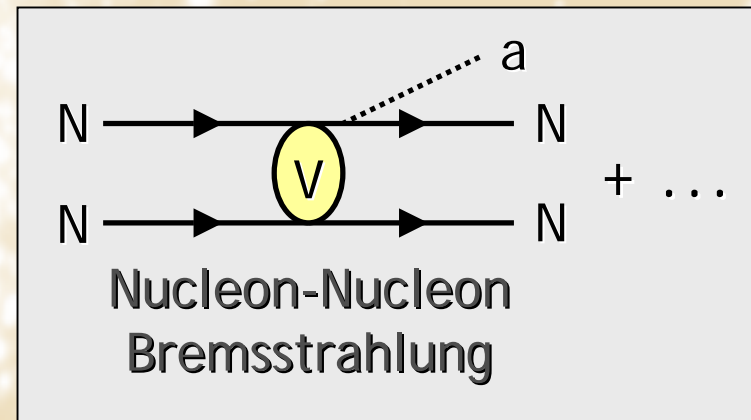
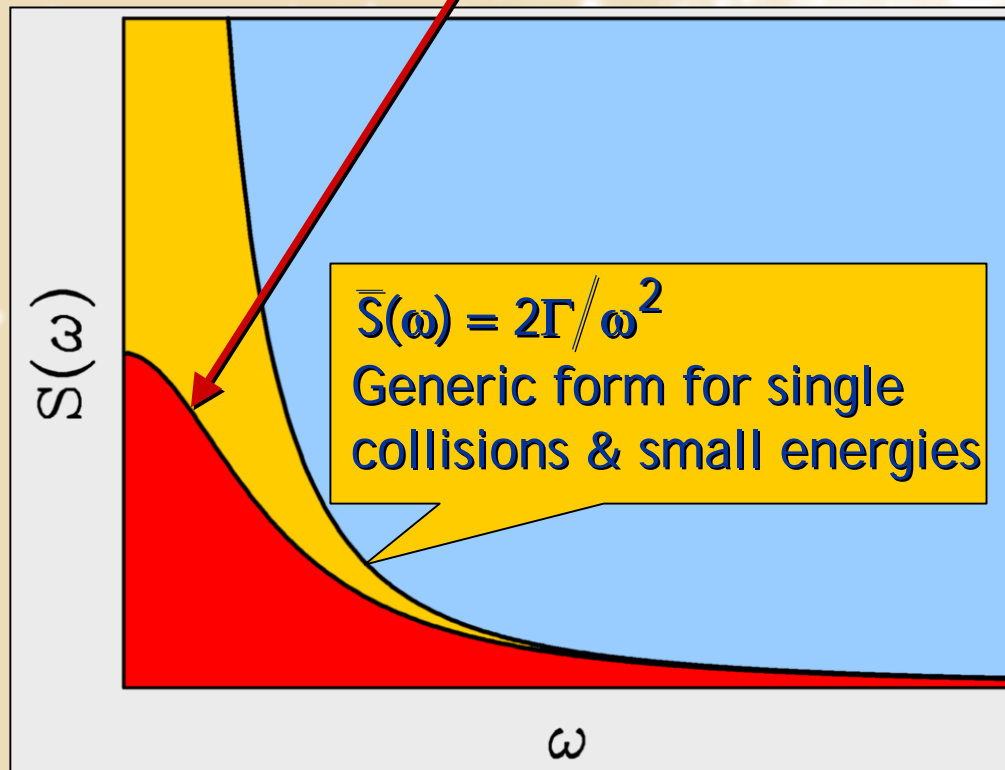
A spin immersed in a bath of scatterers with spin-dependent forces relaxes exponentially for uncorrelated kicks (Markov chain)

$$\bar{R}(t) = e^{-\Gamma t}$$

with  $\Gamma$  the "spin relaxation rate", leading to the Fourier transform

$$\bar{S}(\omega) = \frac{2\Gamma}{\omega^2 + \Gamma^2}$$

Lorentzian structure function,  
includes multiple scattering effects



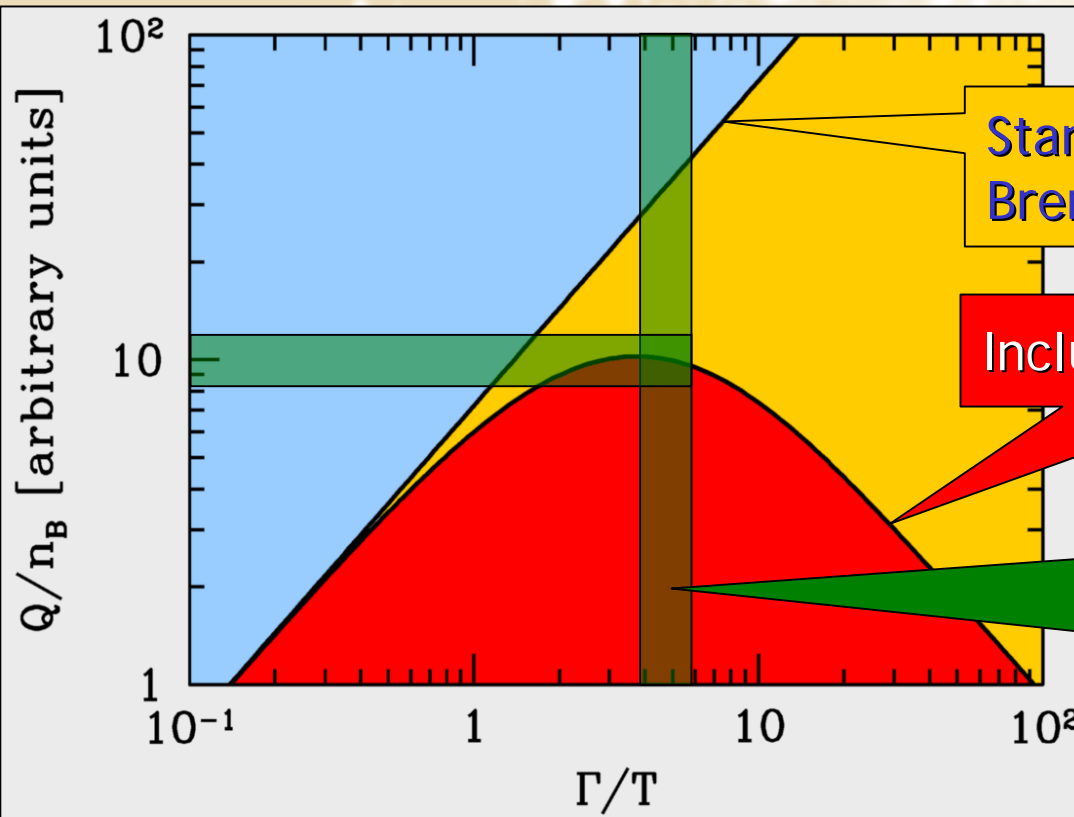
Identify coefficient  $\Gamma$  from  
bremsstrahlung calculation  
with spin relaxation rate

# Axion Emission Rate

$$Q = \left( \frac{C_N}{2f_a} \right)^2 \frac{n_B}{4\pi^2} \int_0^\infty d\omega \omega^4 S(-\omega)$$

Axionic volume energy loss rate of nuclear medium

$$= \left( \frac{C_N}{2f_a} \right)^2 \frac{n_B}{4\pi^2} \int_0^\infty d\omega \omega^4 \frac{2\Gamma}{\omega^2 + \Gamma^2} \frac{2}{1 + e^{\omega/T}} \propto \begin{cases} n_B^2 & \text{for small density} \\ n_B^{-1} & \text{for large density} \end{cases}$$



Standard behavior:  
Bremsstrahlung rate  $\propto \rho^2$

Including LPM effect

One-pion exchange potential in Born approximation:

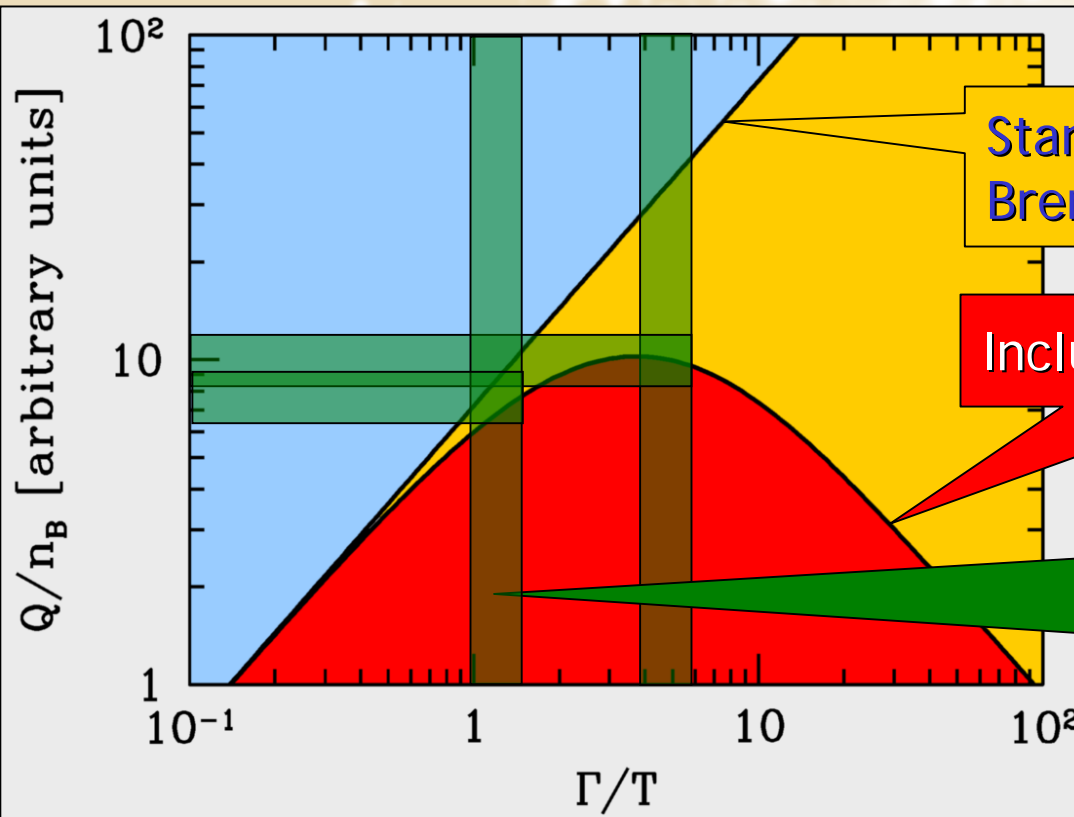
$$\frac{\Gamma}{T} \approx 1.25 \frac{\rho}{10^{14} \text{ g cm}^{-3}} \sqrt{\frac{30 \text{ MeV}}{T}}$$

# Axion Emission Rate

$$Q = \left( \frac{C_N}{2f_a} \right)^2 \frac{n_B}{4\pi^2} \int_0^\infty d\omega \omega^4 S(-\omega)$$

Axionic volume energy loss rate of nuclear medium

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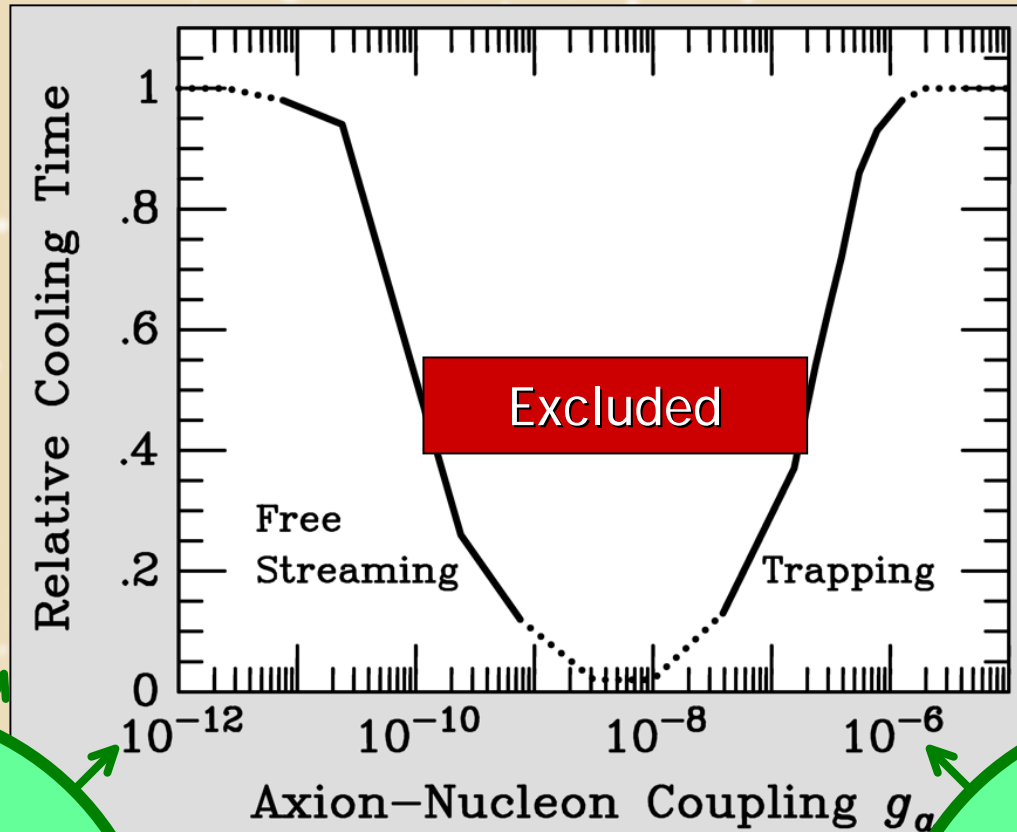
Standard behavior:  
Bremsstrahlung rate  $\propto \rho^2$

Including LPM effect

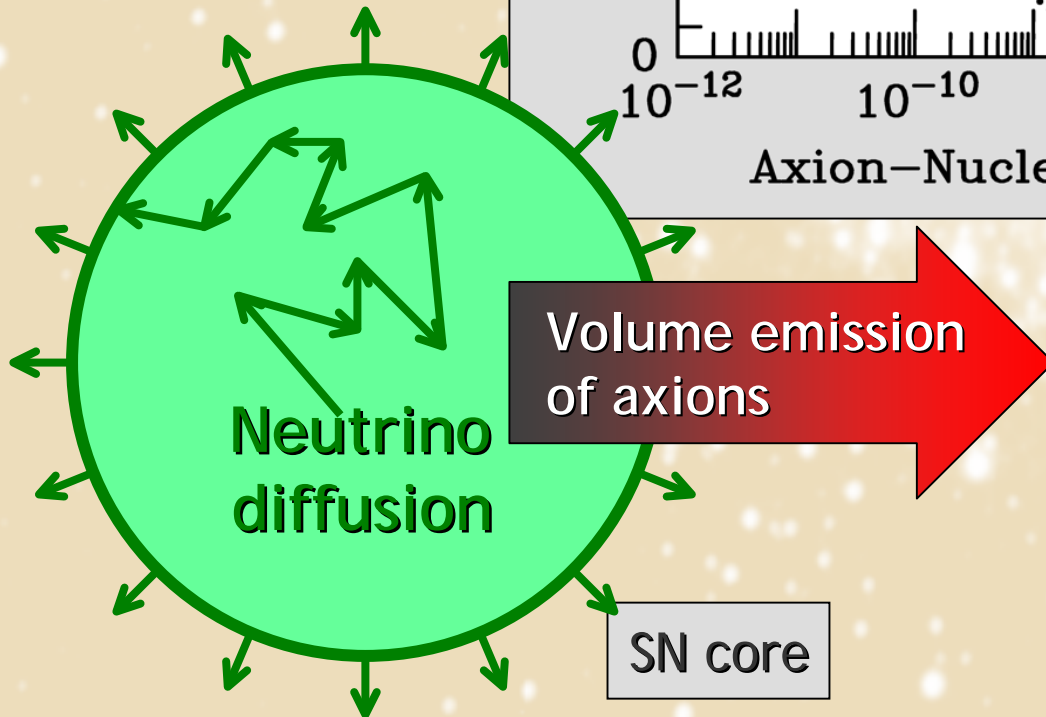
Using phase shifts from  
nuclear scattering data  
[Hanhart, Phillips & Reddy,  
astro-ph/0003445]

# SN 1987A Axion Limits

Free streaming



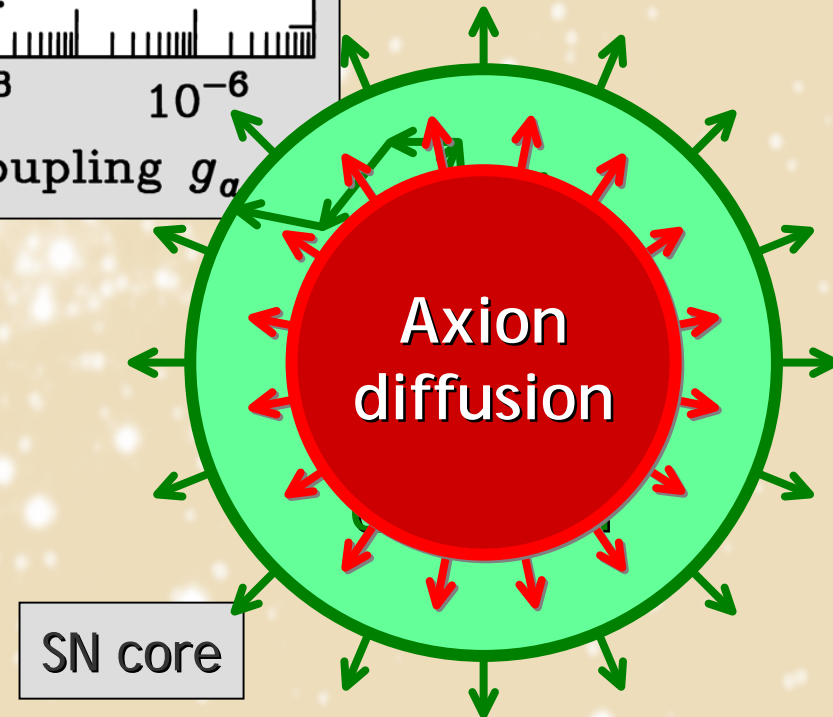
Trapping



Neutrino diffusion

Volume emission of axions

SN core



Axion diffusion

SN core

# Astrophysical Axion Bounds



Experiments

Tele  
scope

Axion dark matter possible  
(Late inflation scenario)

Globular clusters  
(a- $\gamma$ -coupling)

DM o.k.

Too much DM

(String scenario)

Too many  
events

Too much  
energy loss

Direct  
search

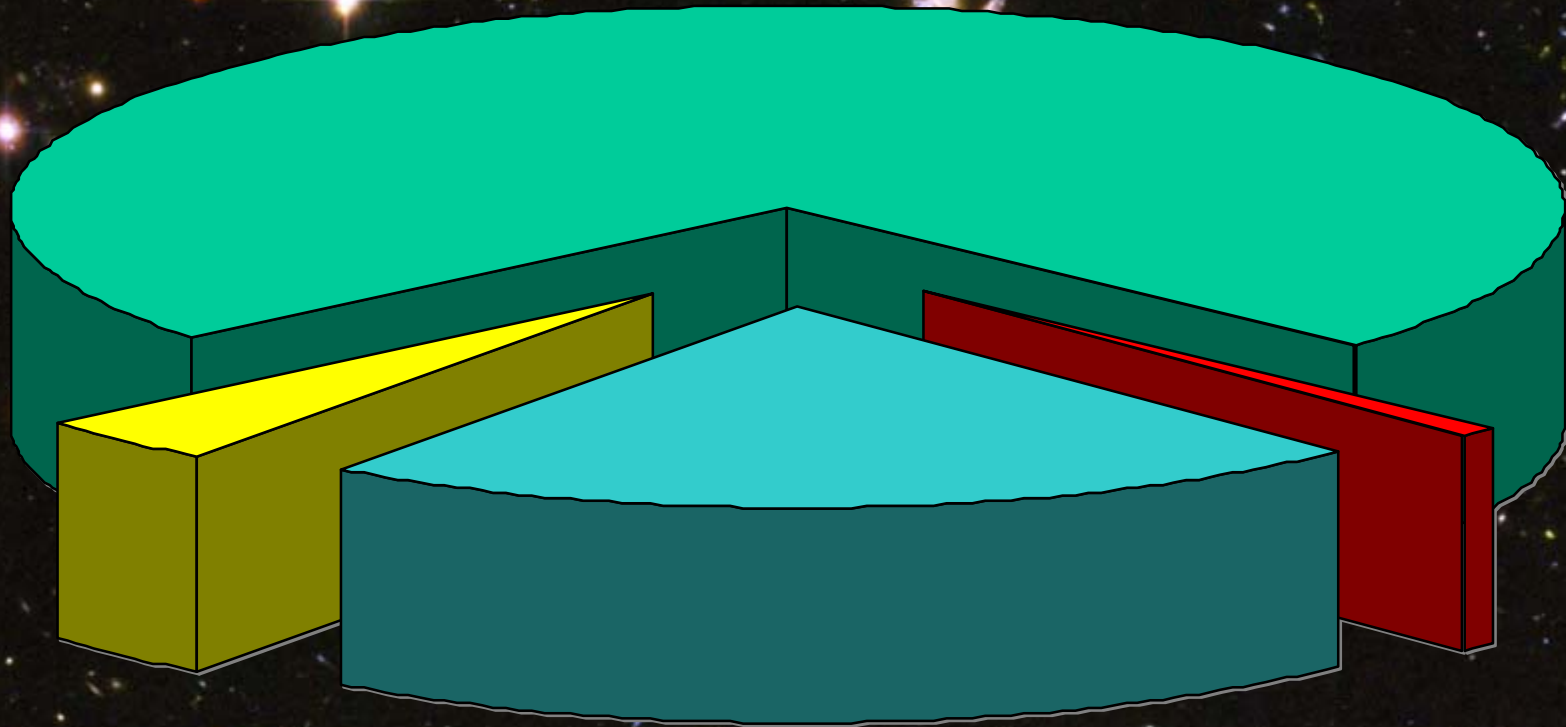
SN 1987A (a-N-coupling)





# Structure-Formation Limits on Hot Dark-Matter Axions

Dark Energy 73%  
(Cosmological Constant)



Ordinary Matter 4%  
(of this only about  
10% luminous)

Dark Matter  
23%

Neutrinos  
0.1-2%



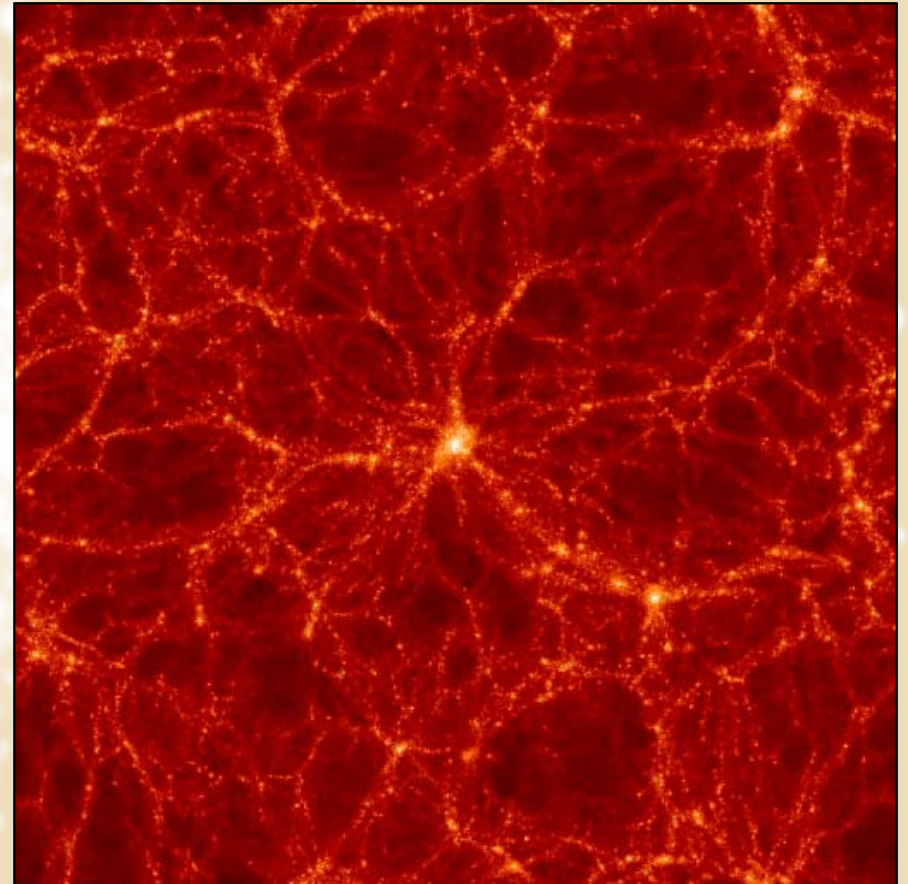
# Formation of Structure

Smooth



Structured

Structure forms by  
gravitational instability  
of primordial  
density fluctuations



# Formation of Structure

Smooth



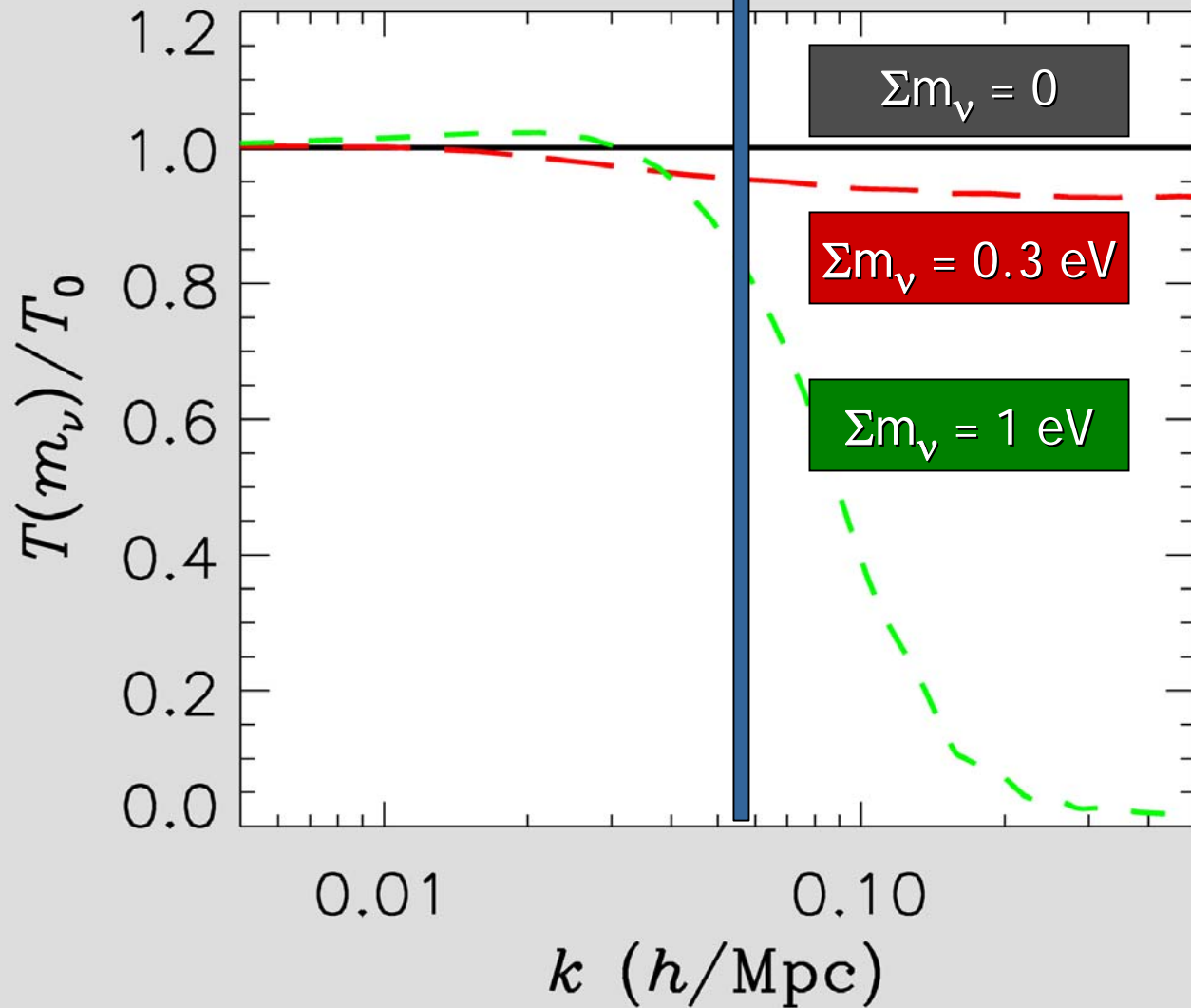
Structured

Structure forms by  
gravitational instability  
of primordial  
density fluctuations

A fraction of hot dark matter  
suppresses small-scale structure

# Neutrino Free Streaming - Transfer Function

Power suppression for  $\lambda_{FS} \lesssim 100 \text{ Mpc}/h$



Transfer function

$$P(k) = T(k) P_0(k)$$

Effect of neutrino free streaming on small scales

$$T(k) = 1 - 8\Omega_\nu/\Omega_M$$

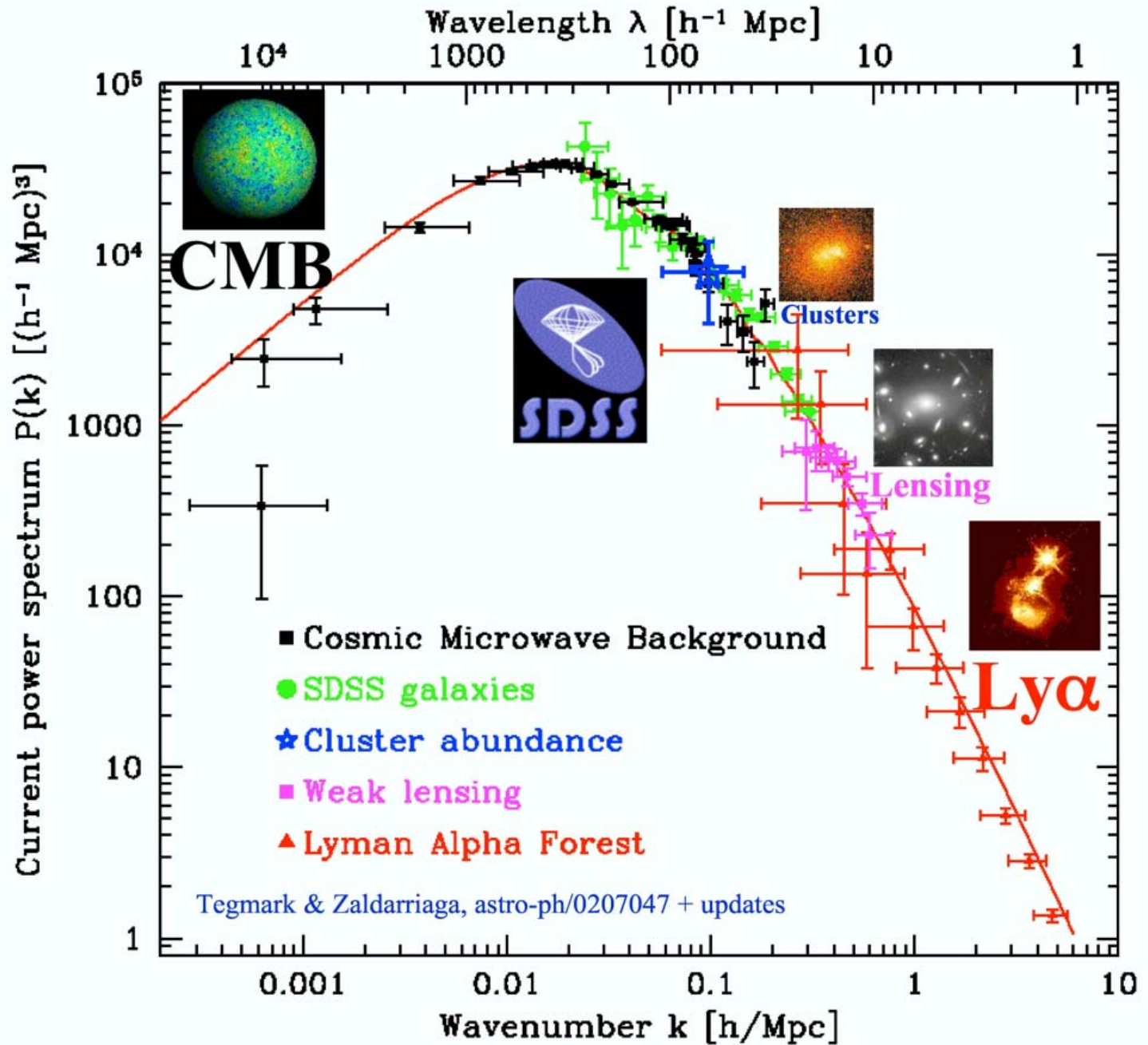
valid for

$$8\Omega_\nu/\Omega_M \ll 1$$

Hannestad, Neutrinos in Cosmology, hep-ph/0404239



# Power Spectrum of Cosmic Density Fluctuations



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TAUP 2003  
September 5, 2003



# Recent Cosmological Limits on Neutrino Masses

	$\Sigma m_\nu / \text{eV}$ (limit 95%CL)	Data / Priors
Ichikawa, Fukugita, Kawasaki 2004 [astro-ph/0409768]	2.0	WMAP
Tegmark et al. 2003 [astro-ph/0310723]	1.8	WMAP, SDSS
Hannestad 2003 [astro-ph/0303076]	1.01	WMAP, CMB, 2dF, HST
Spergel et al. (WMAP) 2003 [astro-ph/0302209]	0.69	WMAP, CMB, 2dF, HST, $\sigma_8$
Barger et al. 2003 [hep-ph/0312065]	0.75	WMAP, CMB, 2dF, SDSS, HST
Crotty et al. 2004 [hep-ph/0402049]	1.0 0.6	WMAP, CMB, 2dF, SDSS & HST, SN
Hannestad 2004 [hep-ph/0409108]	0.65	WMAP, SDSS, SN Ia gold sample, Ly- $\alpha$ data from Keck sample
Seljak et al. 2004 [astro-ph/0407372]	0.42	WMAP, SDSS, Bias, Ly- $\alpha$ data from SDSS sample

# Sensitivity Forecasts for Future LSS Observations

Lesgourgues, Pastor  
& Perotto,  
hep-ph/0403296

Planck & SDSS

$\Sigma m_\nu > 0.21$  eV detectable  
at  $2\sigma$

Ideal CMB & 40 x SDSS

$\Sigma m_\nu > 0.13$  eV detectable  
at  $2\sigma$

Abazajian & Dodelson  
astro-ph/0212216

Future weak lensing  
survey 4000 deg<sup>2</sup>

$\sigma(m_\nu) \sim 0.1$  eV

Kaplinghat, Knox & Song,  
astro-ph/0303344

CMB lensing

$\sigma(m_\nu) \sim 0.15$  eV (Planck)  
 $\sigma(m_\nu) \sim 0.044$  eV (CMBpol)

Wang, Haiman, Hu,  
Khoury & May,  
astro-ph/0505390

Weak-lensing selected  
sample of  $> 10^5$  clusters

$\sigma(m_\nu) \sim 0.03$  eV

# Extending the Mass Bound to Other Low-Mass Particles

Assume a generic hot dark matter particle that was in thermal equilibrium at some cosmological epoch

- Internal particle degrees of freedom (e.g. spin states)  $g_X$
- Mass  $m_X$
- Effective number of thermal degrees of freedom at freeze-out  $g_*$

Contribution to cosmic mass density

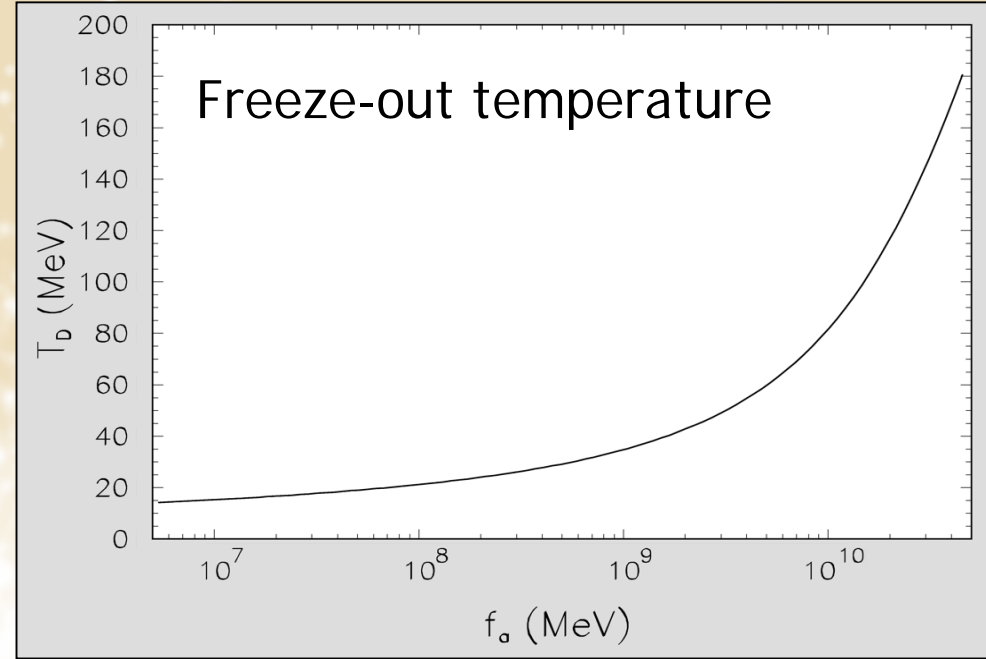
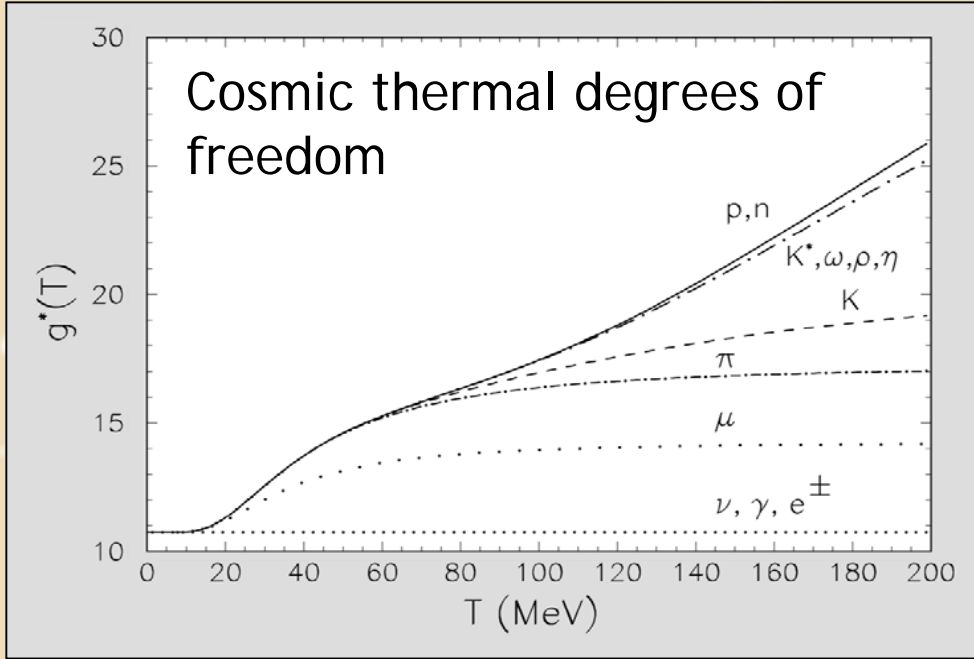
$$\Omega_X h^2 = \frac{m_X g_X}{183 \text{ eV}} \frac{10.75}{g_*} \times \begin{cases} 1 & \text{for fermions} \\ 4/3 & \text{for bosons} \end{cases}$$

Free-streaming length

$$\lambda_{\text{FS}} \approx \frac{20 \text{ Mpc}}{\Omega_X h^2} \left( \frac{T_X}{T_\nu} \right)^4 \left[ 1 + \log \left( 3.9 \frac{\Omega_X}{\Omega_m} \frac{T_\nu^2}{T_X^2} \right) \right]$$

Perform maximum likelihood analysis for different choices of  $g_X$  and  $g_*$  to derive cosmological limit on  $m_X$

# Axion Freeze-Out

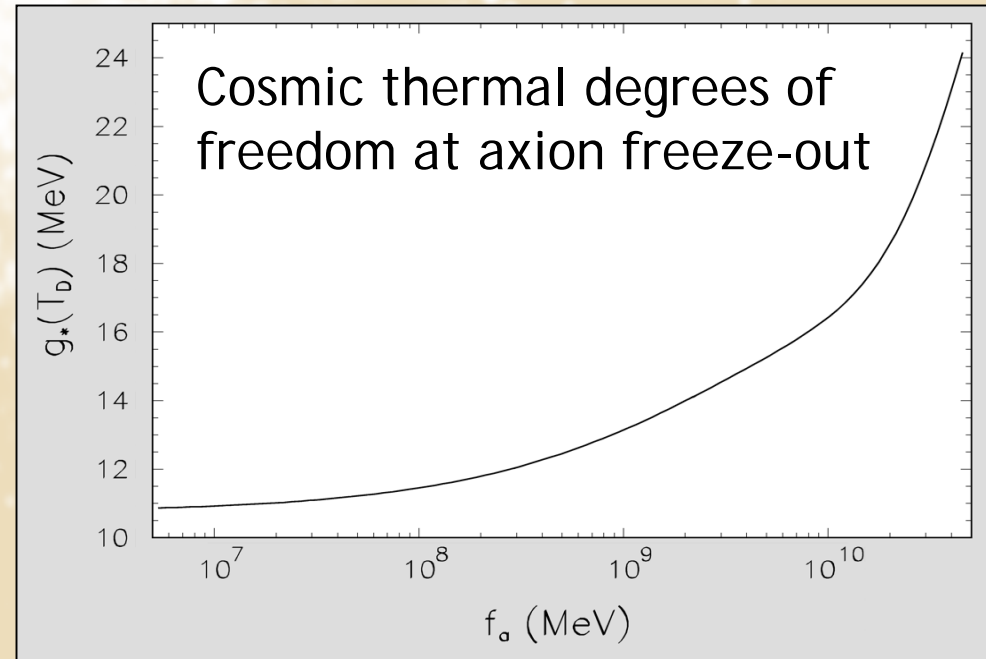


$$L_{a\pi} = \frac{C_{a\pi}}{f_a f_\pi} (\pi^0 \pi^+ \partial_\mu \pi^- + \pi^0 \pi^- \partial_\mu \pi^+ - 2\pi^+ \pi^- \partial_\mu \pi^0) \partial^\mu a$$

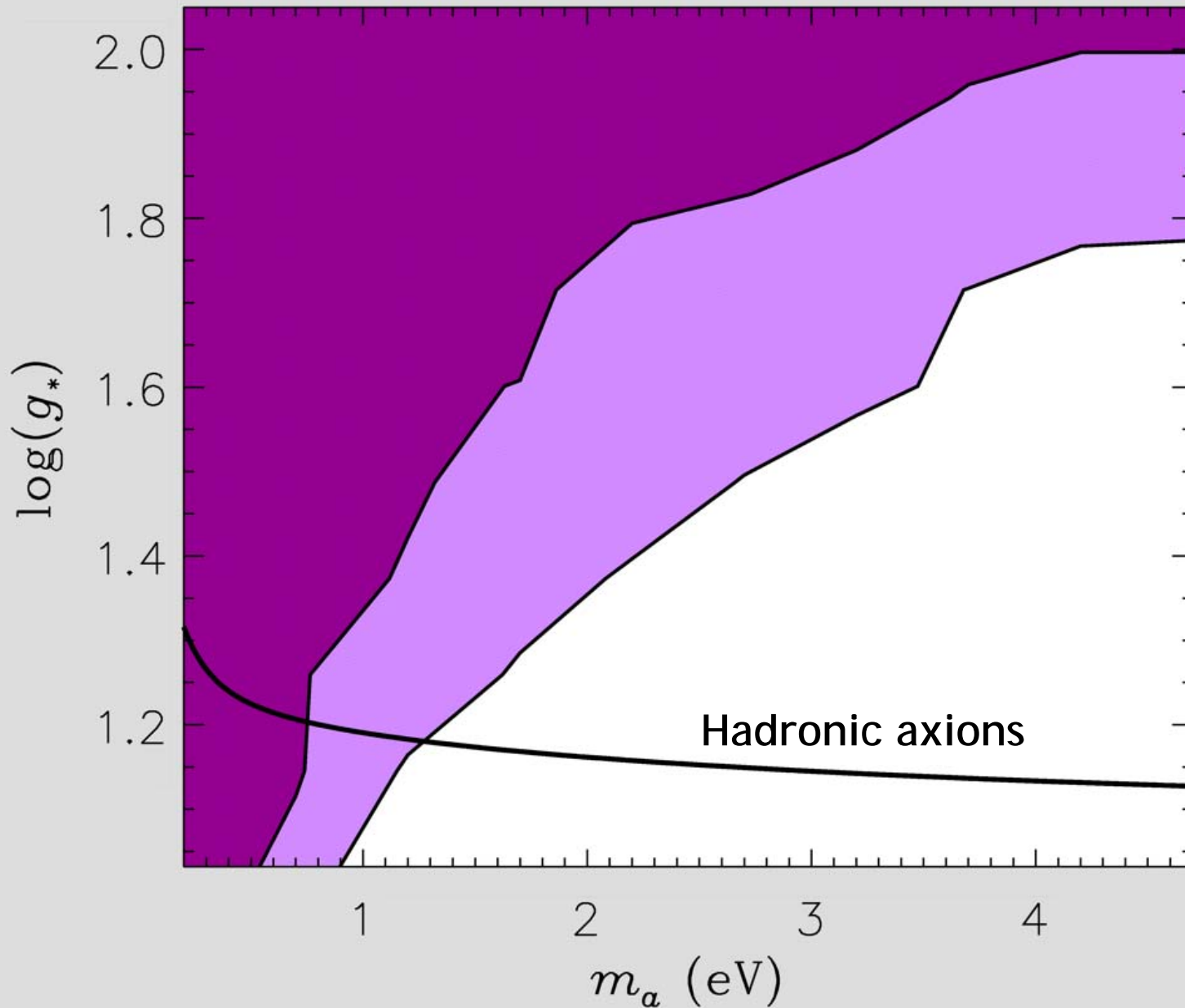
The Feynman diagram shows a vertex where two pions ( $\pi$ ) and one axion ( $a$ ) meet. The pions are represented by lines that cross each other, and the axion is represented by a line that connects to the vertex.

$$C_{a\pi} = \frac{1-z}{3(1+z)} \approx 0.094$$

Chang & Choi, PLB 316 (1993) 51



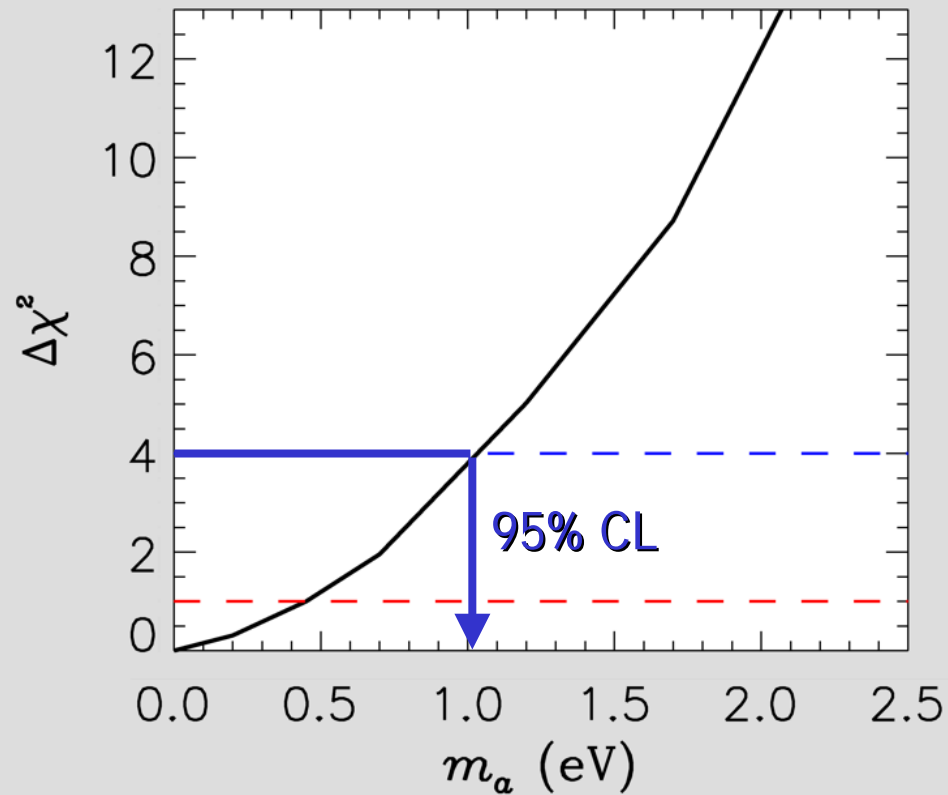
# Structure-Formation Exclusion Range for Axions



Hannestad,  
Mirizzi &  
Raffelt,  
hep-ph/0504059

# Mass Limits on Hot Dark Matter Axions and Neutrinos

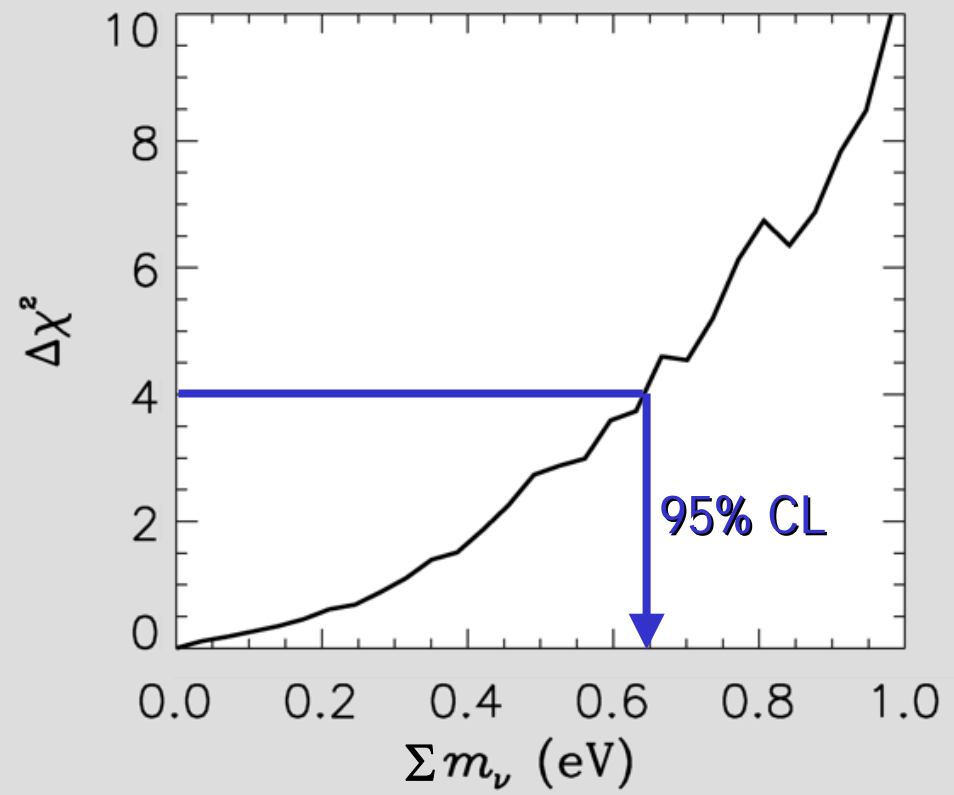
Hannestad, Mirizzi & Raffelt  
hep-ph/0504059



Axions

$$m_a < 1.05 \text{ eV (95% CL)}$$

Hannestad, astro-ph/0409108  
(Seesaw proceedings, Paris, 2004)



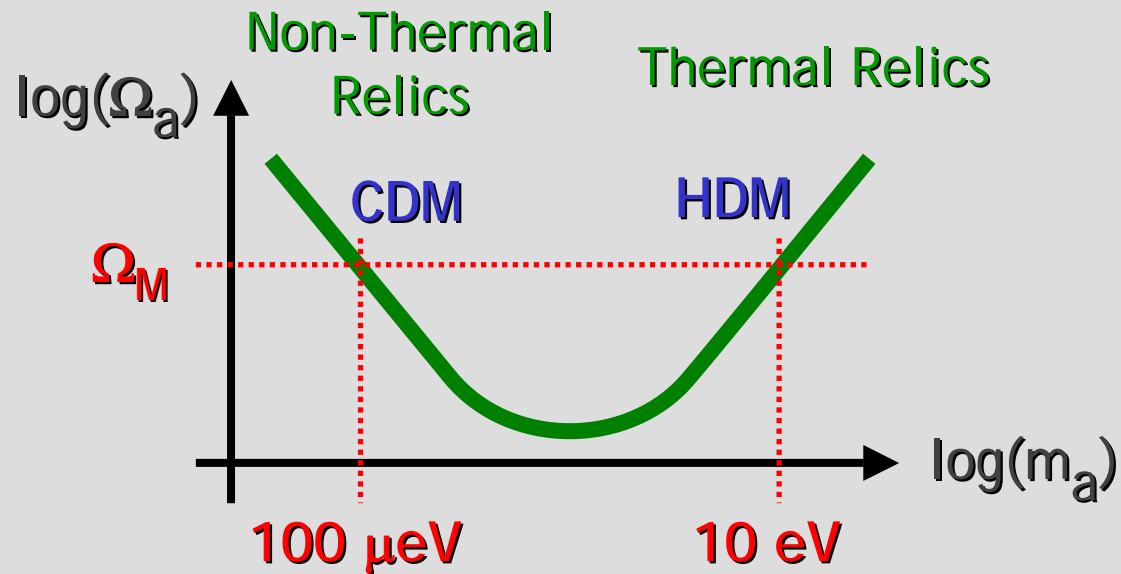
Neutrinos

$$\Sigma m_\nu < 0.65 \text{ eV (95% CL)}$$

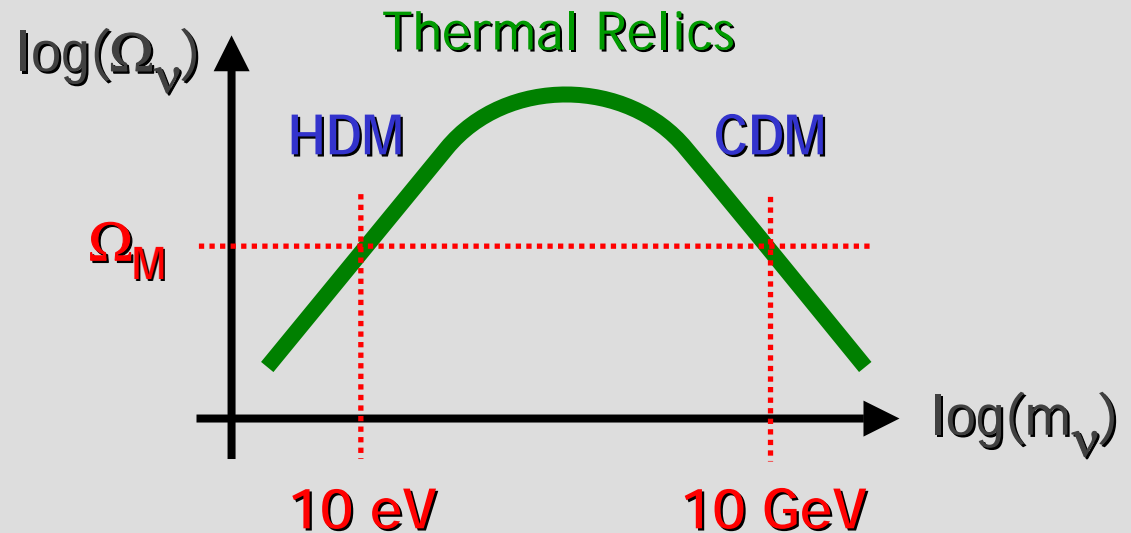


# Lee-Weinberg Curve for Neutrinos and Axions

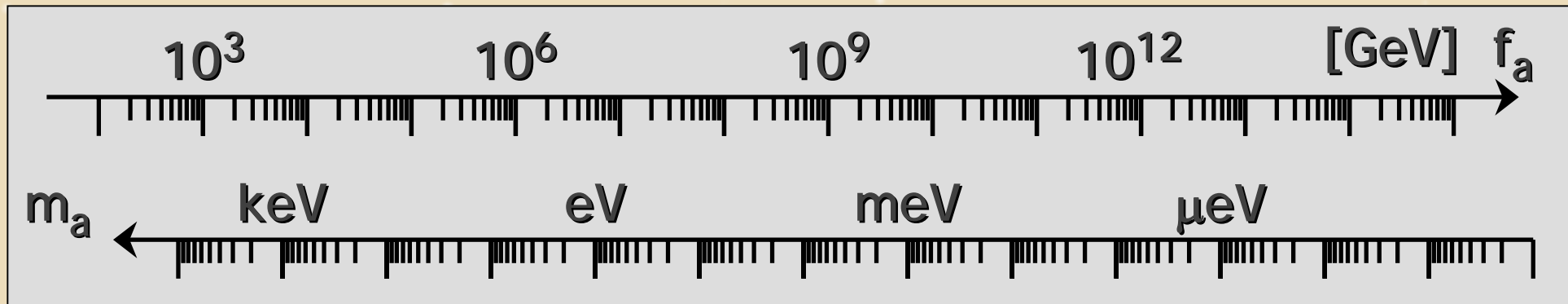
Axions



Neutrinos



# Astrophysical Axion Bounds



Experiments

Tele  
scope

Axion dark matter possible  
(Late inflation scenario)

Globular clusters  
(a- $\gamma$ -coupling)

DM o.k.

Too much DM

(String scenario)

Too many  
events

Too much  
energy loss

Direct  
search

SN 1987A (a-N-coupling)

Hot dark matter limits  
(a- $\pi$ -coupling)