Self-interactions of Higgs bosons in the CP-violating scenarios

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- Radiatively induced CP violation in the MSSM two-doublet Higgs sector appears naturally. The case of no CPV reqiures an artificial fine tuning of the λ_i phases

- Effective field theory approach
  H.Haber, R.Hempfling, Phys.Rev.D48 (1993) 4280
  Advantages
  - boundary condition at M_{SUSY} respected
  - transparent treatment of the general $\lambda_{1,...7}$ potential for construction of the mass eigenstates in the minimum
  - technically somewhat less complicated (probably) in comparison with diagrammatica
  Disadvantages
  - case of nondegenerate squark masses very difficult
  - limited to external momenta zero

- Experimental reconstruction of the Higgs potential is one of the most important problems for the LHC and NLC

Hermitian two-Higgs-doublet potential in the $-\mu^2\varphi^2 + \lambda\varphi^4$ representation

$$U(\Phi_1,\Phi_2) = \quad -\mu_1^2(\Phi_1^+\Phi_1) - \mu_2^2(\Phi_2^+\Phi_2)$$
$$-\mu_{12}^2(\Phi_1^+\Phi_2) - \overset{*}{\mu_{12}^2}(\Phi_2^+\Phi_1)$$
$$+\lambda_1(\Phi_1^+\Phi_1)^2 + \lambda_2(\Phi_2^+\Phi_2)^2 + \lambda_3(\Phi_1^+\Phi_1)(\Phi_2^+\Phi_2) + \lambda_4(\Phi_1^+\Phi_2)(\Phi_2^+\Phi_1)$$
$$+\frac{\lambda_5}{2}(\Phi_1^+\Phi_2)(\Phi_1^+\Phi_2) + \frac{\overset{*}{\lambda_5}}{2}(\Phi_2^+\Phi_1)(\Phi_2^+\Phi_1)$$
$$+\lambda_6(\Phi_1^+\Phi_1)(\Phi_1^+\Phi_2) + \overset{*}{\lambda_6}(\Phi_1^+\Phi_1)(\Phi_2^+\Phi_1)$$
$$+\lambda_7(\Phi_2^+\Phi_2)(\Phi_1^+\Phi_2) + \overset{*}{\lambda_7}(\Phi_2^+\Phi_2)(\Phi_2^+\Phi_1)$$

λ_5, λ_6 λ_7 are complex variables,

no discrete symmetry imposed.

the VEV's

$$<\varphi_1>= \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_1 \end{pmatrix} \qquad <\varphi_2>= \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_2 e^{i\zeta} \end{pmatrix}$$

Mass eigenstates h_1, h_2 h_3

($s_\alpha = \texttt{sin}\alpha$, $c_\beta = \texttt{cos}\beta$ etc.)

$$\Phi_1 = \begin{pmatrix} -i*(-H^+ s_\beta + G^+ c_\beta) \\ \frac{1}{\sqrt{2}}[v_1 + Hc_\alpha - hs_\alpha + i*(A^0 c_\beta + G^{'} s_\beta)] \end{pmatrix}$$

$$\Phi_2 = e^{i\xi} \begin{pmatrix} -i*(H^+ c_\beta + G^+ s_\beta) \\ \frac{1}{\sqrt{2}}[v_2 e^{i\zeta} + Hs_\alpha + hc_\alpha + i*(-A^0 s_\beta + G^{'} c_\beta)] \end{pmatrix}$$

h, H CP-even bosons, A CP-odd boson, $H^\pm$ charged boson, G Goldstone modes of the CP-conserving limit: $\texttt{Im}\mu_{12}^2 = 0$, $\texttt{Im}\lambda_{5,6,7}$ =0, $\xi = \zeta$ =0. With imaginary parts nonzero

$$\begin{pmatrix} h \\ H \\ A \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix}$$

In the CP conserving limit the mixing matrix a_{ij}

not only $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$, but also $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}, \ldots$

$$
\begin{aligned}
U(\Phi_1,\Phi_2) \quad = \quad & c_1 hA + c_2 HA + \\
& \frac{m_h^2}{2}h^2 + \frac{m_H^2}{2}H^2 + \frac{m_A^2}{2}A^2 + m_{H^\pm}^2 H^+H^- \\
& + \texttt{trilinear and quartic terms in } h, H, A, H^\pm
\end{aligned}
$$

where

$$
\begin{aligned}
c_1 \quad &= \quad \frac{v^2}{2}(s_\alpha s_\beta - c_\alpha c_\beta)\mathtt{Im}\bar{\lambda}_5 + v^2\,(s_\alpha c_\beta \mathtt{Im}\bar{\lambda}_6 - c_\alpha s_\beta \mathtt{Im}\bar{\lambda}_7) \\
c_2 \quad &= \quad -\frac{v^2}{2}(s_\alpha c_\beta + c_\alpha s_\beta)\mathtt{Im}\bar{\lambda}_5 - v^2\,(c_\alpha c_\beta \mathtt{Im}\bar{\lambda}_6 + s_\alpha s_\beta \mathtt{Im}\bar{\lambda}_7)
\end{aligned}
$$

To diagonalize $U(\Phi_1,\Phi_2)$ perform orthogonal rotation a_{ij} ($i,j=$1,2,3) in h, H, A space

$$
(h, H, A)\; M^2 \begin{pmatrix} h \\ H \\ A \end{pmatrix} \quad = \quad (h_1, h_2, h_3)\; a^T_{ik}\; M^2_{kl}\; a_{lj} \begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix}
$$

with the mass matrix

$$
M^2 \quad = \quad \frac{1}{2}\begin{pmatrix} m_h^2 & 0 & c_1 \\ 0 & m_H^2 & c_2 \\ c_1 & c_2 & m_A^2 \end{pmatrix}
$$

In the MSSM $\lambda_{5,6,7}$ can be calculated by means of the effective potential method and expressed through the parameters of the squark-Higgs boson sector.

At the one-loop

$$\lambda_5 = -\Delta\lambda_5 \quad = \quad -\frac{3}{96\,\pi^2}\left(h_t^4\left(\frac{\mu A_t}{M_{\rm SUSY}^2}\right)^2 + h_b^4\left(\frac{\mu A_b}{M_{\rm SUSY}^2}\right)^2\right),$$

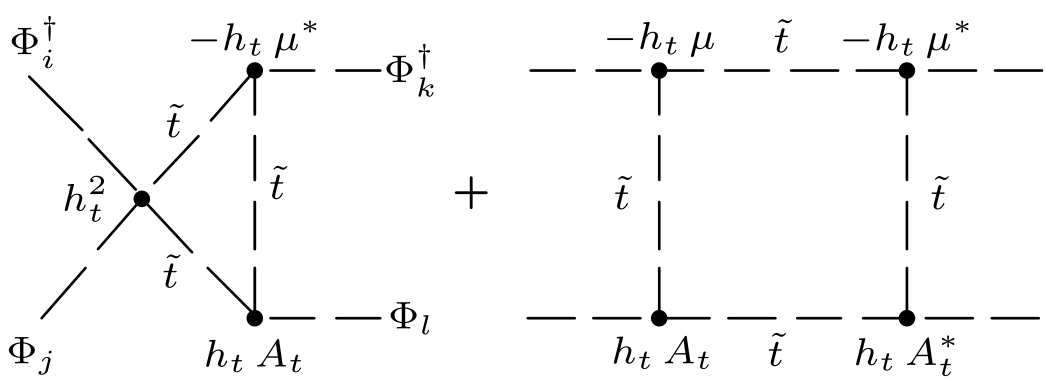
$$\lambda_6 = -\Delta\lambda_6 \quad = \frac{3}{96\,\pi^2}\left[h_t^4\,\frac{|\mu|^2\mu A_t}{M_{\rm SUSY}^4} - h_b^4\,\frac{\mu A_b}{M_{\rm SUSY}^2}\left(6 - \frac{|A_b|^2}{M_{\rm SUSY}^2}\right) + (h_b^2 A_b - h_t^2 A_t)\,\frac{3\,\mu}{M_{\rm SUSY}^2}\,\frac{g_2^2 + g_1^2}{4}\right],$$

$$\lambda_7 = -\Delta\lambda_7 \quad = \frac{3}{96\,\pi^2}\left[h_b^4\,\frac{|\mu|^2\mu A_b}{M_{\rm SUSY}^4} - h_t^4\,\frac{\mu A_t}{M_{\rm SUSY}^2}\left(6 - \frac{|A_t|^2}{M_{\rm SUSY}^2}\right) + (h_t^2 A_t - h_b^2 A_b)\,\frac{3\,\mu}{M_{\rm SUSY}^2}\,\frac{g_2^2 + g_1^2}{4}\right].$$

(Threshold corrections to the boundary condition at the scale M_{SUSY}:

$$\lambda_1^{SUSY} = \lambda_2^{SUSY} = \frac{g_2^2+g_1^2}{8},\ \lambda_3^{SUSY} = \frac{g_2^2-g_1^2}{4},$$

$$\lambda_4^{SUSY} = -\frac{g_2^2}{2}, \qquad \lambda_{5,6,7}^{SUSY} = 0. \qquad)$$



CPX scenario (M.Carena et.al., PL B495 (2000) 155)

Defines a region of MSSM parameter space constrained by

$|\mu| = 4\, M_{SUSY}$, $|A_{t,b}| = 2\, M_{SUSY}$

Moderate $m_{H^\pm} \sim$ 180 - 200 GeV.

Quantum effects of CP-even/CP-odd states mixing depending on $\mathtt{Im}(\mu A_t)/M_{SUSY}^2$ are not small.

In the following $M_{SUSY} = 500\,$GeV, $|A_t| = |A_b| = A = 1000\,$GeV, $|\mu| = 2000\,$GeV.

and

CPsuperH parameter set: $m_Z = 91.19\,$GeV, $m_b = 3\,$GeV, $m_t = 175\,$GeV, $m_W = 79.96\,$GeV, $g_2 = 0.6517$, $g_1 = 0.3573$, $v = 245.4\,$GeV, $G_F = 1.17 \cdot 10^{-5}\,$GeV^{-2}, $\alpha_S(m_t) = 0.1072$, $\sigma = m_t$.

J.S.Lee et.al., CPC 156 (2004) 283

$\lambda_{1,\ldots 7}$ and $\Delta\lambda_{1,\ldots 7}$ patterns.
MSSM, CPX_{500} scenario.

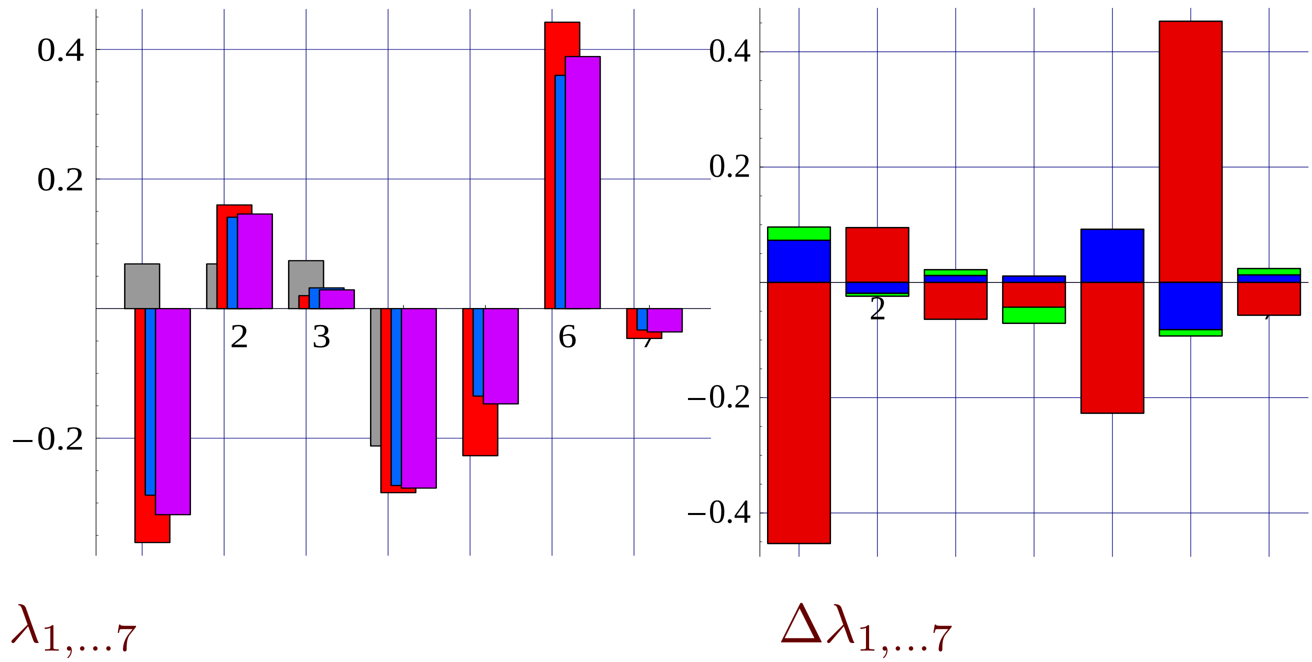


Figure 1: Effective parameters
$\lambda_{1,\ldots 7}$ and $\Delta\lambda_{1,\ldots 7} = \lambda_{1,\ldots 7}^{M_{SUSY}} - \lambda_{1,\ldots 7}$

- tree level$_{M_{SUSY}}$
- .
- one-loop$_{m_{top}}$
- two-loop Yukawa$_{m_{top}175}$
- two-loop Yukawa$_{m_{top}178}$
- one-loop, no D-terms and wfr
- two-loop Yukawa$_{175}$
- D-terms and wfr

The cubic equation for eigenvalues

$$(m_{h_i}^2)^3 + a_2(m_{h_i}^2)^2 + a_1 m_{h_i}^2 + a_0 = 0$$

Squared masses of the physical states h_1, h_2, h_3 - the eigenvalues of mass matrix M^2

$$\begin{aligned}
m_{h1}^2 &= 2\sqrt{(-q)}\cos\left(\frac{\Theta + 2\pi}{3}\right) - \frac{a_2}{3} \\
m_{h2}^2 &= 2\sqrt{(-q)}\cos\left(\frac{\Theta - 2\pi}{3}\right) - \frac{a_2}{3} \\
m_{h3}^2 &= 2\sqrt{(-q)}\cos\left(\frac{\Theta}{3}\right) - \frac{a_2}{3}
\end{aligned}$$

$$\Theta = \arccos \frac{r}{\sqrt{(-q^3)}}$$

$$r = \frac{1}{54}(9a_1a_2 - 27a_0 - 2a_2^3), \; q = \frac{1}{9}(3a_1 - a_2^2)$$

$$\begin{aligned}
a_0 &= c_1^2 m_H^2 + c_2^2 m_h^2 - m_h^2 m_H^2 m_A^2, \\
a_1 &= m_h^2 m_H^2 + m_h^2 m_A^2 + m_H^2 m_A^2 - c_1^2 - c_2^2, \\
a_2 &= -m_h^2 - m_H^2 - m_A^2.
\end{aligned}$$

if $c_{1,2} \to 0$ then $m_{h_1} \to min(m_h,\ m_H,\ m_A)$ and $m_{h_3} \to max(m_h,\ m_H,\ m_A)$.

The normalized eigenvectors of the matrix M^2
$a_{ij} = \frac{a'_{ij}}{n_j}$, $n_i = k_i \sqrt{a'^2_{1i} + a'^2_{2i} + a'^2_{3i}}$.

$$
\begin{aligned}
a'_{11} &= ((m_H^2 - m_{h_1}^2)(m_A^2 - m_{h_1}^2) - c_2^2),\\
a'_{21} &= c_1 c_2,\\
a'_{31} &= -c_1(m_H^2 - m_{h_1}^2),\\
a'_{12} &= -c_1 c_2,\\
a'_{22} &= -((m_h^2 - m_{h_2}^2)(m_A^2 - m_{h_2}^2) - c_1^2),\\
a'_{32} &= c_2(m_h^2 - m_{h_2}^2),\\
a'_{13} &= -c_1(m_H^2 - m_{h_3}^2),\\
a'_{23} &= -c_2(m_h^2 - m_{h_3}^2),\\
a'_{33} &= (m_h^2 - m_{h_3}^2)(m_H^2 - m_{h_3}^2)
\end{aligned}
$$

The cubic equation can be rewritten in the form

$$(m_h^2 - m_{h_i}^2)[(m_H^2 - m_{h_i}^2)(m_A^2 - m_{h_i}^2) - c_2^2] - c_1^2(m_H^2 - m_{h_i}^2) = 0.$$

For example at $i = 1$ **(first eigenvector)**

$$(m_h^2 - m_{h_1}^2)a'_{11} + c_1 a'_{31} = 0$$

so if c_1 **=0 then either (I)** a'_{11} **=0 or (II)** $m_{h_1} = m_h$**. Case (II) takes place when** $\frac{r}{\sqrt{-q^3}}$ **=max.value (**$\varphi \sim \pi/2$**) or min.value (**$\varphi \sim \pi$**), but** $\frac{r}{\sqrt{-q^3}}$ **≠1 or, eqivalently** Θ **≠0**

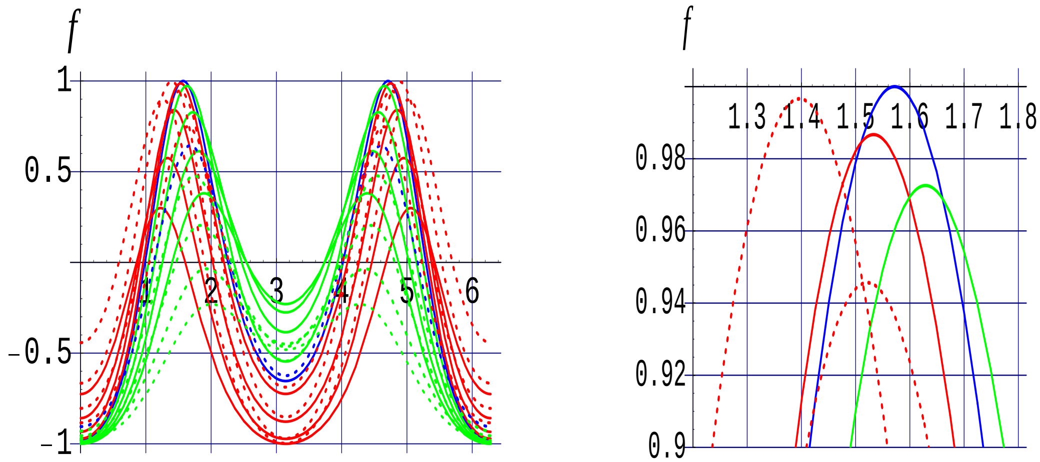


Figure 2: Plots for $f = \frac{r}{\sqrt{(-q^3)}}$ as a function of φ for different of $m_{H^\pm}$ (10 GeV difference for various lines). Blue line – $m_{H^\pm} = 184$ GeV. Dashed – with leading two-loop corrections

Special point, MSSM, CPX$_{500}$ scenario, one-loop:

$$m_{H^\pm}\mathbf{=184\ GeV.}$$

This change of regime (I)→(II) leads to a discontinuties in $a_{ij}(m_{H^\pm})$.

In the MSSM, CPX class of scenarios

$$c_1 = \frac{v^2}{2}(s_\alpha s_\beta - c_\alpha c_\beta)\mathrm{Im}\bar{\lambda}_5 + v^2\,(s_\alpha c_\beta \mathrm{Im}\bar{\lambda}_6 - c_\alpha s_\beta \mathrm{Im}\bar{\lambda}_7) = 0$$

appears in the vicinity of $\varphi = \pi/2$ as a consequence of the MSSM relation between the radiatively induced phases of $\lambda_{5,6,7}$: $2\arg(\lambda_{6,7}) = \arg(\lambda_5)$. In other nonstandard models c_1 =0 may not take place and different structure of discontinuites may appear.

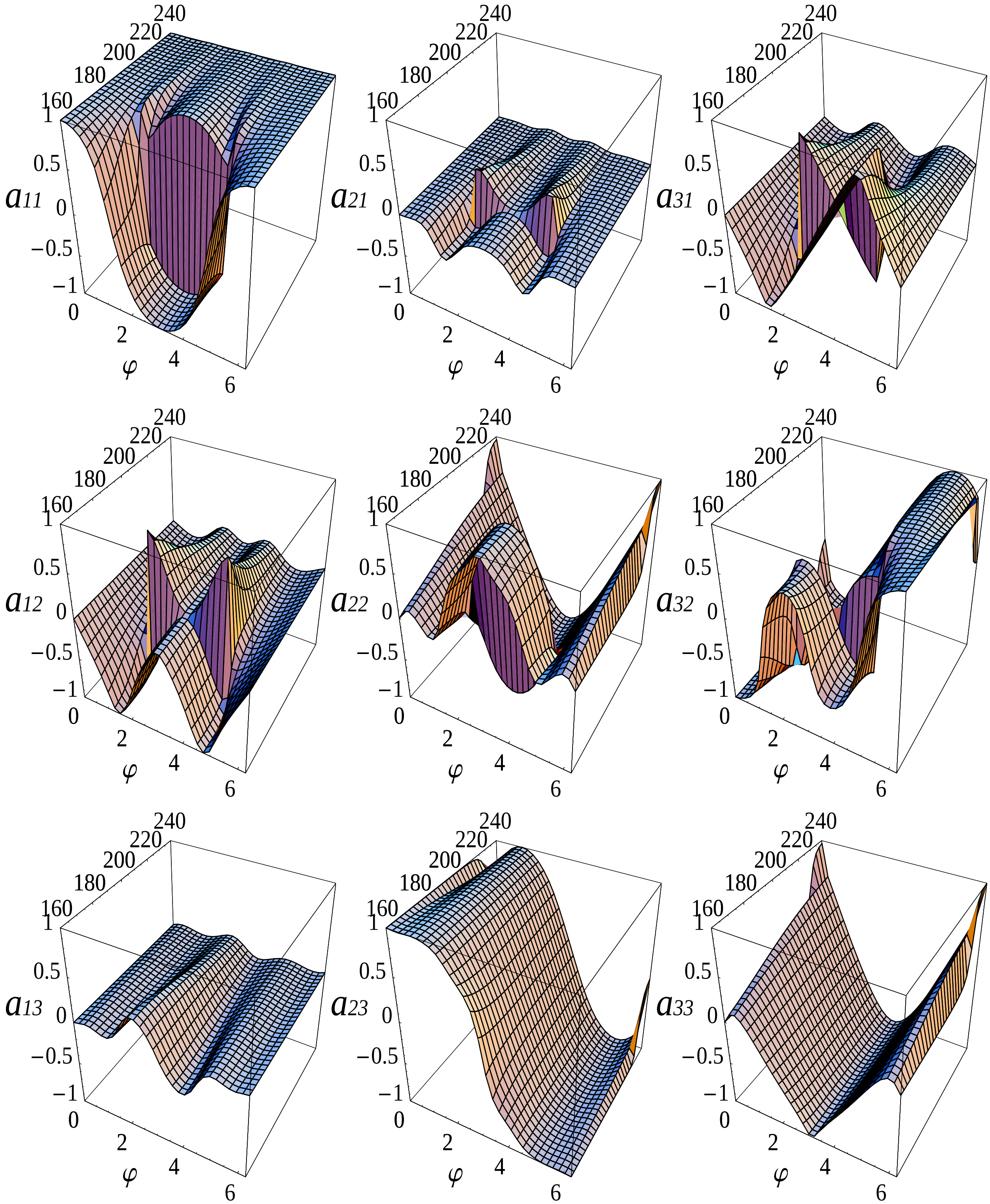


Figure 3: The matrix elements a_{ij} as a two-dimensional functions of the mass $m_{H^\pm}$ (GeV) and the phase φ (rad) at the one-loop approximation for $\lambda_{1,...7}$, MSSM, CPX_{500}

There are no simple representations for the triple and quartic Higgs boson self-interaction vertices.

$$\mathcal{L}_{3H} = v \sum_{i\geq j\geq k=1}^{3} g_{h_i h_j h_k} \frac{1}{N_S^{ijk}} h_i h_j h_k + v \sum_{i=1}^{3} g_{h_i H^+ H^-} h_i H^+ H^- ,$$

where N_S **are the combinatorial factors and**

$$g_{h_i h_j h_k} = \sum_{\alpha\geq\beta\geq\gamma=1}^{3} \{a_{\alpha i} a_{\beta j} a_{\gamma k}\} \, g_{\alpha\beta\gamma}$$

$$g_{h_i H^+ H^-} = - \sum_{\alpha=1}^{3} a_{\alpha i} \, g_{\alpha H^+ H^-}$$

$$\{a_{\alpha i} a_{\beta j} a_{\gamma k}\} \equiv \frac{1}{N_S} \Big(a_{\alpha i} a_{\beta j} a_{\gamma k} + a_{\alpha i} a_{\beta k} a_{\gamma j} + a_{\alpha j} a_{\beta i} a_{\gamma k} + a_{\alpha j} a_{\beta k} a_{\gamma i} + a_{\alpha k} a_{\beta i} a_{\gamma j} + a_{\alpha k} a_{\beta j} a_{\gamma i} \Big) ,$$

where $N_S = 6$ **at** $i = j = k$**,** $N_S = 1$ **at** $(i, j, k) = (3, 2, 1)$**, and** $N_S = 2$ **in all other cases. There are no discontinutes in** $g_{h_i h_j h_k}$ **because** a_{ij} **products are always in combinations of "compensating sign".**

In the λ_i basis

$$
\begin{aligned}
g_{1H^+H^-} &= \mathtt{Re}\Delta\lambda_5\, s_\beta c_\beta\, c_{\alpha+\beta} - \mathtt{Re}\Delta\lambda_6\, c_\alpha\, s_\beta^2\, c_\beta + \mathtt{Re}\Delta\lambda_6\, s_\alpha\, s_\beta^3 \\
&\quad - \mathtt{Re}\Delta\lambda_6\, s_\alpha\, s_{2\beta}\, c_\beta + \mathtt{Re}\Delta\lambda_7\, c_\beta\, (s_\alpha\, s_\beta\, c_\beta \\
&\quad - c_\alpha\, \left(c_\beta^2 - 2\, s_\beta^2\right)\Big) - 2\, s_\alpha\, s_\beta^2\, c_\beta\, \lambda_1 + 2\, c_\alpha\, s_\beta\, c_\beta^2\, \lambda_2 \\
&\quad - c_\beta^3\, s_\alpha\, \lambda_3 + c_\alpha\, s_\beta^3\, \lambda_3 - c_\beta\, s_\beta\, \lambda_4\, c_{\alpha-\beta} \\
g_{2H^+H^-} &= \mathtt{Re}\Delta\lambda_5\, s_\beta c_\beta\, s_{\alpha+\beta} + 2\, \mathtt{Re}\Delta\lambda_6\, c_\alpha\, s_\beta\, c_\beta^2 - \mathtt{Re}\Delta\lambda_6\, c_\alpha\, s_\beta^3 \\
&\quad - \mathtt{Re}\Delta\lambda_6\, s_\alpha\, s_\beta^2 c_\beta - \mathtt{Re}\Delta\lambda_7\, c_\beta\, (c_\alpha\, s_\beta\, c_\beta \\
&\quad + s_\alpha\, \left(c_\beta^2 - 2\, s_\beta^2\right)\Big) + 2\, c_\alpha\, s_\beta^2\, c_\beta \lambda_1 + 2\, s_\alpha\, s_\beta\, c_\beta^2 \lambda_2 \\
&\quad + c_\alpha\, c_\beta^3 \lambda_3 + s_\alpha\, s_\beta^3 \lambda_3 - c_\beta\, s_\beta\, \lambda_4\, s_{\alpha+\beta} \\
g_{3H^+H^-} &= c_\beta^2\, \mathtt{Im}\Delta\lambda_7 - s_\beta\, c_\beta\, \mathtt{Im}\Delta\lambda_5 + s_\beta^2\, \mathtt{Im}\Delta\lambda_6
\end{aligned}
$$

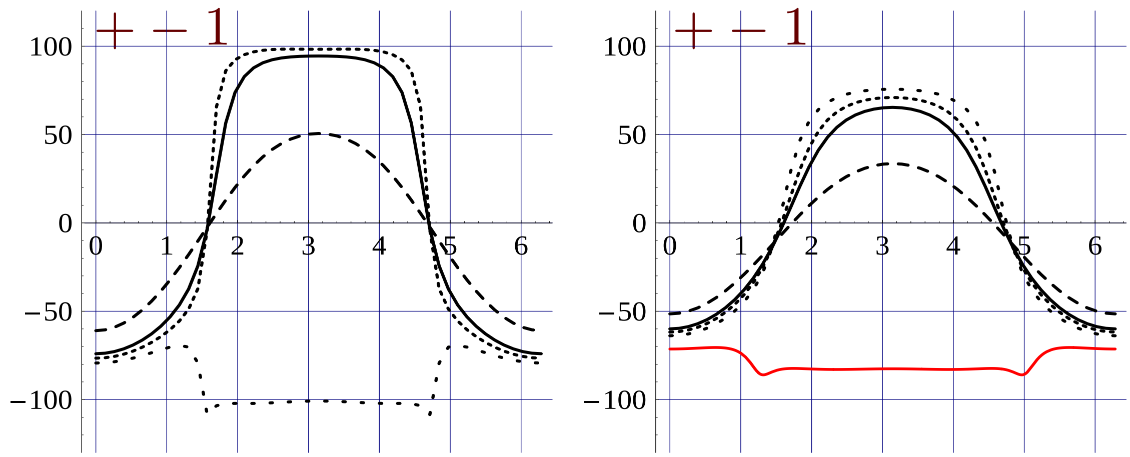


Figure 4: The triple Higgs boson interaction vertex $v \cdot g_{H^+ \; H^- \; h_1}$ (GeV) vs the phase $\mathrm{Arg}(\mu A)$ (the left figure for 1-loop approximation and the right figure with additional leading OCD Yukawa corrections to λ_i) at parameter values of CPX$_{500}$: $M_{SUSY} = 500$ GeV, tgβ =5, $A_{t,b}$ =1000 GeV, $\mu = 2000$ GeV. Long dashed line – $m_{H^\pm} = 300$ GeV, solid line – $m_{H^\pm} = 200$ GeV, short dashed line – $m_{H^\pm} = 190$ GeV, rare dotted line – $m_{H^\pm} = 180$ GeV; red line on the right plot – $m_{H^\pm} = 150$ GeV for the intense-coupling regime.

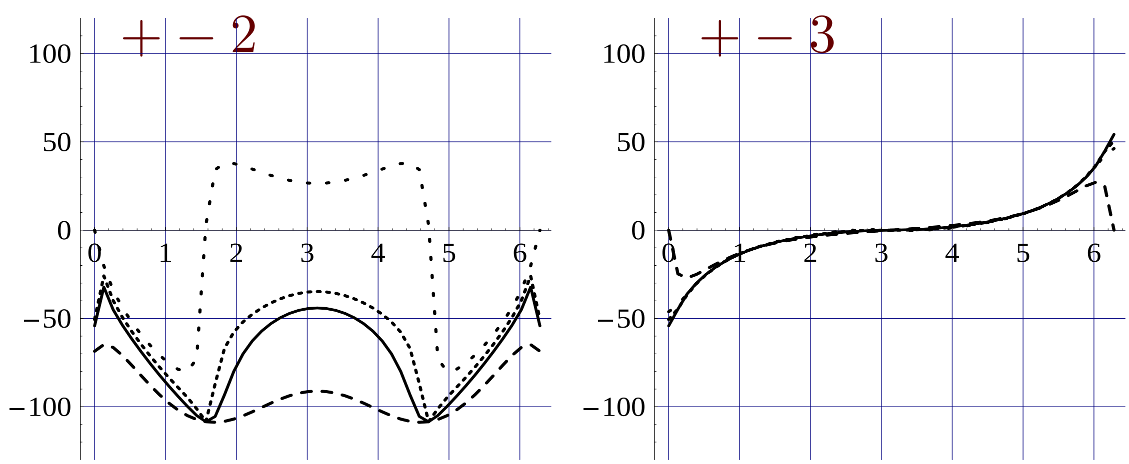


Figure 5: The triple Higgs boson interaction vertices $v \cdot g_{H^+ \; H^- \; h_2}$ (GeV, the left figure) and $v \cdot g_{H^+ \; H^- \; h_3}$ (the right figure) vs the phase $\mathrm{Arg}(\mu A)$ for 1-loop approximation to lambda-couplings at the same values of parameters and notations as fig. 4. Curves on the right plot coincide, except for the case $m_{H^\pm} = 300$ GeV

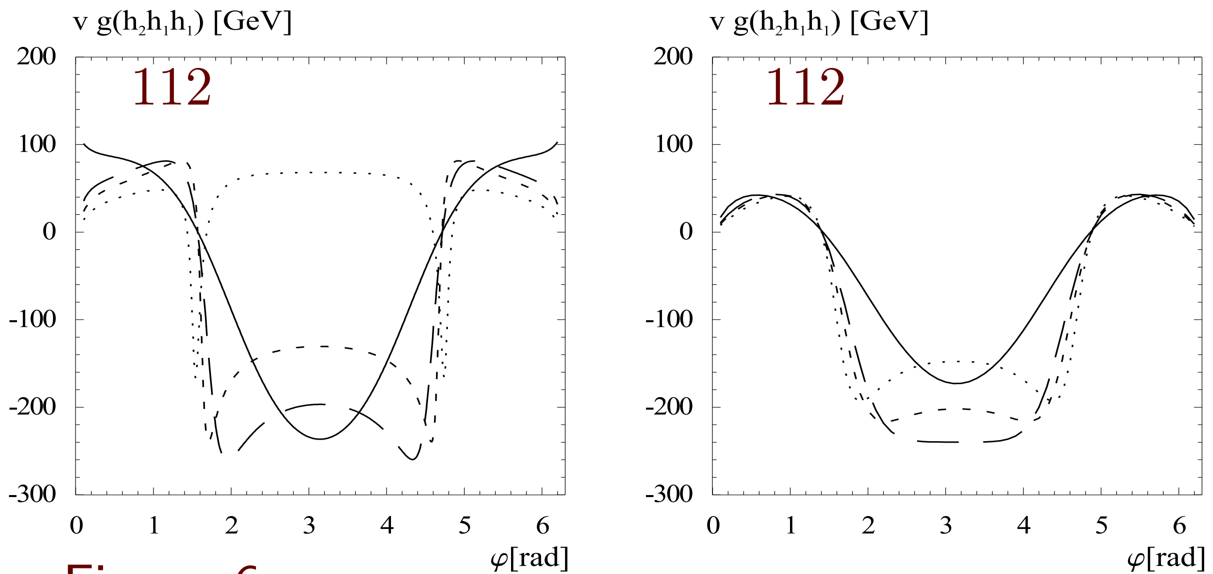


Figure 6: Light Higgs boson interaction vertex $v \cdot g_{h_1 h_1 h_2}$ (GeV) vs the phase $\arg(\mu A)$ at parameter values $M_{SUSY} = 500$ GeV, tg$\beta = 5$, $A_{t,b} = 1000$ GeV, $\mu = 2000$ GeV. Solid line – $m_{H^\pm} = 300$ GeV, long dashed line – $m_{H^\pm} = 200$ GeV, short dashed line – $m_{H^\pm} = 190$ GeV, dotted line – $m_{H^\pm} = 180$ GeV. (a) – effective one-loop potential, (b) – leading two-loop corrections included.

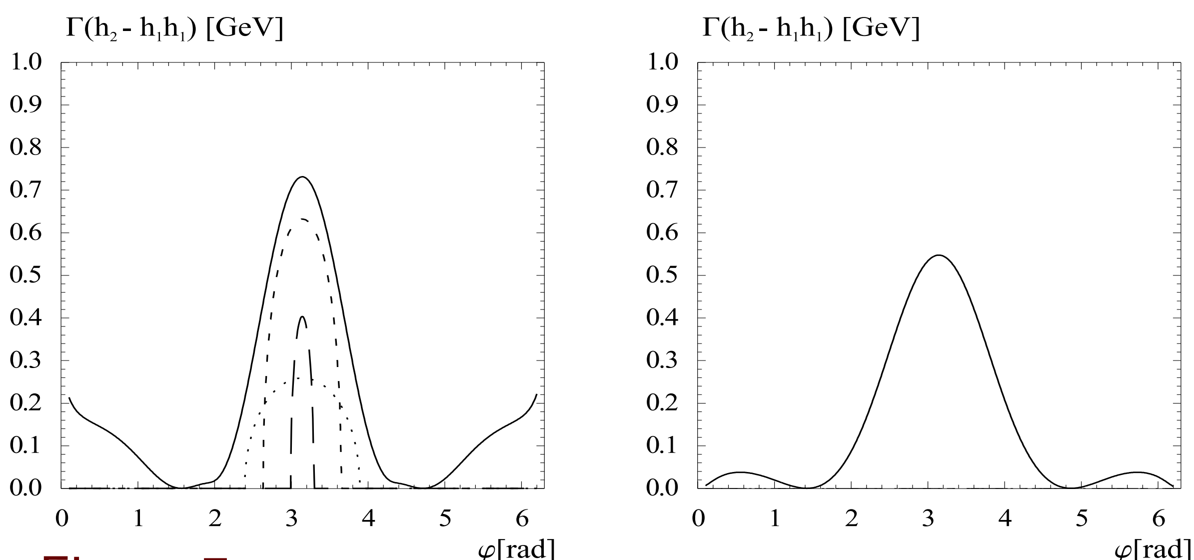


Figure 7: The decay width $h_2 \to h_1 h_1$ (GeV) vs the phase $\mathrm{Arg}(\mu A)$ at parameter values $M_{SUSY} = 500$ GeV, tg$\beta = 5$, $A_{t,b} = 1000$ GeV, $\mu = 2000$ GeV. Solid line – $m_{H^\pm} = 300$ GeV, long dashed line – $m_{H^\pm} = 200$ GeV, short dashed line – $m_{H^\pm} = 190$ GeV, pointed curve – $m_{H^\pm} = 180$ GeV. (a) – effective one-loop approach, (b) – with leading two-loop corrections

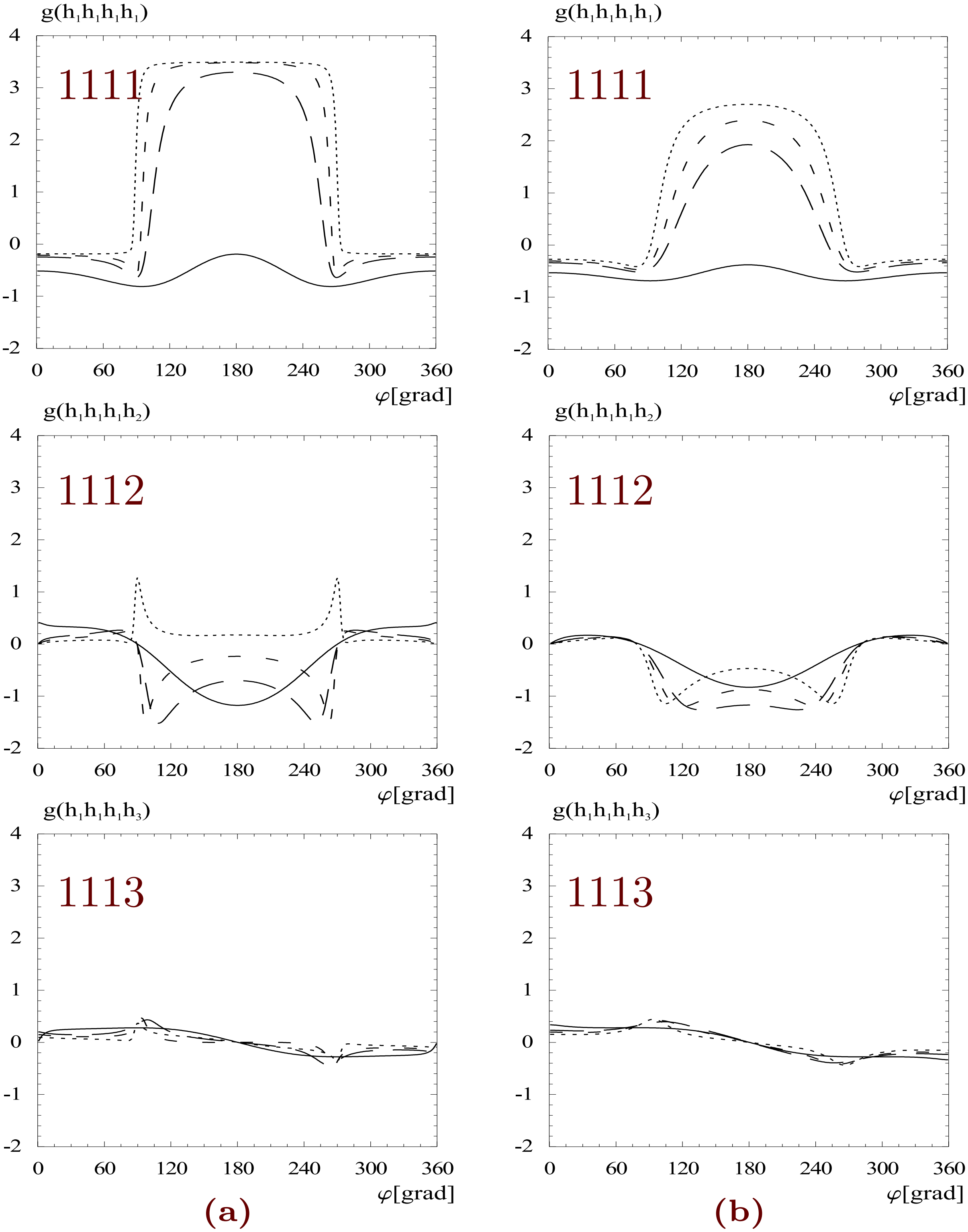


Figure 8: Quartic interaction vertices $g_{h_1 h_1 h_1 h_i}$, i =1,2,3, vs the phase $\mathrm{Arg}(\mu A)$ Solid line – $m_{H^\pm}$ = 300 GeV, long dashed line – $m_{H^\pm}$ = 200 GeV, short dashed line – $m_{H^\pm}$ = 190 GeV, pointed curve – $m_{H^\pm}$ = 180 GeV. (a) – effective one-loop potential, (b) – with leading two-loop corrections included.

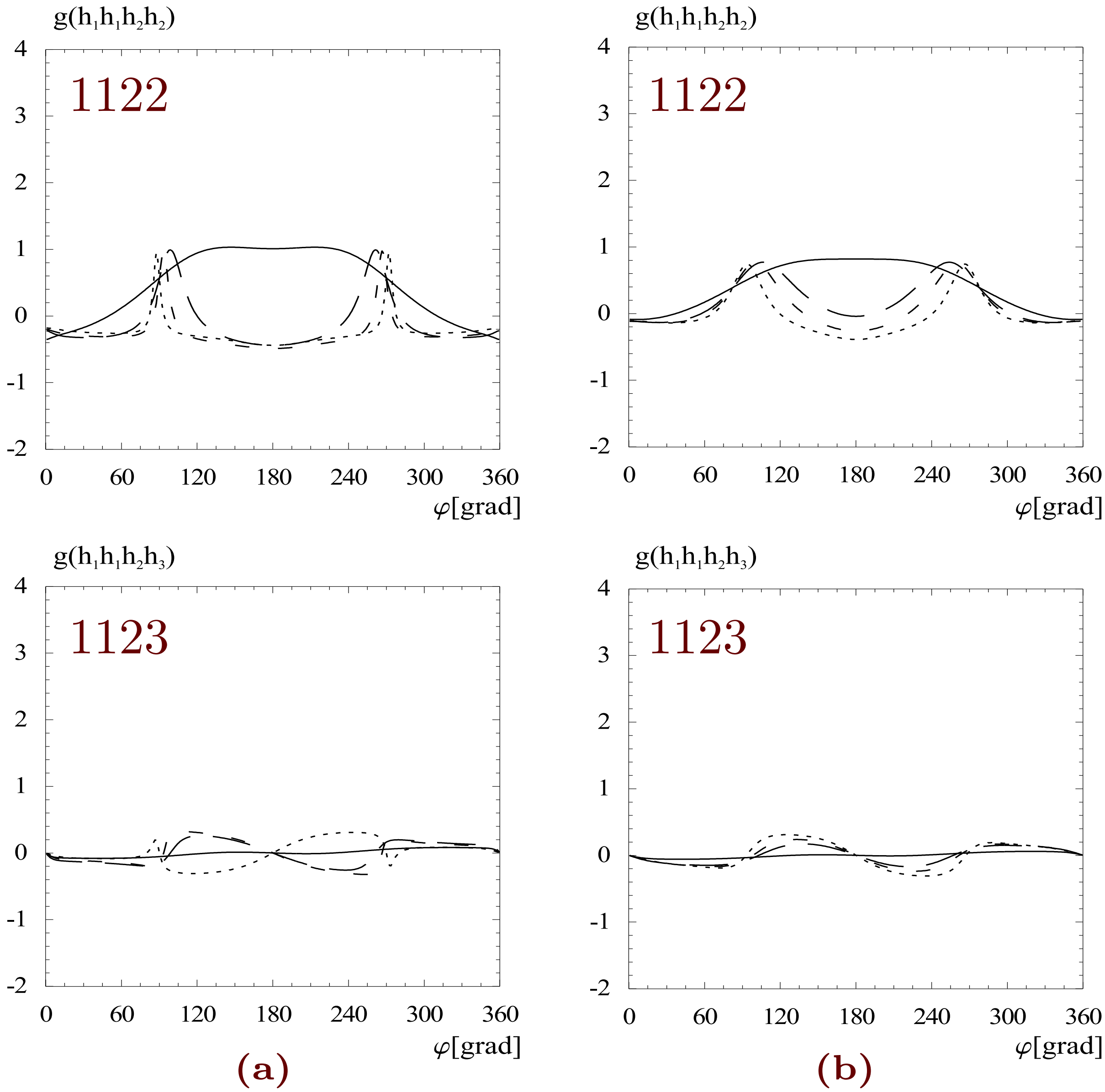


Figure 9: Quartic interaction vertices $g_{h_1 h_1 h_2 h_i}$, i =2,3, vs the phase $\mathrm{Arg}(\mu A)$ Solid line – $m_{H^\pm}$ = 300 GeV, long dashed line – $m_{H^\pm}$ = 200 GeV, short dashed line – $m_{H^\pm}$ = 190 GeV, pointed curve – $m_{H^\pm}$ = 180 GeV. (a) – effective one-loop potential, (b) – with leading two-loop corrections included.

Summary and work in progress

- In order to keep (1) mass ordering $m_{h_1} < m_{h_2} < m_{h_3}$ of Higgs bosons, (2) matching of (h_1, h_2, h_3) states to the (h, H A) states of the CP-conserving limit $\varphi = 0$ and (3) definite (always left, $\det\|a_{ij}\| = 1$) orientation of the eigenvector basis at any values of model parameters $m_{H^\pm}$, $\mathrm{tg}\beta$, $\varphi = \arg(\mu A_{t,b})$ a sign prescription for the normalized eigenvectors should be accepted. For example, sign of $\mathbf{e_1}$

$$k_1: \left\{ \begin{array}{l} 1)\text{ if } m_H(\varphi=0) > m_A(\varphi=0) \; and \\ \left\{ \begin{array}{l} 1.1)\text{ if } m_{h_3}(\varphi=0) \geq m_{H^\pm} \; and \\ \left\{ \begin{array}{l} 1.1.1)\text{ if sign}(c_1)\text{ sign}(\pi-\varphi) = 1 \text{ then } \; k_1 = \text{sign}(m_{h_3} - m_{H^\pm}). \\ 1.1.2)\text{ if sign}(c_1)\text{ sign}(\pi-\varphi) = -1, \text{ then } \; k_1 = -1. \end{array} \right. \\ 1.2)\text{ if } m_{h_3}(\varphi=0) < m_{H^\pm} \quad \text{then } k_1 = 1. \end{array} \right. \\ 2)\text{ if } m_H(\varphi=0) \leq m_A(\varphi=0) \qquad \text{then } \; k_1 = 1. \end{array} \right.$$

  The only discontinuity of $a_{ij}(\varphi, m_{H^\pm})$ is then the discontinuity in $m_{H^\pm}$, which takes place at 184 GeV at the one-loop effective λ_i in the MSSM, CPX$_{500}$ scenario. There are no discontinuites in the trilinear and quartic couplings.

- Triple and quartic couplings of physical Higgs bosons vary very strongly (by a factor of 5-10 or more) with $\varphi\,[0, 2\pi]$ and $m_{H^\pm}$ (MSSM, CPX$_{500}$ scenario). Small variations are only at $\varphi \sim 0$ (CP-conserving limit). High sensitivity of self-couplings to radiative corrections is observed. At the phase $\varphi \simeq \pi/2$ (c_1 =0, $m_{H^\pm}$ moderately small, the case of strong mixing) many of them change sign bypassing zero, so in the vicinity of $\varphi \simeq \pi/2$ the physical scalars are extremely weakly self-interacting.

- With Yukawa term of type II the $t\bar{t}h_1$ vertex has the form

  $$t\bar{t}h_1 \qquad -\frac{m_{top}}{v\,\sin\beta}(a_{21}\,\sin\alpha + a_{11}\,\cos\alpha - i\,a_{31}\cos\beta\,\gamma_5)$$

  So experimental reconstruction of the channel $pp \to t\bar{t}h_1$ at LHC and analogous will give valuable information about mixing a_{ij}.

  Caltech-MSU group: J.Albert, V.Litvin, H.Newman, M.D.

  Reconstruction of Gunion-Xe observables for complete tree levels set of diagrams, $pp \to t\bar{t}h_1$.