The Significance of $\tan\beta$ in the Two-Higgs-Doublet Model

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This talk is based on work that appears in:

1. S. Davidson and H.E. Haber, "Basis-independent methods for the two-Higgs-doublet model," *Phys. Rev.* **D72**, 035004 (2005). [See also G. Branco, L. Lavoura and J.P. Silva, *CP Violation* (Oxford University Press, Oxford, England, 1999), chapters 22 and 23.]

2. H.E. Haber and D. O'Neil, in preparation.

3. J.F. Gunion, H.E. Haber and J. Kalinowski, in preparation.

Outline

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Motivation

The Higgs sector of the minimal supersymmetric extension of the Standard Model (MSSM) is a constrained 2HDM. However, at one-loop all possible 2HDM interactions allowed by gauge invariance are generated (due to SUSY-breaking interactions).

Thus, the Higgs sector of the MSSM is in reality the most general 2HDM model (albeit with certain relations among the Higgs sector parameters determined by the fundamental parameters of the broken supersymmetric model).

The general 2HDM consists of two identical (hypercharge-one) scalar doublets Φ_1 and Φ_2. One can always redefine the basis, so the parameter $\tan\beta \equiv v_2/v_1$ is not meaningful!

To determine the physical quantities, one must develop basis-independent techniques.

The General Two-Higgs-Doublet Model

Consider the 2HDM potential in a *generic* basis:

$$\mathcal{V} = m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - [m_{12}^2 \Phi_1^\dagger \Phi_2 + \text{h.c.}] + \tfrac{1}{2}\lambda_1 (\Phi_1^\dagger \Phi_1)^2$$

$$+ \tfrac{1}{2}\lambda_2 (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1)$$

$$+ \left\{ \tfrac{1}{2}\lambda_5 (\Phi_1^\dagger \Phi_2)^2 + \left[\lambda_6 (\Phi_1^\dagger \Phi_1) + \lambda_7 (\Phi_2^\dagger \Phi_2)\right] \Phi_1^\dagger \Phi_2 + \text{h.c.} \right\}$$

A basis change consists of a U(2) transformation $\Phi_a \longrightarrow U_{a\bar{b}} \Phi_b$ (and $\Phi_{\bar{a}}^\dagger = \Phi_{\bar{b}}^\dagger U_{b\bar{a}}^\dagger$). Rewrite $\mathcal{V}$ in a U(2)-covariant notation:

$$\boxed{\mathcal{V} = Y_{a\bar{b}} \Phi_{\bar{a}}^\dagger \Phi_b + \tfrac{1}{2} Z_{a\bar{b}c\bar{d}} (\Phi_{\bar{a}}^\dagger \Phi_b)(\Phi_{\bar{c}}^\dagger \Phi_d)}$$

where $Z_{a\bar{b}c\bar{d}} = Z_{c\bar{d}a\bar{b}}$ and hermiticity implies $Y_{a\bar{b}} = (Y_{b\bar{a}})^*$ and $Z_{a\bar{b}c\bar{d}} = (Z_{b\bar{a}d\bar{c}})^*$. The barred indices help keep track of which indices transform with U and which transform with $U^\dagger$. For example, $Y_{a\bar{b}} \longrightarrow U_{a\bar{c}} Y_{c\bar{d}} U_{d\bar{b}}^\dagger$ and $Z_{a\bar{b}c\bar{d}} \longrightarrow U_{a\bar{e}} U_{f\bar{b}}^\dagger U_{c\bar{g}} U_{h\bar{d}}^\dagger Z_{e\bar{f}g\bar{h}}$.

The most general $\mathrm{U(1)_{EM}}$-conserving vev is:

$$\langle\Phi_a\rangle = \frac{v}{\sqrt{2}}\begin{pmatrix} 0 \\ \widehat{v}_a \end{pmatrix}, \qquad \text{with} \qquad \widehat{v}_a \equiv e^{i\eta}\begin{pmatrix} c_\beta \\ s_\beta\, e^{i\xi} \end{pmatrix},$$

where $v \equiv 2m_W/g = 246$ GeV. The overall phase η is arbitrary (and can be removed with a $\mathrm{U(1)_Y}$ hypercharge transformation). If we define $V_{a\bar{b}} \equiv \hat{v}_a\hat{v}^*_{\bar{b}}$, then the scalar potential minimum condition is given by the invariant condition:

$$\mathrm{Tr}\,(VY) + \tfrac{1}{2}v^2 Z_{a\bar{b}c\bar{d}}V_{b\bar{a}}V_{d\bar{c}} = 0\,.$$

The orthonormal eigenvectors of $V_{a\bar{b}}$ are $\hat{v}_b$ and $\widehat{w}_b \equiv -\epsilon_{bc}\,\widehat{v}^*_{\bar{c}}$ (with $\epsilon_{12} = -\epsilon_{21} = 1$, $\epsilon_{11} = \epsilon_{22} = 0$). Note that $\hat{v}^*_{\bar{b}}\hat{w}_b = 0$. Under a U(2) transformation, $\hat{v}_a \longrightarrow U_{a\bar{b}}\hat{v}_b$, but:

$$\widehat{w}_a \longrightarrow (\det\, U)^{-1}\, U_{a\bar{b}}\,\widehat{w}_b\,,$$

where $\det U$ is a pure phase. In fact, the overall phase in the definition of $\hat{w}$ is completely arbitrary. Keeping track of this phase ambiguity is important for identifying true invariants.

Remark: $\mathrm{U(2)} \cong \mathrm{SU(2)}\times\mathrm{U(1)_Y}$. The parameters m^2_{11}, m^2_{22}, m^2_{12}, and $\lambda_1, \ldots, \lambda_7$ would change under the "flavor"-SU(2) transformation, while $\hat{v}$ transforms under the full U(2) group.

The Higgs basis

Define new Higgs doublet fields:

$$H_1 \equiv \hat{v}_{\bar{a}}^* \Phi_a \,, \qquad\qquad e^{i\chi} H_2 \equiv \hat{w}_{\bar{a}}^* \Phi_a \,.$$

Equivalently, $\Phi_a = H_1 \hat{v}_a + H_2 \hat{w}_a e^{i\chi}$. The phase χ parameterizes a class of bases called the *Higgs bases*. In addition, χ represents the ambiguity in the overall phase in the definition of $\hat{w}$. From the definitions of H_1 and H_2, it follows that

$$\langle H_1^0 \rangle = \frac{v}{\sqrt{2}} \,, \qquad\qquad \langle H_2^0 \rangle = 0 \,.$$

The field H_1 defined above is *invariant*. However, under a U(2) transformation,

$$e^{i\chi} H_2 \longrightarrow (\det U) e^{i\chi} H_2 \,.$$

For example, under the U(2) transformation $U = \mathrm{diag}\;(1\,,\, e^{i\theta})$, one can transform among different Higgs bases. Henceforth, quantities that are invariant under SU(2) but not under U(2) will be called *pseudo-invariants*.

If we rewrite the Higgs potential $\mathcal{V}$ in the Higgs basis, we find:

$$
\begin{aligned}
\mathcal{V} = {} & Y_1 H_1^\dagger H_1 + Y_2 H_2^\dagger H_2 + [Y_3 e^{i\chi} H_1^\dagger H_2 + \text{h.c.}] + \tfrac{1}{2} Z_1 (H_1^\dagger H_1)^2 \\
& + \tfrac{1}{2} Z_2 (H_2^\dagger H_2)^2 + Z_3 (H_1^\dagger H_1)(H_2^\dagger H_2) + Z_4 (H_1^\dagger H_2)(H_2^\dagger H_1) \\
& + \left\{ \tfrac{1}{2} Z_5 e^{2i\chi} (H_1^\dagger H_2)^2 + \left[Z_6 e^{i\chi} (H_1^\dagger H_1) + Z_7 e^{i\chi} (H_2^\dagger H_2) \right] H_1^\dagger H_2 + \text{h.c.} \right\} ,
\end{aligned}
$$

where (with $W_{a\bar{b}} \equiv \hat{w}_a \hat{w}_{\bar{b}}^* = \delta_{a\bar{b}} - V_{a\bar{b}}$),

$$
\begin{aligned}
Y_1 &\equiv \mathrm{Tr}\,(YV)\,, & Y_2 &\equiv \mathrm{Tr}\,(YW)\,, \\
Z_1 &\equiv Z_{a\bar{b}c\bar{d}} V_{b\bar{a}} V_{d\bar{c}}\,, & Z_2 &\equiv Z_{a\bar{b}c\bar{d}} W_{b\bar{a}} W_{d\bar{c}}\,, \\
Z_3 &\equiv Z_{a\bar{b}c\bar{d}} V_{b\bar{a}} W_{d\bar{c}}\,, & Z_4 &\equiv Z_{a\bar{b}c\bar{d}} V_{b\bar{c}} W_{d\bar{a}}
\end{aligned}
$$

are invariant quantities, whereas the following pseudo-invariants

$$
\begin{aligned}
Y_3 &\equiv Y_{a\bar{b}}\, \widehat{v}_{\bar{a}}^*\, \widehat{w}_b\,, & Z_5 &\equiv Z_{a\bar{b}c\bar{d}}\, \widehat{v}_{\bar{a}}^*\, \widehat{w}_b\, \widehat{v}_{\bar{c}}^*\, \widehat{w}_d\,, \\
Z_6 &\equiv Z_{a\bar{b}c\bar{d}}\, \widehat{v}_{\bar{a}}^*\, \widehat{v}_b\, \widehat{v}_{\bar{c}}^*\, \widehat{w}_d\,, & Z_7 &\equiv Z_{a\bar{b}c\bar{d}}\, \widehat{v}_{\bar{a}}^*\, \widehat{w}_b\, \widehat{w}_{\bar{c}}^*\, \widehat{w}_d\,.
\end{aligned}
$$

transform as $[Y_3, Z_6, Z_7] \to (\det\ U)^{-1}[Y_3, Z_6, Z_7]$ and $Z_5 \to (\det\ U)^{-2} Z_5$. The (pseudo-)invariants in the generic basis are [with $\lambda_{345} \equiv \lambda_3 + \lambda_4 + \mathrm{Re}(\lambda_5\, e^{2i\xi})$]:

$$
\begin{aligned}
Y_1 &= m_{11}^2 c_\beta^2 + m_{22}^2 s_\beta^2 - \mathrm{Re}(m_{12}^2 e^{i\xi}) s_{2\beta}\,, \\
Y_2 &= m_{11}^2 s_\beta^2 + m_{22}^2 c_\beta^2 + \mathrm{Re}(m_{12}^2 e^{i\xi}) s_{2\beta}\,, \\
Y_3\, e^{i\xi} &= \tfrac{1}{2}(m_{22}^2 - m_{11}^2) s_{2\beta} - \mathrm{Re}(m_{12}^2 e^{i\xi}) c_{2\beta} - i\,\mathrm{Im}(m_{12}^2 e^{i\xi})\,,
\end{aligned}
$$

$$
\begin{aligned}
Z_1 =& \lambda_1 c_\beta^4 + \lambda_2 s_\beta^4 + \tfrac{1}{2}\lambda_{345} s_{2\beta}^2 + 2 s_{2\beta}\left[c_\beta^2 \mathrm{Re}(\lambda_6 e^{i\xi}) + s_\beta^2 \mathrm{Re}(\lambda_7 e^{i\xi})\right]\,, \\
Z_2 =& \lambda_1 s_\beta^4 + \lambda_2 c_\beta^4 + \tfrac{1}{2}\lambda_{345} s_{2\beta}^2 - 2 s_{2\beta}\left[s_\beta^2 \mathrm{Re}(\lambda_6 e^{i\xi}) + c_\beta^2 \mathrm{Re}(\lambda_7 e^{i\xi})\right]\,, \\
Z_3 =& \tfrac{1}{4} s_{2\beta}^2 \left[\lambda_1 + \lambda_2 - 2\lambda_{345}\right] + \lambda_3 - s_{2\beta} c_{2\beta} \mathrm{Re}[(\lambda_6 - \lambda_7) e^{i\xi}]\,, \\
Z_4 =& \tfrac{1}{4} s_{2\beta}^2 \left[\lambda_1 + \lambda_2 - 2\lambda_{345}\right] + \lambda_4 - s_{2\beta} c_{2\beta} \mathrm{Re}[(\lambda_6 - \lambda_7) e^{i\xi}]\,, \\
Z_5\, e^{2i\xi} =& \tfrac{1}{4} s_{2\beta}^2 \left[\lambda_1 + \lambda_2 - 2\lambda_{345}\right] + \mathrm{Re}(\lambda_5 e^{2i\xi}) + i c_{2\beta} \mathrm{Im}(\lambda_5 e^{2i\xi})\,, \\
& - s_{2\beta} c_{2\beta} \mathrm{Re}[(\lambda_6 - \lambda_7) e^{i\xi}] - i s_{2\beta} \mathrm{Im}[(\lambda_6 - \lambda_7) e^{i\xi})]\,, \\
Z_6\, e^{i\xi} =& -\tfrac{1}{2} s_{2\beta}\left[\lambda_1 c_\beta^2 - \lambda_2 s_\beta^2 - \lambda_{345} c_{2\beta} - i \mathrm{Im}(\lambda_5 e^{2i\xi})\right] + c_\beta c_{3\beta} \mathrm{Re}(\lambda_6 e^{i\xi})\,, \\
& + s_\beta s_{3\beta} \mathrm{Re}(\lambda_7 e^{i\xi}) + i c_\beta^2 \mathrm{Im}(\lambda_6 e^{i\xi}) + i s_\beta^2 \mathrm{Im}(\lambda_7 e^{i\xi})\,, \\
Z_7\, e^{i\xi} =& -\tfrac{1}{2} s_{2\beta}\left[\lambda_1 s_\beta^2 - \lambda_2 c_\beta^2 + \lambda_{345} c_{2\beta} + i \mathrm{Im}(\lambda_5 e^{2i\xi})\right] + s_\beta s_{3\beta} \mathrm{Re}(\lambda_6 e^{i\xi}) \\
& + c_\beta c_{3\beta} \mathrm{Re}(\lambda_7 e^{i\xi}) + i s_\beta^2 \mathrm{Im}(\lambda_6 e^{i\xi}) + i c_\beta^2 \mathrm{Im}(\lambda_7 e^{i\xi})\,.
\end{aligned}
$$

The Higgs mass-eigenstate basis

Starting in the Higgs basis,

$$H_1 = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}\left(v + \varphi_1^0 + iG^0\right) \end{pmatrix}, \qquad e^{i\chi}H_2 = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}\left(\varphi_2^0 + iA\right) \end{pmatrix},$$

where φ_1^0, φ_2^0 are CP-even neutral Higgs fields, A is a CP-odd neutral Higgs field, and H^+ is the physical charged Higgs boson, with mass $m^2_{H^\pm} = Y_2 + \frac{1}{2}Z_3 v^2$. If the Higgs sector is CP-violating, then φ_1^0, φ_2^0, and A all mix to produce three physical neutral Higgs states of indefinite CP. After employing the potential minimum conditions: $Y_1 = -\frac{1}{2}Z_1 v^2$ and $Y_3 = -\frac{1}{2}Z_6 v^2$, the resulting neutral Higgs squared-mass matrix is:

$$\mathcal{M}^2 = \begin{pmatrix} Z_1 v^2 & \mathrm{Re}(Z_6)v^2 & -\mathrm{Im}(Z_6)v^2 \\ \mathrm{Re}(Z_6)v^2 & m_A^2 + \mathrm{Re}(Z_5)v^2 & -\frac{1}{2}\mathrm{Im}(Z_5)v^2 \\ -\mathrm{Im}(Z_6)v^2 & -\frac{1}{2}\mathrm{Im}(Z_5)v^2 & m_A^2 \end{pmatrix}$$

where m_A^2 is the following auxiliary quantity:

$$m_A^2 \equiv Y_2 + \tfrac{1}{2}[Z_3 + Z_4 - \mathrm{Re}(Z_5)]v^2 \,.$$

The squared-mass matrix is real symmetric; thus it is diagonalized by an orthogonal transformation $\mathcal{M}_D^2 = R\mathcal{M}^2 R^T$, where $RR^T = I$. A convenient form for R is:

$$R = \begin{pmatrix} c_{12} & -s_{12} & 0 \\ s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c_{13} & 0 & -s_{13} \\ 0 & 1 & 0 \\ s_{13} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & -s_{23} \\ 0 & s_{23} & c_{23} \end{pmatrix}$$

$$= \begin{pmatrix} c_{12}c_{13} & -c_{23}s_{12} - c_{12}s_{13}s_{23} & -c_{12}c_{23}s_{13} + s_{12}s_{23} \\ c_{13}s_{12} & c_{12}c_{23} - s_{12}s_{13}s_{23} & -c_{23}s_{12}s_{13} - c_{12}s_{23} \\ s_{13} & c_{13}s_{23} & c_{13}c_{23} \end{pmatrix},$$

where $c_{ij} \equiv \cos\theta_{ij}$ and $s_{ij} \equiv \sin\theta_{ij}$. The neutral Higgs mass eigenstates are denoted by h_1, h_2 and h_3:

$$\begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix} = R \begin{pmatrix} \varphi_1^0 \\ \varphi_2^0 \\ A \end{pmatrix} .$$

Since the mass-eigenstates h_i do not depend on the initial basis choice, they are U(2)-invariants. It follows that θ_{12} and θ_{13} are U(2)-invariants, whereas θ_{23} transforms as $e^{i\theta_{23}} \longrightarrow (\det\, U)^{-1} e^{i\theta_{23}}$. One can also check that the physical Higgs squared-masses (which are the eigenvalues of $\mathcal{M}^2$) are invariant quantities.

Example: Higgs self-couplings

Lightest neutral Higgs boson cubic self-coupling:

$$g(h_1h_1h_1) = -\,3v\Big\{Z_1c_{12}^3c_{13}^3 + (Z_3+Z_4)c_{12}c_{13}|s_{123}|^2 + \tfrac{1}{2}c_{12}c_{13}[Z_5(e^{2i\theta_{23}}\,s_{123}^2)^* + \text{c.c.}]$$

$$-\,\tfrac{3}{2}c_{12}^2c_{13}^2[Z_6(e^{i\theta_{23}}\,s_{123})^* + \text{c.c.}] - \tfrac{1}{2}|s_{123}|^2[Z_7(e^{i\theta_{23}}\,s_{123})^* + \text{c.c.}]\Big\}$$

Lightest neutral Higgs boson quartic self-coupling:

$$g(h_1h_1h_1h_1) = -\,3\Big\{Z_1c_{12}^4c_{13}^4 + Z_2|s_{123}|^4 + 2(Z_3+Z_4)c_{12}^2c_{13}^2|s_{123}|^2$$

$$+\,c_{12}^2c_{13}^2[Z_5(e^{2i\theta_{23}}\,s_{123}^2)^* + \text{c.c.}] - 2c_{12}^3c_{13}^3[Z_6(e^{i\theta_{23}}\,s_{123})^* + \text{c.c.}]$$

$$-\,2c_{12}c_{13}|s_{123}|^2[Z_7(e^{i\theta_{23}}\,s_{123})^* + \text{c.c.}]\Big\}$$

where $s_{123} \equiv s_{12} - ic_{12}s_{13}$.

Note that these quantities depend on U(2)-invariants. In particular $Z_5e^{-2i\theta_{23}}$, $Z_6e^{-i\theta_{23}}$ and $Z_7e^{-i\theta_{23}}$ are U(2)-invariants!

The Higgs-fermion Yukawa couplings

In the generic basis, the Higgs-fermion Yukawa Lagrangian is:

$$-\mathcal{L}_Y = \overline{Q}^0_L \widetilde{\Phi}_1 \eta_1^{U,0} U^0_R + \overline{Q}^0_L \Phi_1 (\eta_1^{D,0})^* D^0_R + \overline{Q}^0_L \widetilde{\Phi}_2 \eta_2^{U,0} U^0_R + \overline{Q}^0_L \Phi_2 (\eta_2^{D,0})^* D^0_R + \text{h.c.}\,,$$

where $\widetilde{\Phi}_i \equiv i\sigma_2 \Phi_i^*$, Q^0_L is the weak isospin quark doublet, and U^0_R, D^0_R are weak isospin quark singlets in an interaction eigenstate basis, and $\eta_1^{U,0}, \eta_2^{U,0}, \eta_1^{D,0}, \eta_2^{D,0}$ are matrices in flavor space. Under a U(2) transformation, if we demand that $(\eta_i^{Q,0})_a \longrightarrow U_{a\bar{b}}(\eta_i^{Q,0})_b$ (for $Q = U, D$ and $i = 1, 2$) then $\mathcal{L}_{\rm Y}$ is invariant.

Identify the fermion mass eigenstates by employing the appropriate bi-unitary transformation of the quark mass matrices involving unitary matrices V_L^U, V_L^D, V_R^U, V_R^D, where $K \equiv V_L^U V_L^{D\,\dagger}$ is the CKM matrix. Then, define $\eta^Q \equiv (\eta_1^Q, \eta_2^Q)$, where $\eta_i^Q \equiv V_L^Q \eta_i^{Q,0} V_R^{Q\,\dagger}$, for $Q = U$, D. Define $\eta^Q \equiv (\eta_1^Q\,,\ \eta_2^Q)$ and introduce the quantities:

$$\kappa^Q \equiv \widehat{v}^* \cdot \eta^Q\,, \qquad\qquad \rho^Q \equiv \widehat{w}^* \cdot \eta^Q\,.$$

Under a U(2) transformation, κ^Q is invariant, whereas $\rho^Q \to (\det\, U)\rho^Q$.

The quark mass terms are identified by replacing the scalar fields with their vev's. V_L^U, V_L^D, V_R^U and V_R^D are then chosen so that κ^D and κ^U are diagonal with real non-negative entries. The resulting quark mass matrices are then diagonal:

$$M_D = \frac{v}{\sqrt{2}}\kappa^D \,, \qquad M_U = \frac{v}{\sqrt{2}}\kappa^U \,.$$

Converting the Higgs-fermion interaction to the Higgs basis, one finds:

$$\begin{aligned}
-\mathcal{L}_Y = {} & \frac{1}{v}\overline{D}M_D D\varphi_1^0 + \frac{1}{\sqrt{2}}\overline{D}\left[(\rho^D)^* P_R + (\rho^D)^T P_L\right] D\varphi_2^0 + \frac{i}{\sqrt{2}}\overline{D}\left[(\rho^D)^* P_R - (\rho^D)^T P_L\right] DA + \frac{i}{v}\overline{D}M_D\gamma_5 DG^0 \\
& + \frac{1}{v}\overline{U}M_U U\varphi_1^0 + \frac{1}{\sqrt{2}}\overline{U}\left[\rho^U P_R + \rho^{U\dagger} P_L\right] U\varphi_2^0 - \frac{i}{\sqrt{2}}\overline{U}\left[\rho^U P_R - \rho^{U\dagger} P_L\right] UA - \frac{i}{v}\overline{U}M_D\gamma_5 UG^0 \\
& + \left\{\overline{U}\left[K(\rho^D)^* P_R - \rho^{U\dagger} K P_L\right] DH^+ + \frac{\sqrt{2}}{v}\overline{U}\left[KM_D P_R - M_U K P_L\right] DG^+ + \text{h.c.}\right\} ,
\end{aligned}$$

where $P_{R,L} = \frac{1}{2}(1 \pm \gamma_5)$. The physical couplings of the quarks to the Higgs fields are then obtained by expressing φ_1^0, φ_2^0 and A in terms of the neutral Higgs mass eigenstates. Note that under a U(2) transformation, $H^+ \longrightarrow (\det\, U)H^+$, so that the combination $(\rho^Q)^* H^+$ is invariant. The Goldstone couplings are also invariant.

The final form for the Yukawa couplings of the neutral Higgs bosons to the quarks is:

$$
\begin{aligned}
-\mathcal{L}_Y =& \frac{1}{v}\overline{D}\left[M_D c_{12}c_{13} - \frac{v}{\sqrt{2}}\left[(\rho^D e^{i\theta_{23}} s_{123})^* P_R + (\rho^D e^{i\theta_{23}} s_{123})^T P_L\right]\right] D h_1 \\
&+ \frac{1}{v}\overline{D}\left[M_D s_{12}c_{13} + \frac{v}{\sqrt{2}}\left[(\rho^D e^{i\theta_{23}} c_{123})^* P_R + (\rho^D e^{i\theta_{23}} c_{123})^T P_L)\right]\right] D h_2 \\
&+ \frac{1}{v}\overline{D}\left[M_D s_{13} + \frac{iv}{\sqrt{2}} c_{13}\left[(\rho^D e^{i\theta_{23}})^* P_R - (\rho^D e^{i\theta_{23}})^T P_L\right]\right] D h_3 \\
&+ \frac{1}{v}\overline{U}\left[M_U c_{12}c_{13} - \frac{v}{\sqrt{2}}\left[(\rho^U e^{i\theta_{23}} s_{123}) P_R + (\rho^U e^{i\theta_{23}} s_{123})^\dagger P_L\right]\right] U h_1 \\
&+ \frac{1}{v}\overline{U}\left[M_U s_{12}c_{13} + \frac{v}{\sqrt{2}}\left[(\rho^U e^{i\theta_{23}} c_{123}) P_R + (\rho^U e^{i\theta_{23}} c_{123})^\dagger P_L)\right]\right] U h_2 \\
&+ \frac{1}{v}\overline{U}\left[M_U s_{13} - \frac{iv}{\sqrt{2}} c_{13}\left[(\rho^U e^{i\theta_{23}}) P_R - (\rho^U e^{i\theta_{23}})^\dagger P_L\right]\right] U h_3 \,,
\end{aligned}
$$

where $s_{123} \equiv s_{12} - ic_{12}s_{13}$ and $c_{123} \equiv c_{12} + is_{12}s_{13}$. The Higgs-fermion Yukawa couplings depend only on invariant quantities: the diagonal quark mass matrices, $\rho^Q e^{i\theta_{23}}$, and the invariant angles θ_{12} and θ_{13}. In general, the ρ^Q are complex non-diagonal matrices. Hence, $\mathcal{L}_{\rm Y}$ exhibits tree-level Higgs-mediated flavor-changing neutral currents.

The significance of $\tan\beta$

So far, $\tan\beta$ has been completely absent from the Higgs couplings. This must be so, since $\tan\beta$ is basis-dependent in a general 2HDM. However, a particular 2HDM may single out a preferred basis, in which case $\tan\beta$ would be promoted to an observable. To simplify the discussion, we focus on a one-generation model, where the Yukawa coupling matrices are simply numbers.

As an example, the MSSM Higgs sector is a type-II 2HDM, *i.e.*, $\eta_1^U = \eta_2^D = 0$. A basis-independent condition for type-II is: $\eta^U \cdot \eta^{D\,*} = 0$. In the preferred basis, $\hat{v} = (\cos\beta\,,\ \sin\beta\, e^{i\xi})$ and $\hat{w} = (-\sin\beta e^{-i\xi}\,,\ \cos\beta)$. Evaluating $\kappa^Q = \hat{v}^* \cdot \eta^Q$ and $\rho^Q = \hat{w}^* \cdot \eta^Q$ in the preferred basis, it follows that:

$$e^{-i\xi}\tan\beta = -\frac{\rho^{D\,*}}{\kappa^D} = \frac{\kappa^U}{\rho^U}\,,$$

where $\kappa^Q = \sqrt{2}m_Q/v$. These two definitions are consistent if $\kappa^U\kappa^D + \rho^U\rho^{D\,*} = 0$ is satisfied. But this is equivalent to the type-II condition, $\eta^U \cdot \eta^{D\,*} = 0$.

Since ρ^Q is a pseudo-invariant, we can eliminate ξ by rephasing Φ_2. Hence,

$$\tan\beta = \frac{|\rho^D|}{\kappa^D} = \frac{\kappa^U}{|\rho^U|}\,,$$

with $0 \leq \beta \leq \pi/2$. Indeed, $\tan\beta$ is now a physical parameter, and the $|\rho^Q|$ are no longer independent:

$$|\rho^D| = \frac{\sqrt{2}m_d\tan\beta}{v}\,, \qquad |\rho^U| = \frac{\sqrt{2}m_u\cot\beta}{v}\,.$$

In the more general (type-III) 2HDM, $\tan\beta$ is not a meaningful parameter. Nevertheless, one can introduce three $\tan\beta$-like parameters:*

$$\tan\beta_d \equiv \frac{|\rho^D|}{\kappa^D}\,, \qquad \tan\beta_u \equiv \frac{\kappa^U}{|\rho^U|}\,, \qquad \tan\beta_e \equiv \frac{|\rho^E|}{\kappa^E}\,,$$

the last one corresponding to the Higgs-lepton interaction. In a type-III 2HDM, there is no reason for the three parameters above to coincide.

* Interpretation: In the Higgs basis, up and down-type quarks interact with both Higgs doublets. But, clearly there exists some basis (*i.e.*, a rotation by angle β_u from the Higgs basis) for which only one of the two up-type quark Yukawa couplings is non-vanishing. This defines the physical angle β_u.

The MSSM Higgs sector is a type-III 2HDM

The tree-level Higgs potential of the MSSM satisfies:

$$\lambda_1=\lambda_2=-\lambda_{345}=\tfrac{1}{4}(g^2+g'^2)\,,\lambda_4=-\tfrac{1}{2}g^2\,,\lambda_5=\lambda_6=\lambda_7=0\,.$$

But, one-loop radiative corrections generate corrections to these relations, due to SUSY-breaking. *E.g.*, at one-loop, λ_5, λ_6, $\lambda_7\neq 0$.

For MSSM Higgs couplings to fermions, Yukawa vertex corrections modify the effective Lagrangian that describes the coupling of the Higgs bosons to the third generation quarks:

$$-\mathcal{L}_{\rm eff}=\epsilon_{ij}\left[(h_b+\delta h_b)\bar{b}_R H_d^i Q_L^j+(h_t+\delta h_t)\bar{t}_R Q_L^i H_u^j\right]+\Delta h_b \bar{b}_R Q_L^k H_u^{k*}+\Delta h_t \bar{t}_R Q_L^k H_d^{k*}+{\rm h.c.}$$

Indeed, this is a general type-III model. For example, in some MSSM parameter regimes (corresponding to large $\tan\beta$ and large supersymmetry-breaking scale compared to v),[†]

$$\Delta h_b\simeq h_b\left[\frac{2\alpha_s}{3\pi}\mu M_{\tilde g}\,I(M^2_{\tilde b_1},M^2_{\tilde b_2},M^2_{\tilde g})+\frac{h_t^2}{16\pi^2}\mu A_t\,I(M^2_{\tilde t_1},M^2_{\tilde t_2},\mu^2)\right]\,.$$

This leads to a modification of the tree-level relation between m_b and h_b. In addition, it leads to a splitting of $\tan\beta_d$ and $\tan\beta_u$.

[†] $I(a,b,c)=[ab\ln(a/b)+bc\ln(b/c)+ca\ln(c/a)]/(a-b)(b-c)(a-c)$.

For illustrative purposes, we neglect CP violation in the following simplified discussion. The tree-level relation between m_b and h_b is modified:

$$m_b = \frac{h_b v}{\sqrt{2}} \cos\beta(1 + \Delta_b) \,,$$

where $\Delta_b \equiv (\Delta h_b/h_b) \tan\beta$. That is, Δ_b is $\tan\beta$-enhanced, and governs the leading one-loop correction to the physical Higgs couplings to third generation quarks. In typical models at large $\tan\beta$, Δ_b can be of order 0.1 or larger and of either sign.

In the approximation scheme above, $\kappa^U \simeq h_t \sin\beta$, $\rho^U \simeq h_t \cos\beta$, and

$$\begin{aligned}
\kappa^D &\simeq h_b \cos\beta(1 + \Delta_b) \,, \\
\rho^D &\simeq -\, h_b \sin\beta \left(1 - \frac{\Delta_b}{\tan^2\beta}\right) \simeq -h_b \sin\beta \,.
\end{aligned}$$

It follows that:

$$\tan\beta_d \simeq \frac{\tan\beta}{1 + \Delta_b} \,, \qquad\qquad \tan\beta_u \simeq \tan\beta \,.$$

Thus, supersymmetry-breaking loop-effects can yield observable differences between $\tan\beta$-like parameters that are defined in terms of basis-independent quantities.

Some limiting cases

1. The CP-conserving limit

In the case, $h_1 = h$ and $h_2 = H$ are the CP-even Higgs bosons (with $m_h < m_H$) and $h_3 = -A$ is the CP-odd Higgs boson.[‡] The Higgs mixing angles reduce to:

$$c_{12} = \sin(\beta - \alpha)\,, \quad s_{12} = \cos(\beta - \alpha)\,, \quad c_{13} = -c_{23} = 1\,, \quad s_{13} = s_{23} = 0\,.$$

2. The decoupling limit

Recall that the auxiliary parameter $m_A^2 = m_{H^\pm}^2 + \frac{1}{2}[Z_4 - \mathrm{Re}(Z_5)]v^2$. In the decoupling limit, m_A^2, $m_{H^\pm}^2 \gg v^2$, with $|Z_i| \lesssim \mathcal{O}(1)$. One can show that in this limit:

$$s_{12} \sim s_{13} \sim \mathcal{O}(v^2/m_A^2)\,, \qquad c_{12} \sim c_{13} \sim 1\,,$$

whereas neither c_{23} nor s_{23} is particularly close to zero or one. In this limit, $h_1 \simeq h$, while h_2 and h_3 are states of indefinite CP ($i.e.$, strongly-mixed combinations of H and A).

[‡]Making contact with the standard notation of the CP-conserving 2HDM, the Higgs mixing matrix has $\det R = -1$. Hence, we need to insert the extra minus sign in the relation between h_3 and A.

Lessons and future work

• If phenomena consistent with the 2HDM is found, we will not know a priori the underlying structure that governs the model. In this case, one needs a model-independent analysis of the data that allows for the most general CP-violating Model-III.

• Instead of claiming that you have measured $\tan\beta$ (which can only be done in the context of a simplified version of the model), measure the physical parameters of the model. Examples include the $\tan\beta$-like parameters introduced in the one-generation model. (For three generations, the formalism becomes unwieldly. However, one has good reason to assume that the third generation quark–Higgs Yukawa couplings dominate.)

• Which $\tan\beta$-like parameters will be measured in precision Higgs studies at the linear collider (ILC)? How can one best treat the full three-generation model to one-loop order? What simplifications can be exploited in the MSSM?

• Compute the one-loop radiative corrections to various Higgs processes in terms of the physical $\tan\beta$-like parameters in the MSSM.