Little Higgs models

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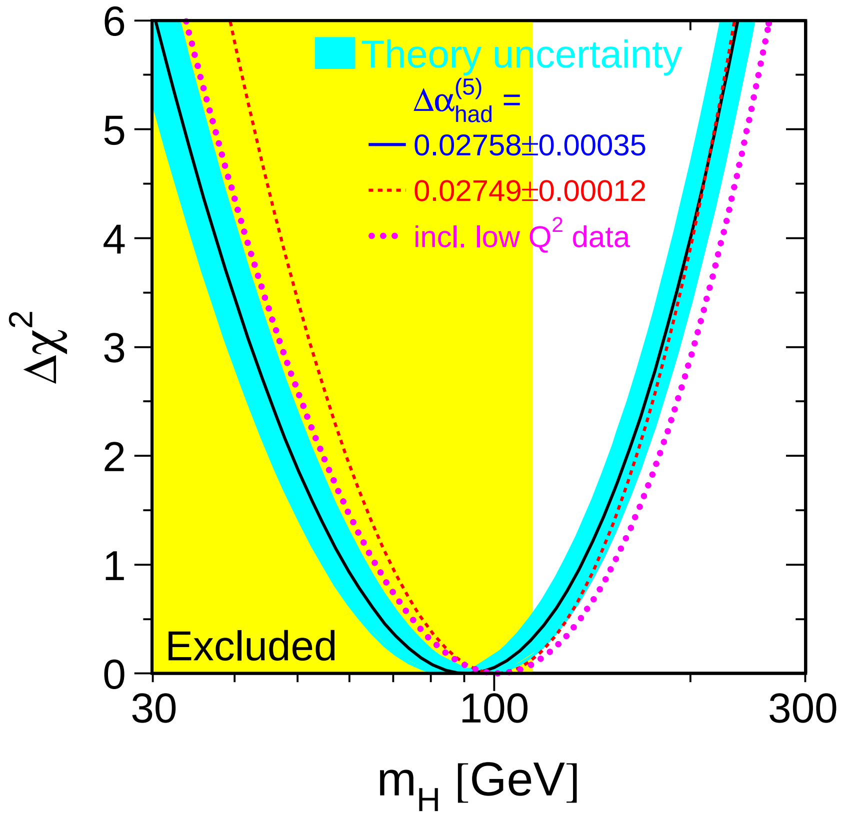
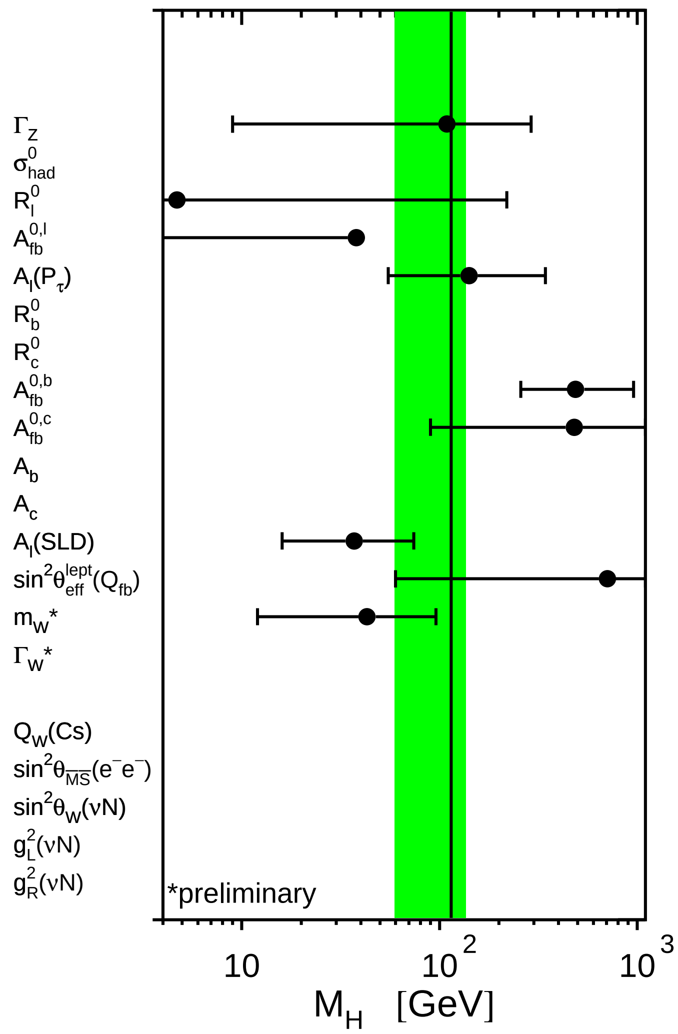
Introduction

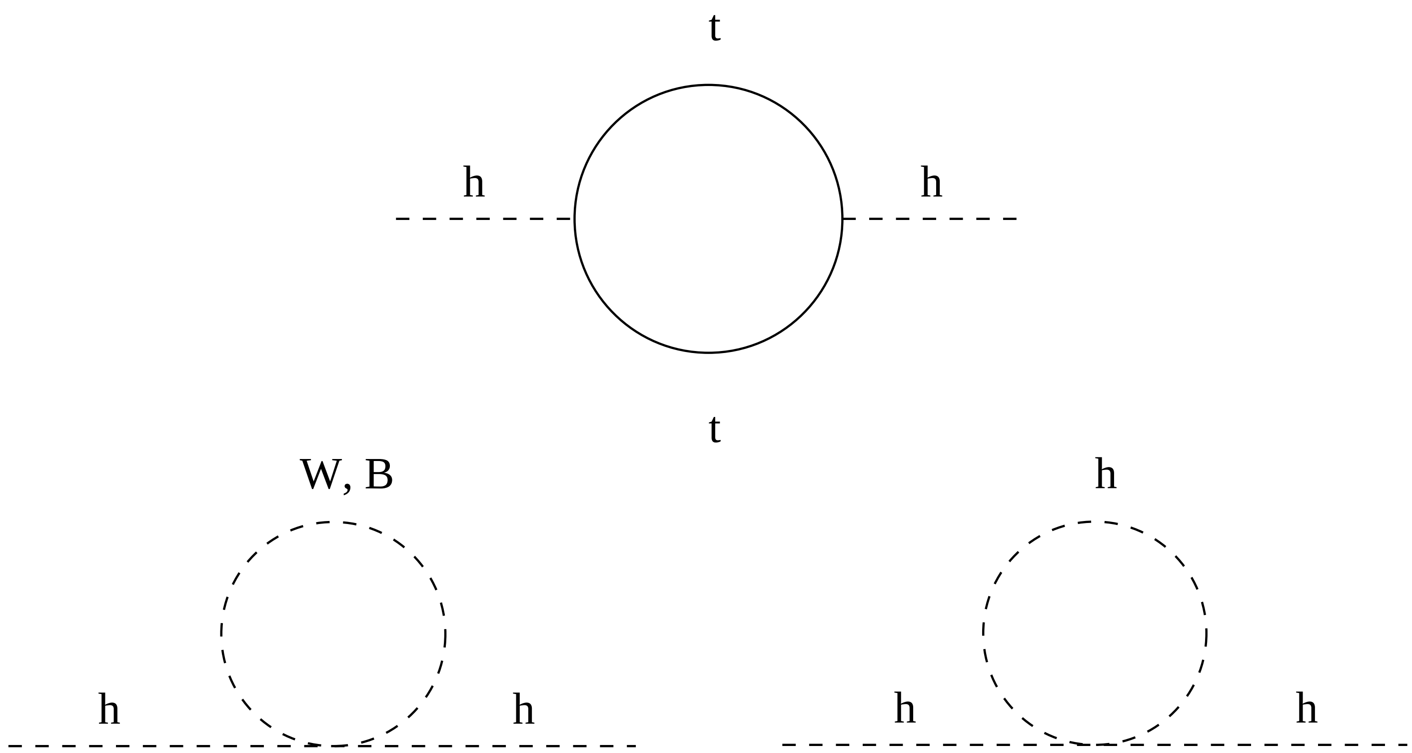




Precision EW data indicate SM with a light Higgs $m_H < 219$ GeV at 95% CL (`lepewwg.web.cern.ch/LEPEWWG`)

The naturalness problem : Higgs mass is quadratically sensitive to the cut-off scale Λ

$$
\begin{aligned}
m_h^2 &= m_{H0}^2 - \frac{3}{8\pi^2}\, y_t^2 \Lambda^2 + \frac{1}{16\pi^2} g^2 \Lambda^2 + \frac{1}{16\pi^2} \lambda^2 \Lambda^2 \\
(200 \text{ GeV})^2 &= m_{H0}^2 + \left[-(2 \text{ TeV})^2 + (700 \text{ GeV})^2 + (500 \text{ GeV})^2 \right] \left(\frac{\Lambda}{10 \text{ TeV}} \right)^2
\end{aligned}
$$



Little Higgs approach to EW symmetry breaking

- The idea is to consider the Higgs fields as Nambu Goldstone Bosons of a global symmetry which is spontaneously broken at some higher scale f by an expectation value (Dimopoulos, Preskill 1982; Georgi, Kaplan 1984; Banks 1984).
- The Higgs field gets mass radiatively through symmetry breaking at the electroweak scale by collective breaking. It is protected by the approximate global symmetry and it remains light.
- The cancellation of the quadratic divergence is realized between particles of the same statistics (Arkani-Hamed, Cohen, Georgi, hep-ph/0105239).

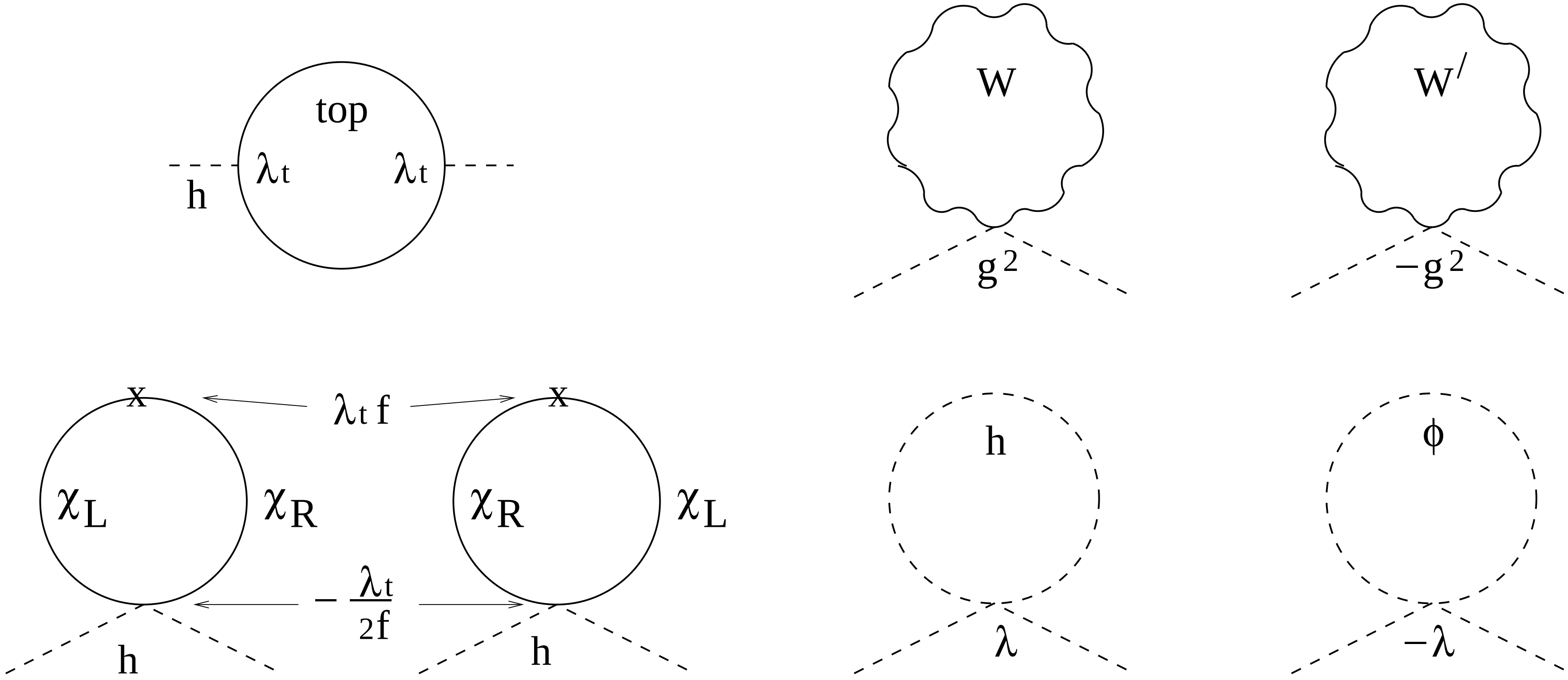


Figure 1: *Cancellation of top, gauge and Higgs loops by particles of the same statistics. Figures from M.Schmaltz hep-ph/0210415.*

Collective symmetry breaking

As an example consider the light fields Φ_i in $SU(3)$ vectors

$$\Phi_1 = e^{i\theta/f} \left(\begin{array}{c} 0 \\ 0 \\ f \end{array} \right) \qquad\qquad \Phi_2 = e^{-i\theta/f} \left(\begin{array}{c} 0 \\ 0 \\ f \end{array} \right) .$$

θ are the pseudo-Goldstone bosons (neglecting a singlet η)

$$\theta = \frac{1}{\sqrt{2}} \left(\begin{array}{ccc} 0 & 0 & \\ & & h \\ 0 & 0 & \\ & h^\dagger & 0 \end{array} \right) ,$$

i.e. a complex Higgs doublet which is charged under the unbroken $SU(2)$ gauge group. The $SU(2)$ gauge interactions for h stem from the $SU(3)$ gauge couplings of the Φ's

$$\left[(\partial_\mu + igA_\mu)\Phi_1\right]^\dagger (\partial_\mu + igA_\mu)\Phi_1 + \left[(\partial_\mu + igA_\mu)\Phi_2\right]^\dagger (\partial_\mu + igA_\mu)\Phi_2 \ .$$

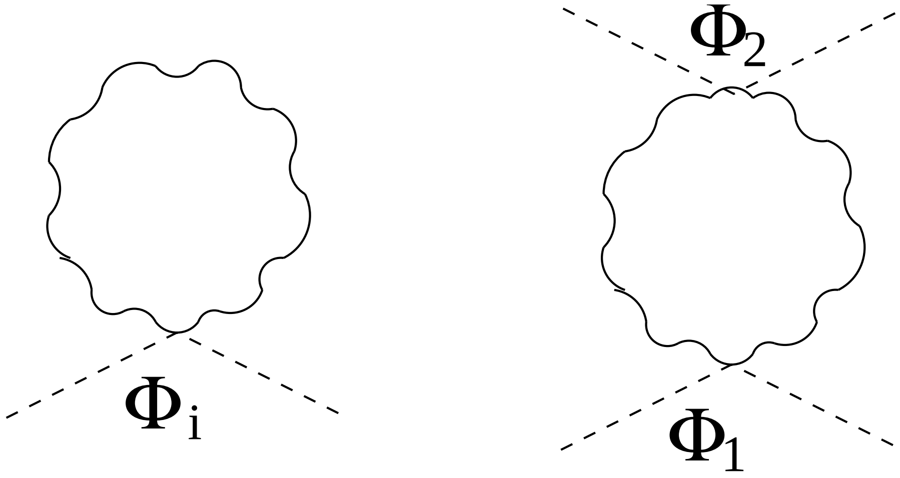


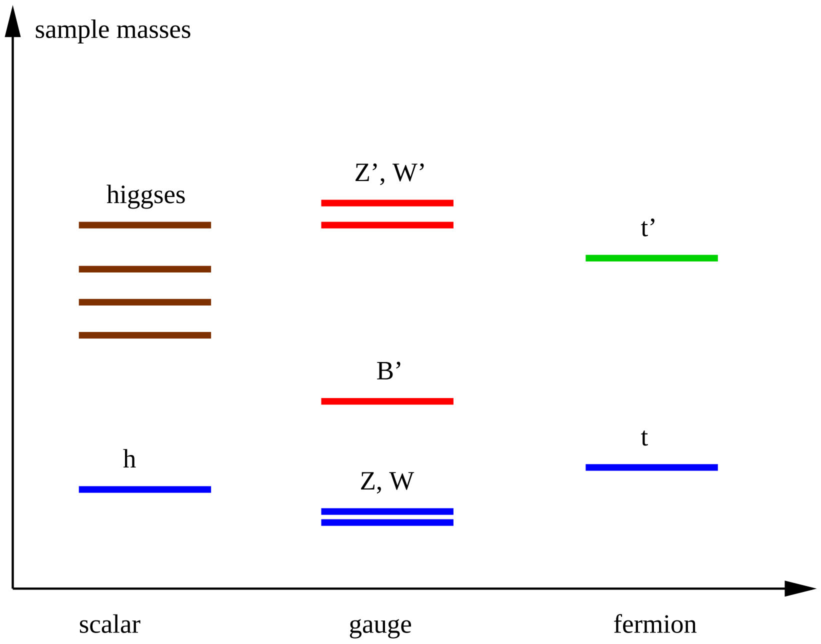
Figure 2: *Quadratically and logarithmically divergent diagrams from the* $SU(3)$ *gauge interactions. The first diagram does not give a Higgs mass.*

Only the logarithmically divergent second diagram gives rise to an operator which explicitly breaks $[SU(3)]^2 \to SU(3)$ and therefore contains a mass for h

$$\frac{g^4}{16\pi^2} \left|\Phi_1^\dagger \Phi_2\right|^2 \log(\Lambda^2/f^2) = \frac{g^4 f^2}{16\pi^2} \log(\Lambda^2/f^2)\ h^\dagger h\ ,$$

which is of order 100 GeV for $f \sim 1$ TeV.

Littlest Higgs*



Model parameters

$\tan\theta = s/c = g_2/g_1$, $\tan\theta' = s'/c' = g_2'/g_1'$ new $SU(2)$ and $U(1)$ mixing

f symmetry breaking scale $\mathcal{O}$ TeV $\qquad v'$ scalar triplet vev

m_H Higgs mass $\qquad m_T$ heavy vector top mass

* Arkani-Hamed et al. hep-ph/0206021

The model is based on $SU(5) \to SO(5)$ global symmetry breaking (24-10=14 Goldstone bosons) by a vev of the order f

$$\langle \Sigma \rangle = \begin{pmatrix} 0 & 0 & \mathbb{1}_2 \\ 0 & 1 & 0 \\ \mathbb{1}_2 & 0 & 0 \end{pmatrix} .$$

4 are eaten by the gauge bosons of the broken gauge group. The Goldstone boson matrix is

$$\Pi = \begin{pmatrix} 0 & h^\dagger/\sqrt{2} & \phi^\dagger \\ h/\sqrt{2} & 0 & h^*/\sqrt{2} \\ \phi & h^T/\sqrt{2} & 0 \end{pmatrix} .$$

h transforms as a doublet and ϕ as a triplet. The gauge symmetry breaking is $[SU(2) \times U(1)]^2 \to SU(2) \times U(1)$.

The scalar sigma model field can be written as

$$\Sigma = e^{i\Pi/f}\langle\Sigma\rangle e^{i\Pi^T/f} = e^{2i\Pi/f}\langle\Sigma\rangle$$

The kinetic term for the scalar fields is given by

$$\mathcal{L}_{kin} = \frac{1}{2}\frac{f^2}{4}\mathrm{Tr}[D_\mu\Sigma D^\mu\Sigma] \ ,$$

with the covariant derivative defined as

$$D_\mu\Sigma = \partial_\mu\Sigma - i(A_\mu\Sigma + \Sigma A_\mu^T) \ .$$

A_μ is the gauge boson matrix:

$$A_\mu = g_1 W_\mu^{1a} Q_1^a + g_2 W_\mu^{2a} Q_2^a + g_1' B_\mu^1 Y_1 + g_2' B_\mu^2 Y_2 \ ,$$

Q_i^a are the generators of the two $SU(2)$, Y_i those of the two $U(1)$ groups.

After symmetry breaking the gauge boson matrix can be diagonalized by the following transformations:

$$
\begin{aligned}
W &= sW_1 + cW_2 \qquad W' = -cW_1 + sW_2 \\
B &= s'B_1 + c'B_2 \qquad B' = -c'B_1 + s'B_2 \ .
\end{aligned}
$$

s, c, s', and c' denote the sines and cosines of two mixing angles, respectively. They can be expressed with the help of the coupling constants:

$$
\begin{aligned}
c' &= g'/g'_2 \qquad\qquad s' = g'/g'_1 \\
c &= g/g_2 \qquad\qquad s = g/g_1 \ ,
\end{aligned}
$$

with the usual SM couplings g, g', related to g_1, g_2, g'_1 and g'_2 by

$$
\frac{1}{g^2} = \frac{1}{g_1^2} + \frac{1}{g_2^2}, \qquad \frac{1}{{g'}^2} = \frac{1}{{g'_1}^2} + \frac{1}{{g'_2}^2} \ .
$$

Up to the order v^2/f^2 the equations of motion give :

$$\begin{aligned}
W'^{\pm\mu} &= g\,x_W\,W^{\pm\mu} + \frac{x_W^F}{\sqrt{2}}\left(J^{\pm\mu} - (1-c_L)J_3^{\pm\mu}\right) \\
W'^{3\mu} &= y_W(g\,W^{3\mu} + g'\,B^{\mu}) + x_W^F\left(J^{0\mu} - s_L^2\bar{t}_L\gamma^\mu t_L\right) \\
B'^{\mu} &= x_B(g\,W^{3\mu} + g'\,B^{\mu}) + x_B^F\Bigg[(J^\mu_{em} + J^{0\mu}) - \frac{5c'^2}{2(3c'^2-2s'^2)}s_L^2\bar{t}_L\gamma^\mu t_L \\
&- \frac{1}{3c'^2-2s'^2}s_R^2\bar{t}_R\gamma^\mu t_R\Bigg]\ ,
\end{aligned}$$

where :

$$\begin{aligned}
x_W^F &= -\frac{4c^3 s}{g f^2} \qquad x_B^F = \frac{4c's'}{g' f^2}(3c'^2 - 2s'^2) \\
x_W &= y_W = \frac{cs}{2g}(c^2-s^2)\frac{v^2}{f^2} \\
x_B &= \frac{2c's'}{g'}(c'^2-s'^2)\frac{v^2}{f^2}\ .
\end{aligned}$$

In terms of the model parameters we obtain:

$$\frac{G_F}{\sqrt{2}} = \frac{\alpha\pi(g^2+g'^2)}{2g^2g'^2m_Z^2}\left(1 - c^2(c^2-s^2)\frac{v^2}{f^2} + 2c^4\frac{v^2}{f^2} - \frac{5}{4}(c'^2-s'^2)^2\frac{v^2}{f^2}\right)$$

and defining the Weinberg angle as

$$\frac{G_F}{\sqrt{2}} = \frac{\alpha\pi}{2s_\theta^2c_\theta^2m_Z^2} \ .$$

we have

$$\begin{aligned} m_Z^2 &= (g^2+g'^2)\frac{v^2}{4}\left[1 - \frac{v^2}{f^2}\left(\frac{1}{6} + \frac{(c^2-s^2)^2}{4} + \frac{5}{4}(c'^2-s'^2)\right) + 8\frac{v'^2}{v^2}\right] \ , \\ m_W^2 &= \frac{g^2v^2}{4}\left[1 - \frac{v^2}{f^2}\left(\frac{1}{6} + \frac{(c^2-s^2)^2}{4}\right) + 4\frac{v'^2}{v^2}\right] \ . \end{aligned}$$

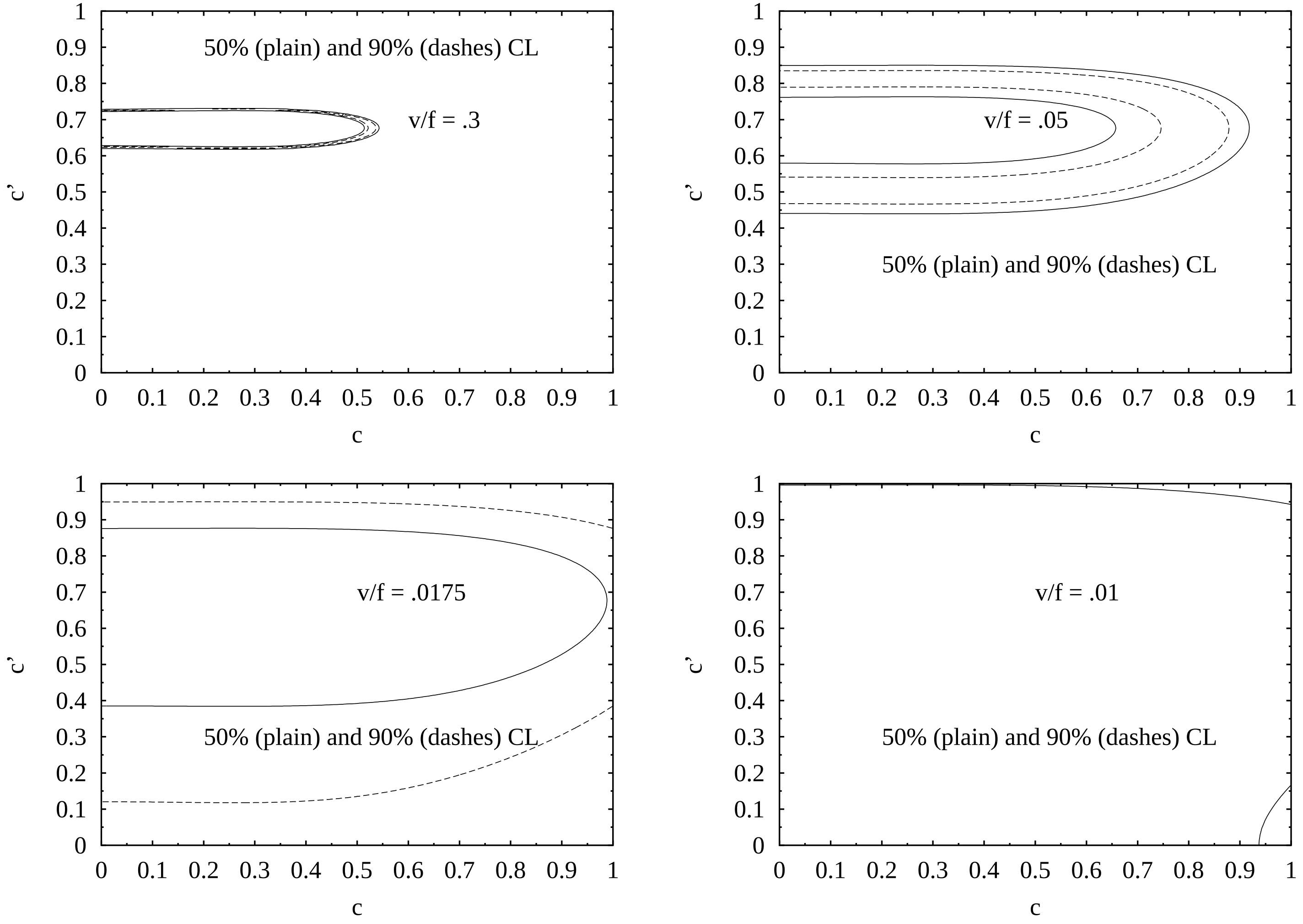


Figure 3: *90% and 50% CL exclusion contours in the plane c-c'. The value of the triplet vev v' is fixed to $v'^2/v^2 = v^2/(17f^2)$. The allowed region lies inside the 90% and 50% bands, respectively. From hep-ph/0311038.*

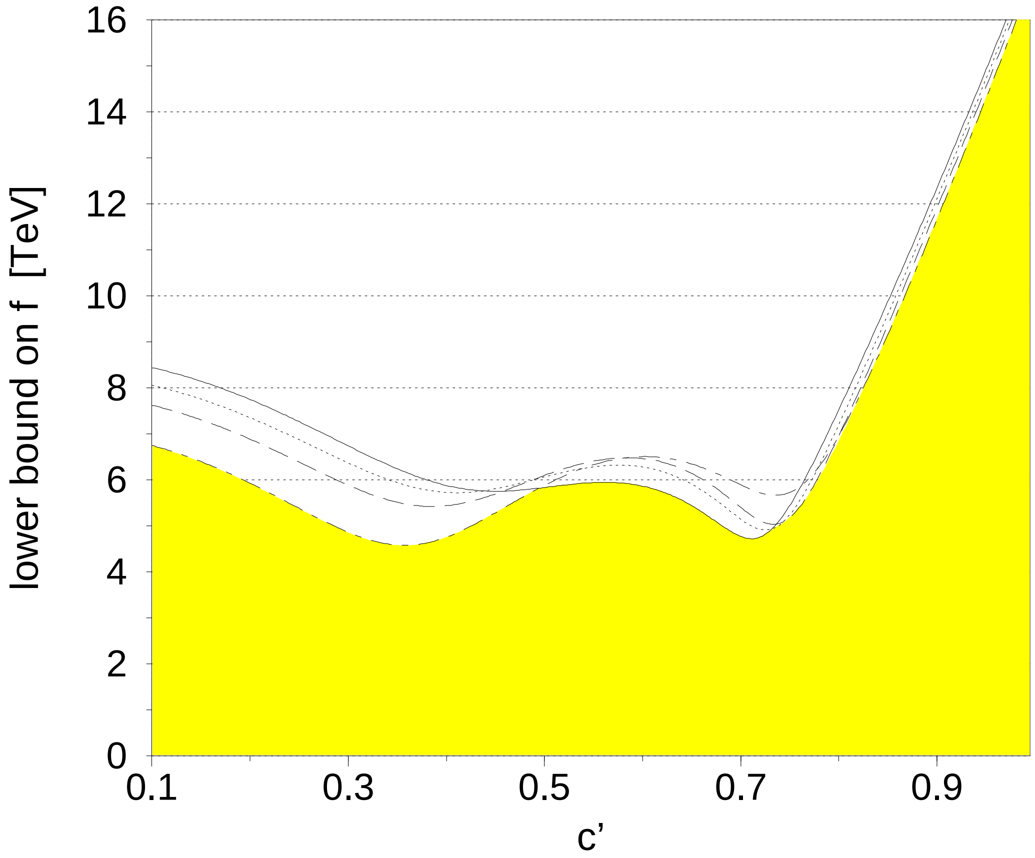


Figure 4: *The region below the contours is excluded to 95% C.L. for* c *equal to* 0.1 *(solid),* 0.5 *(dotted),* 0.7 *(dashed),* 0.99 *(dot-dashed). The yellow region is excluded for any choice of* c*. From hep-ph/0305157 based on hep-ph/0211124, hep-ph/0303236 by Csáki et al.*

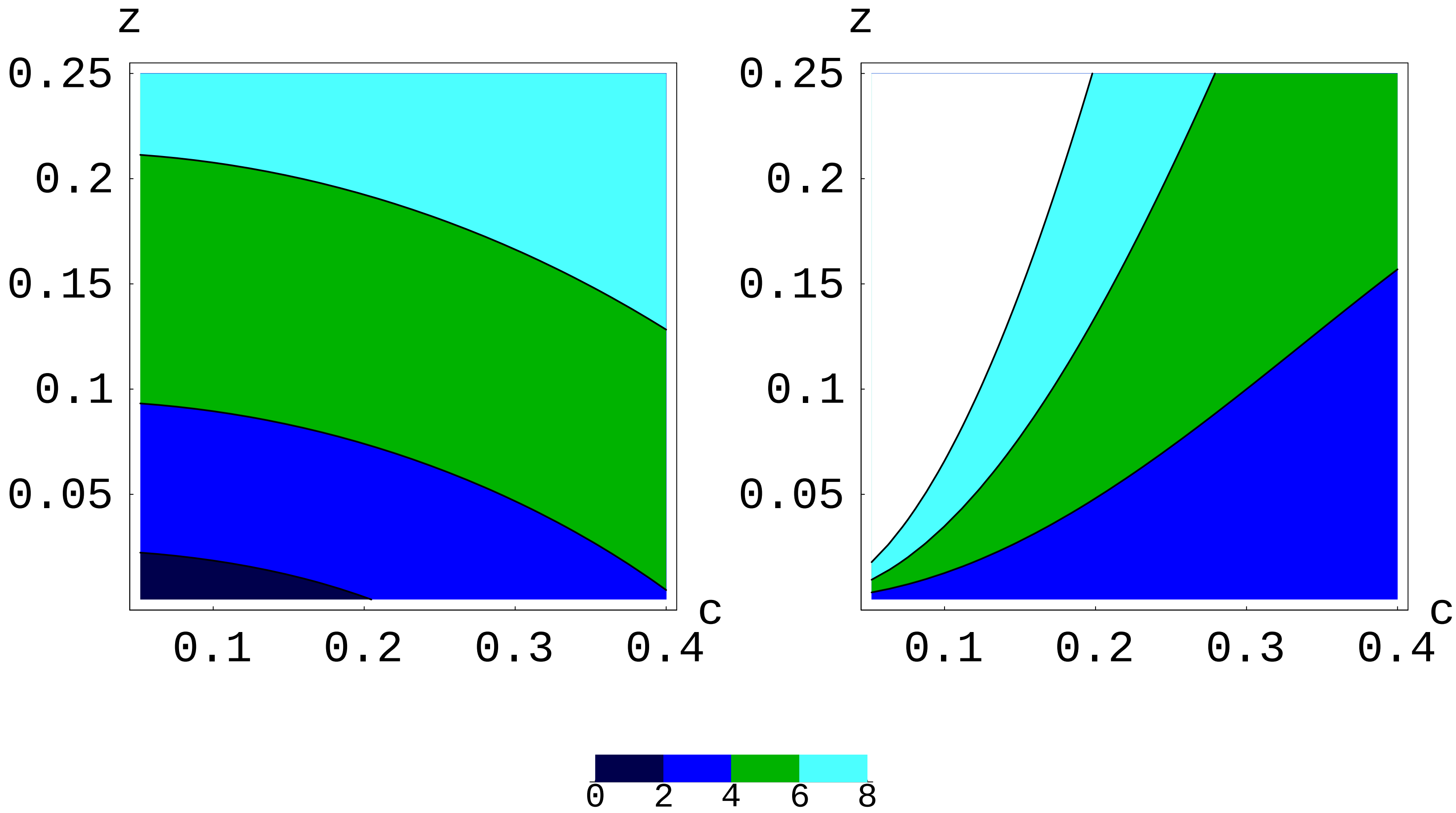


Figure 5: *95% CL lower bounds in TeV on $M_{t'}$ (left) and $M_{W'}$ (right) in the $SU(5)$ model as functions of c and $z \equiv \lambda^2 f^2/M_\phi^4$ for $Y_1^f = Y_2^f$ and $s' = c'$. In the plots, $c \in [g/4\pi, 0.4]$ and $z \in [0, 1/4]$. From hep-ph/0506206 by Han and Skiba.*

Little Higgs with custodial $SU(2)^*$

The model is based on a $SO(9)/[SO(5) \times SO(4)]$ coset space, with $SU(2)_L \times SU(2)_R \times SU(2) \times U(1)$ subgroup of $SO(9)$ gauged. The vev is

$$\langle \Sigma \rangle = \begin{pmatrix} 0 & 0 & \mathbb{1}_4 \\ 0 & 1 & 0 \\ \mathbb{1}_4 & 0 & 0 \end{pmatrix}$$

breaking the $SO(9)$ global symmetry down to an $SO(5) \times SO(4)$ subgroup. This coset space has $20 = (36 - 10 - 6)$ light scalars. Of these 20 scalars, 6 will be eaten in the higgsing of the gauge groups down to $SU(2)_W \times U(1)_Y$. The remaining 14 scalars are : a single higgs doublet h, an electroweak singlet ϕ^0, and three triplets ϕ^{ab}.

* S.Chang hep-ph/0306034

The equations of motion up to the order v^2/f^2 are

$$
\begin{aligned}
W'^{\,1,2} &= -\frac{v^2\,cs}{2f^2}\left(c^2-s^2\right)\,W^{1,2}+\frac{s^3c}{f^2\,g}\,J^{1,2}\\
W'^{\,3} &= -\frac{v^2\,cs}{2f^2}\left(c^2-s^2\right)\,(W^3-\frac{g'}{g}\,B)+\frac{s^3c}{f^2\,g}\,J^3\\
B' &= \frac{v^2\,c's'}{2f^2}\left(c'^{\,2}-s'^{\,2}\right)(\frac{g}{g'}\,W^3-B)+\frac{s'^{\,3}c'}{f^2\,g'}\,J^0\\
W_R^{1,2} &= \frac{v^2}{2f^2}\,W^{1,2}\,.
\end{aligned}
$$

The expression for G_F in terms of the model parameters is

$$
\frac{G_F}{\sqrt{2}}=\frac{\alpha\pi(g^2+g'^2)^2}{2g^2g'^2}\left(1+\frac{v^2}{f^2}\frac{s^2(c^2-s^2)-s^4}{2}\right)\,,
$$

In this case the masses of Z- and W-bosons are given by

$$
m_Z^2 = (g^2+g'^2)\frac{v^2}{4}
$$

$$m_W^2 \quad = \quad \frac{g^2 v^2}{4}\left(1 + 2\frac{v'^2}{v^2}\right) \ .$$

The corrections to the ϵ parameters to the order v^2/f^2 are

$$\begin{aligned}
\epsilon_1 &= \frac{v^2}{4f^2}\left[4s'^{\,2}\left(c'^{\,2} - s'^{\,2}\right) + 2c^2 s^2 - s^4\right] + 2\frac{v'^2}{v^2} \\
\epsilon_2 &= \frac{v^2}{4c_{2\theta}\, f^2}\Big[4s'^{\,2}\left(c'^{\,2} - s'^{\,2}\right) c_\theta^2 c_{2\theta} + 2s^2\left(c^2 - s^2\right)\left(c_\theta^4 - 3c_\theta^2 s_\theta^2 + 2c_\theta^2 - s_\theta^2\right) \\
&+ s^4 (c_\theta^4 + s_\theta^4)\Big] \\
\epsilon_3 &= \frac{v^2}{2s_\theta^2\, f^2}\left[s^2\left(c^2 - s^2\right)\left(-c_{2\theta} + 2s_\theta^2 c_\theta^2\right) - s^4 c_\theta^2 s_\theta^2\right]
\end{aligned}$$

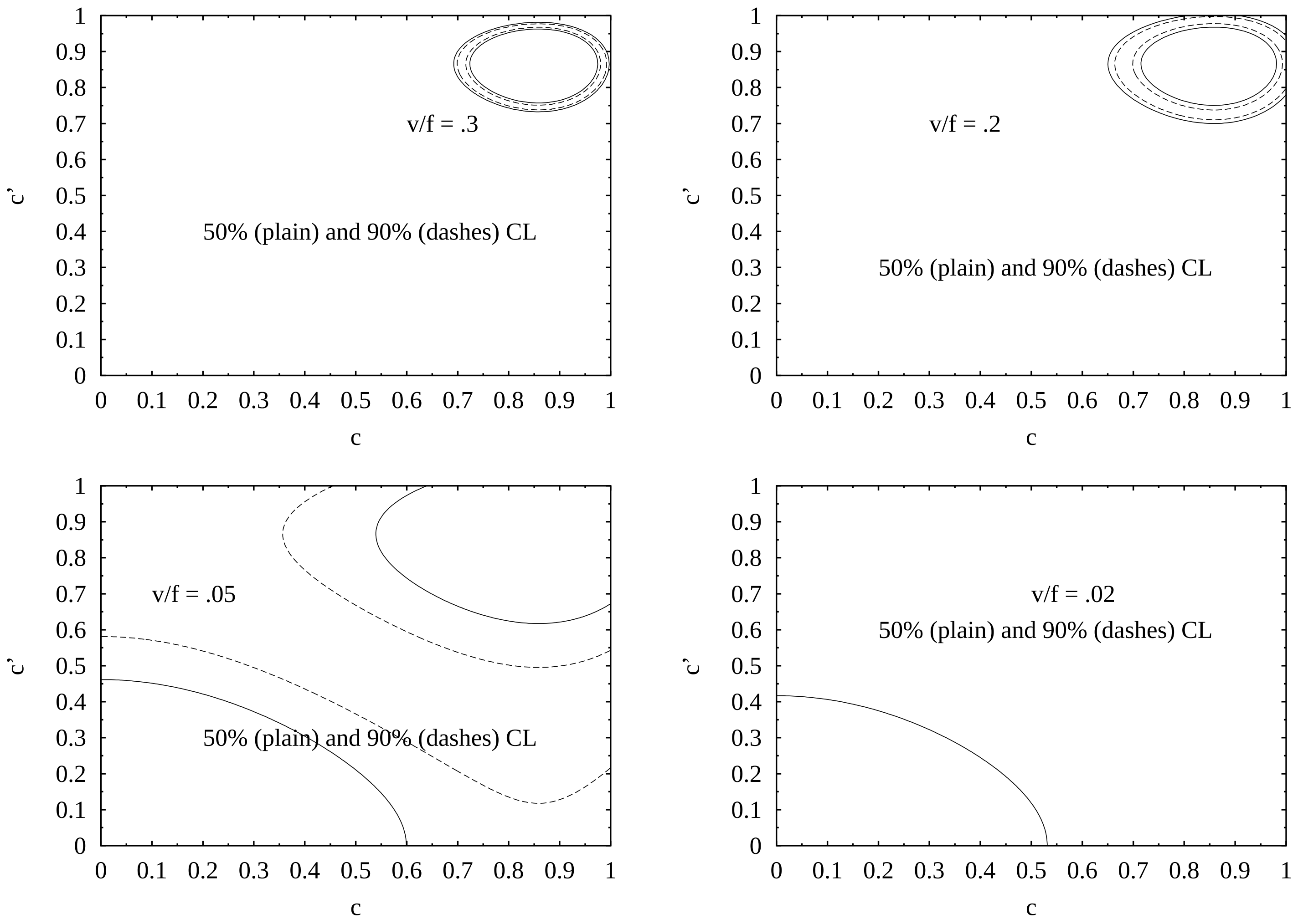


Figure 6: *90% and 50% CL exclusion contours in the plane* c-c' *of the* $SO(9)/[SO(5) \times SO(4)]$ *model. The value of the triplet vev* v' *is fixed to* $v'^2/v^2 = v^2/(17f^2)$. *The allowed region lies inside the 90% and 50% bands, respectively.*

$g - 2$ of the muon

The relevant one-loop Feynman diagrams are

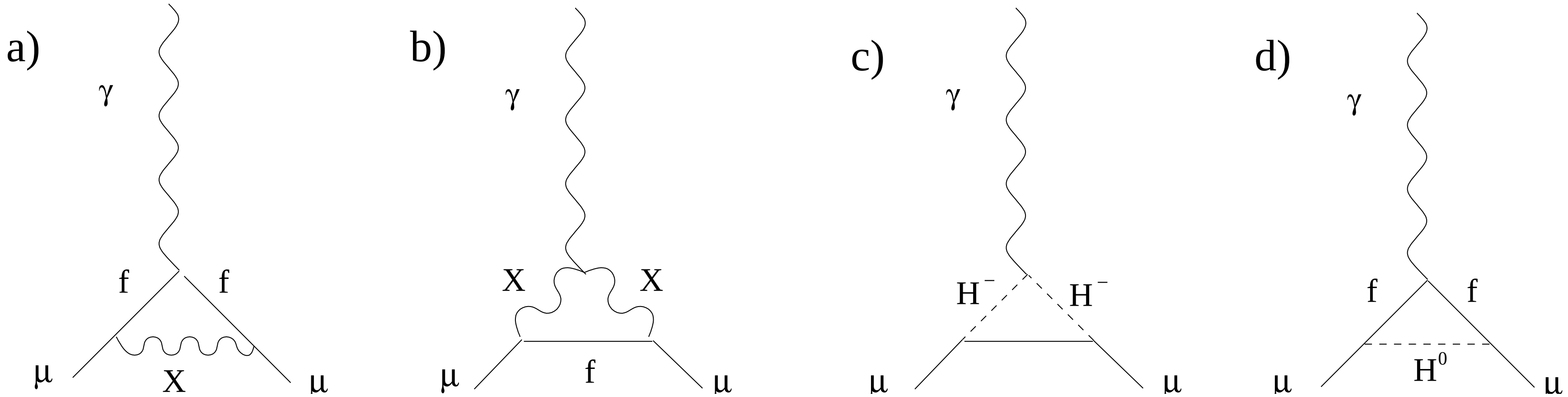


Figure 7: *Loop graphs contributing to the weak correction to Δg. a) and b) correspond to the exchange of a vector boson X while c) and d) are the Higgs sector contributions.*

The difference between experiment and the standard model prediction for a_μ is

$$\delta a_\mu = a_\mu^{exp} - a_\mu^{\mathrm{SM}} = 17(18) \times 10^{(-10)} \ .$$

The numerical results within the littlest Higgs model are relatively insensitive to the choice of parameter values of the model. We obtain a difference from the standard model value of at most $\delta a_\mu = a_\mu^{\mathrm{LH}} - a_\mu^{\mathrm{SM}}$ of the order of 1×10^{-10}. The contributions of the additional heavy particles are thereby completely negligible and the dominant contributions arise from the corrections to the light Z and W couplings. Similar results are obtained in the custodial model.

Weak charge of cesium atoms

At low energy, parity violation in atoms is due to the electron-quark effective Lagrangian

$$\mathcal{L}_{eff} = \frac{G_F}{\sqrt{2}} (\bar{e}\gamma_\mu \gamma_5 e)(C_{1u}\bar{u}\gamma^\mu u + C_{1d}\bar{d}\gamma^\mu d) \ .$$

The experimentally measured quantity is the so-called “weak charge” defined as

$$Q_W = -2\left(C_{1u}(2Z+N) + C_{1d}(Z+2N)\right) \ ,$$

where Z, N are the number of protons and neutrons of the atom, respectively.

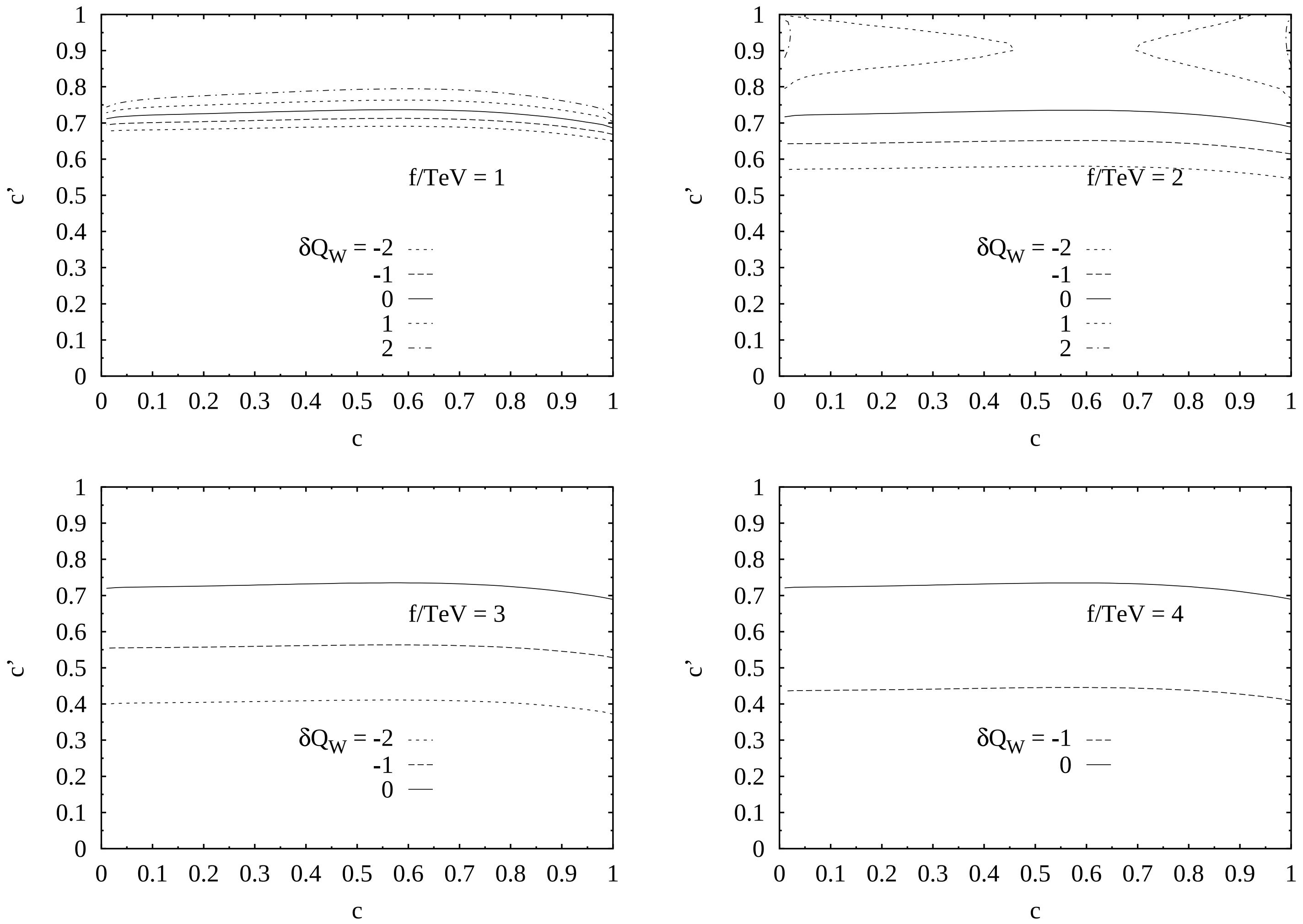


Figure 8: *Corrections to the weak charge of cesium atoms as a function of c and c' in the littlest Higgs model.*

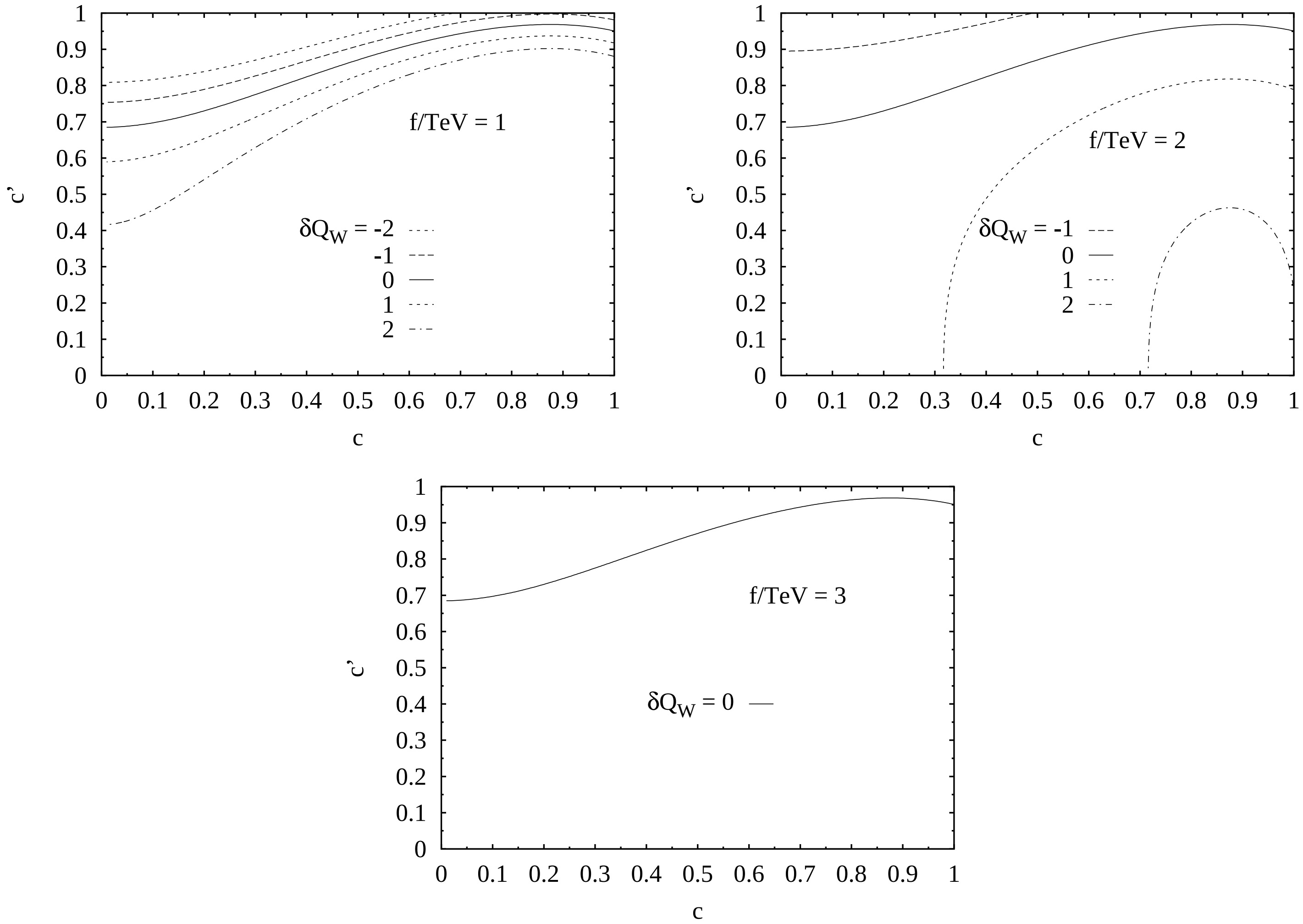


Figure 9: *Corrections to the weak charge of cesium atoms as a function of* c *and* c' *in the little Higgs model with approximate custodial symmetry.*

Collider phenomenology: A_H

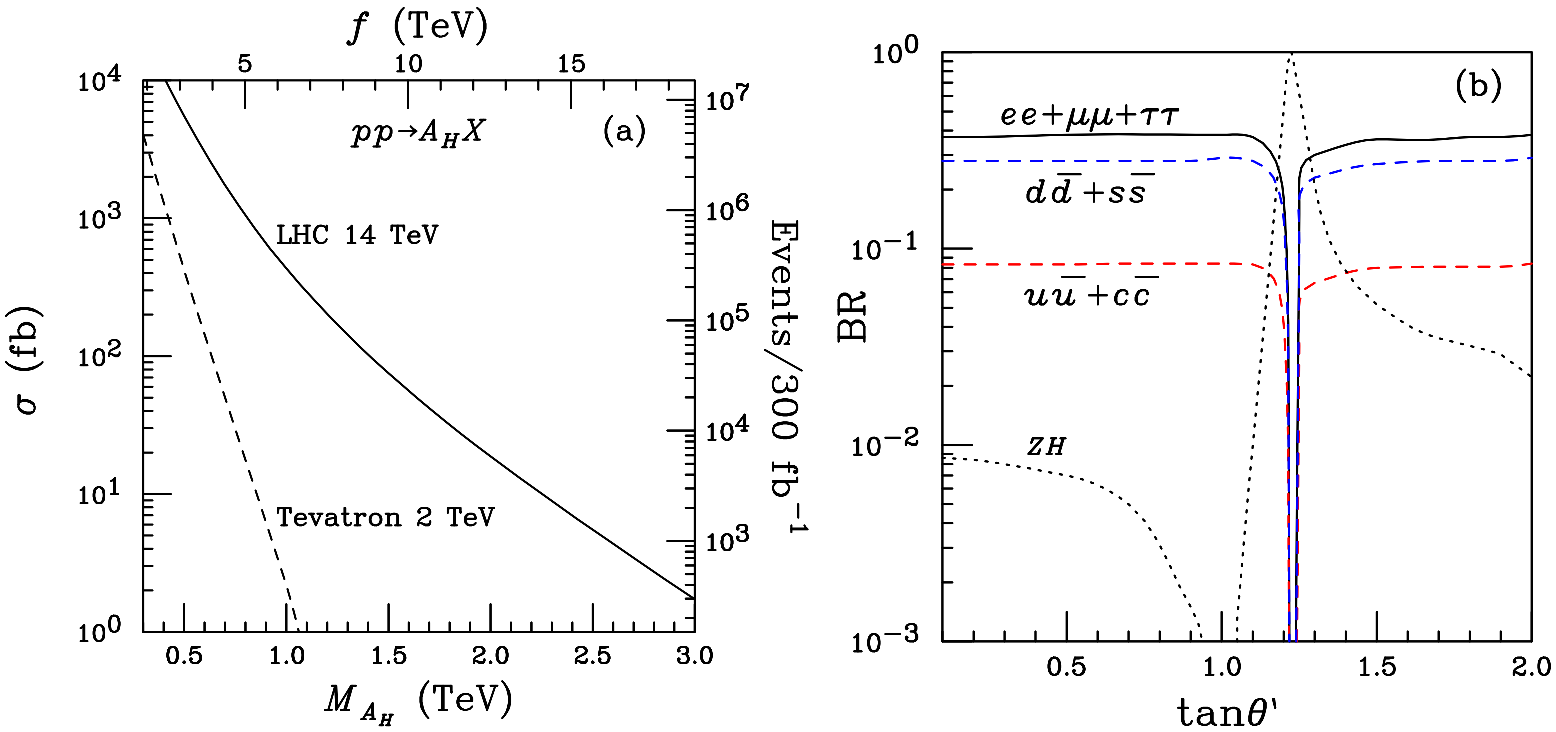


Figure 10: *(a) Total cross section for A_H production versus its mass M_{A_H} at the Tevatron (dashed) and the LHC (solid) for $\tan\theta' = 1$. The scale f corresponding to $\tan\theta' = 1$ is given on the top axis; (b) A_H decay branching fractions versus $\tan\theta'$. Figures from hep-ph/0301040.*

Collider phenomenology: Z_H

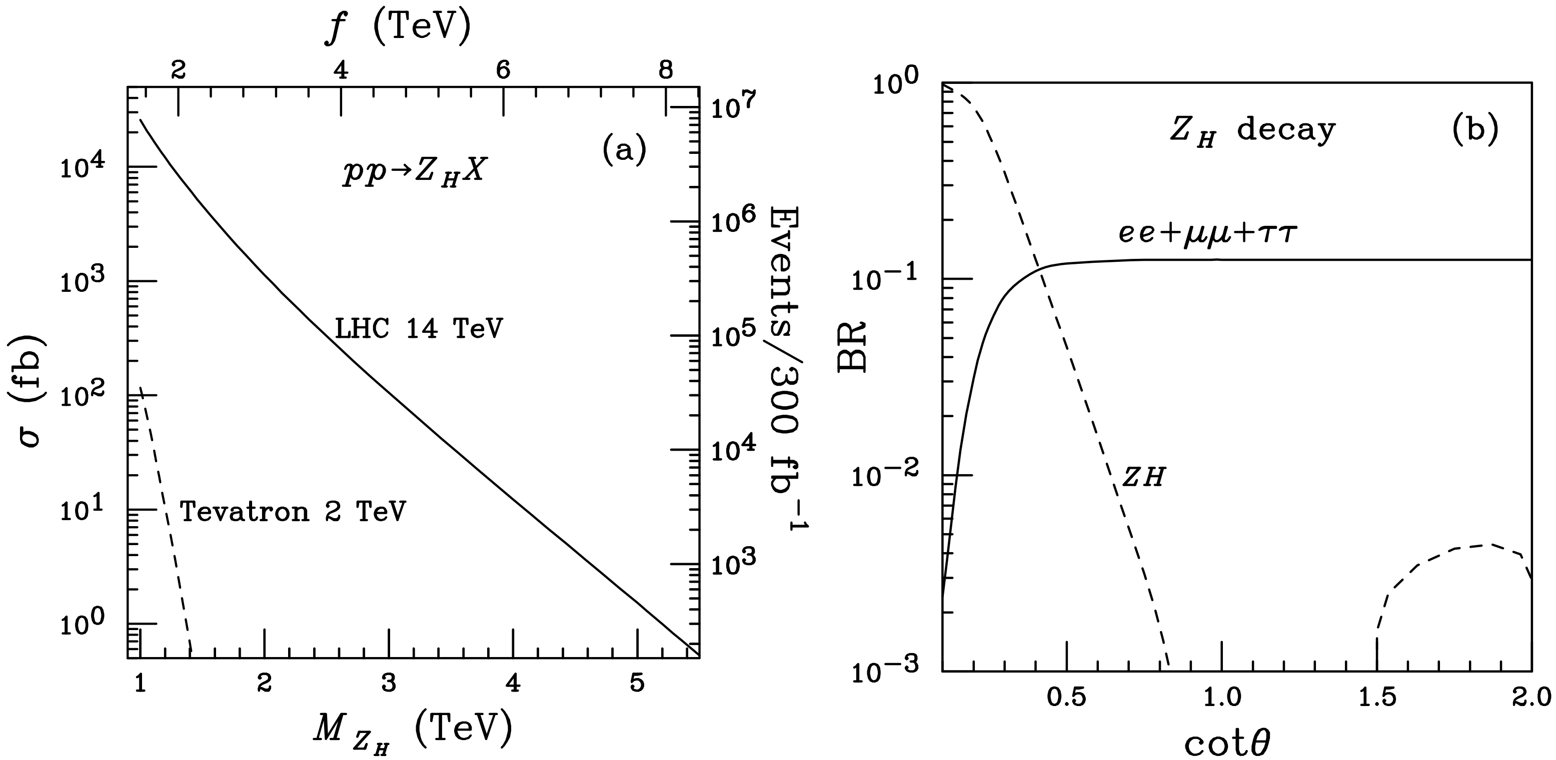


Figure 11: *(a) Total cross section for Z_H production versus its mass at the Tevatron (dashed) and the LHC (solid) for $\cot\theta = 1$. The scale f corresponding to $\cot\theta = 1$ is given on the top axis; (b) Z_H decay branching fractions versus $\cot\theta$. Figures from hep-ph/0301040.*

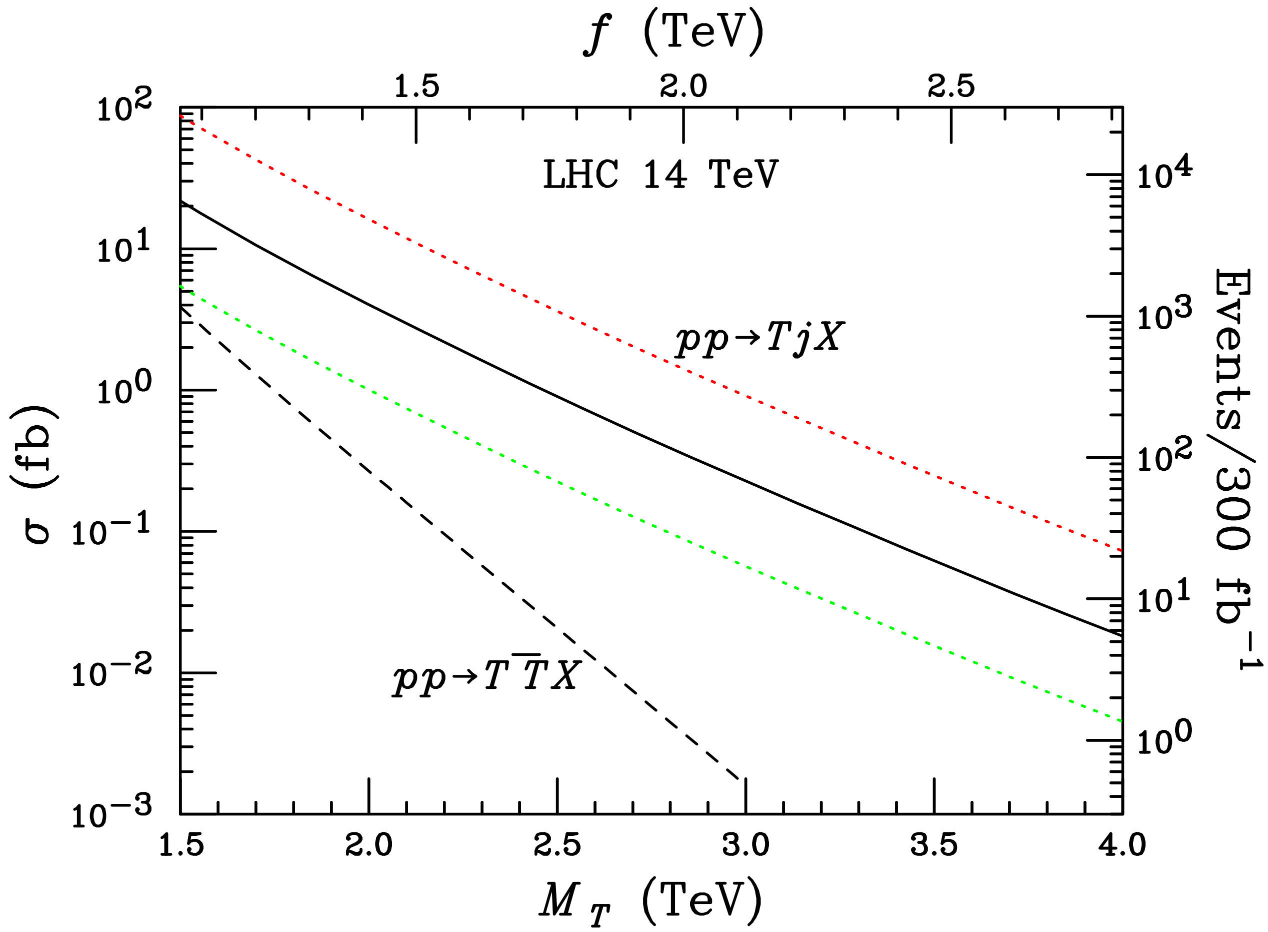


Figure 12: *Total cross sections for $T\bar{T}$ production (dashed) and T+jet production (solid and dotted) via t-channel W-exchange versus mass M_T at the LHC. The solid line is for the couplings $\lambda_1 = \lambda_2$; the dotted are for $\lambda_1/\lambda_2 = 2$ and 1/2. Figure from hep-ph/0301040.*

Conclusions

In the model without custodial symmetry a considerable fine tuning is necessary in order to satisfy the constraints imposed by LEP data. This problem is to a large extent avoided for the model with approximate custodial symmetry.

Low energy precision data does not change the above conclusions. For $g - 2$ of the muon the corrections are small. The weak charge does not allow for establishing new constraints either, even if the corrections are not negligible.

However the TeV region is expected to be rich in LH scenarios : new vectors and scalars, extended top sector.