CP and vacuum in 2HDM

or

Symmetries of 2HDM

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Scalar sector:

- *Freedom in description of Higgs sector - reparametrization (rephasing)*
- *Vacuum in 2HDM: Charged or neutral? Real or complex?*
- *Stability of vacuum*

Spontaneous electroweak symmetry breaking in SM

A spontaneous electroweak symmetry breaking of $SU(2) \times U(1)$ (EWSB). In the Standard Model (SM) - Higgs mechanism: the single scalar isodoublet with hypercharge $Y = 1$

$$\mathcal{L}_H = (D_\mu \phi)^\dagger D_\mu \phi - V, \quad V = \lambda(\phi^\dagger \phi)^2/2 - m^2 \phi^\dagger \phi/2$$

Parameters: λ, m

Minimum of the potential $\rightarrow$ vacuum with v.e.v: $v/\sqrt{2} = \sqrt{m^2/2\lambda}$

Electric charge operator: $\hat{Q} = T_3 + Y/2$

Vacuum: $\hat{Q}\langle\phi\rangle = 0$, so $\hat{Q}\begin{pmatrix} u \\ v \end{pmatrix} = 0 \rightarrow u = 0$.

- $U(1)_{em}$ invariance of vacuum $\Leftrightarrow$ massless photon. Charge conservation.
- Charge conjugation invariance or C (CP) - invariance of V.

C: $\phi \rightarrow e^{i\alpha}\phi^*$, α-arbitrary, C:$(\phi^\dagger\phi) \rightarrow (\phi^\dagger\phi)^* = (\phi^\dagger\phi)$

Symmetries of Two Higgs Doublet Model

I. F. Ginzburg, M. Krawczyk, hep-ph/0408011 (PRD); I. F. Ginzburg at PLC2005

• 2HDM contains two fields, ϕ_1 and ϕ_2, with identical quantum numbers: weak isodoublets ($T = 1/2$) with hypercharges $Y = +1$

•Global transformations which mix these fields and change the relative phases are allowed without changing physical picture

• One of the reason for introducing 2HDM was to describe phenomenon of CP violation Lee' 73; Glashow and Weinberg'77-
CP violation and the flavour changing neutral currents (FCNC) can be naturally suppressed by imposing in Lagrangian a Z_2 symmetry, that is the invariance of the Lagrangian under the interchange

$$(\phi_1 \leftrightarrow \phi_1, \phi_2 \leftrightarrow -\phi_2) \quad \text{or} \quad (\phi_1 \leftrightarrow -\phi_1, \phi_2 \leftrightarrow \phi_2).$$

This symmetry forbids the $\phi_1 \leftrightarrow \phi_2$ transition.

2HDM Potential: quartic and quadratic terms separated:

$$
\begin{aligned}
V = & \tfrac{1}{2}\lambda_1(\phi_1^\dagger\phi_1)^2 + \tfrac{1}{2}\lambda_2(\phi_2^\dagger\phi_2)^2 \\
& + \lambda_3(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) + \lambda_4(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1) \\
& + \tfrac{1}{2}\left[\lambda_5(\phi_1^\dagger\phi_2)^2 + \text{h.c.}\right] \\
& + \left\{\left[\lambda_6(\phi_1^\dagger\phi_1) + \lambda_7(\phi_2^\dagger\phi_2)\right](\phi_1^\dagger\phi_2) + \text{h.c.}\right\} \\
& - \tfrac{1}{2}\left\{m_{11}^2(\phi_1^\dagger\phi_1) + \left[m_{12}^2(\phi_1^\dagger\phi_2) + \text{h.c.}\right] + m_{22}^2(\phi_2^\dagger\phi_2)\right\}
\end{aligned}
$$

14 parameters: $\lambda_1, \lambda_2, \lambda_3, \lambda_4$, $\lambda_5, \lambda_6, \lambda_7$, m_{11}^2, m_{22}^2, Re m_{12}^2,Im m_{12}^2

- Z_2 symmetry under $\phi_1 \to -\phi_1$, $\phi_2 \to \phi_2$ (or vice versa):

$\Rightarrow \lambda_6 = \lambda_7 = m_{12}^2 = 0$

- Soft violation of Z$_2$ symmetry - $m_{12}^2(\lambda_6 = \lambda_7 = 0)$

- Hard violation of Z_2 symmetry: quartic terms with λ_6, λ_7

Lee, Diaz-Cruz, Mendez, Haber, Gunion, Branco, Pomarol, Barroso, Grzadkowski, Kalinowski, Rosiek, Santos, Hollik, Djouadi, Illana, Akeroyd, Arhrib, Dubinin, Davidson, Ivanov ...

Reparametrization transformation of fields

Two fields (doublets) with identical quantum numbers:
a global unitary transformations $\hat{\mathcal{F}}$ which mix these fields and change the phases, $\hat{\mathcal{F}} = \hat{\mathcal{F}}(\rho_0; \rho, \tau, \theta)$:

$$\begin{pmatrix} \phi'_1 \\ \phi'_2 \end{pmatrix} = \hat{\mathcal{F}} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}, \quad \hat{\mathcal{F}} = e^{-i\rho_0} \begin{pmatrix} \cos\theta\, e^{i\rho/2} & \sin\theta\, e^{i(\tau-\rho)/2} \\ -\sin\theta\, e^{-i(\tau-\rho)/2} & \cos\theta\, e^{-i\rho/2} \end{pmatrix}$$

ρ_0 - an overall phase
→ PHASE ROTATION AND MIXING OF FIELDS (changing field basis)

The particular case with $\theta = 0$ means no mixing
- a global transformation of fields with the independent phase rotations:

$$\phi_{1,2} \to e^{-i\rho_{1,2}} \phi_{1,2}; \quad \rho_1 = \rho_0 - \rho/2, \;\; \rho_2 = \rho_0 + \rho/2, \;\; \rho = \rho_2 - \rho_1 \,.$$

→ PHASE ROTATION FOR FIELDS (rephasing fields)

Reparametrization transformation of Lagrangians

Let Λ - represents all parameters of $\mathcal{L}$: $(\lambda' s, m_{ij}^2)$
Let Φ -represents fields (ϕ_1, ϕ_2)

Write

$$\boxed{\mathcal{L} = \mathcal{L}(\Lambda \otimes \Phi)}$$

Without changing physical content we transform $(\hat{\mathcal{F}})$ Φ to Φ':
so

$$\boxed{\mathcal{L} \to \mathcal{L}'(\Lambda \times \Phi')}$$

however since $\Phi' = \hat{\mathcal{F}}\Phi$, we induced $\Lambda \to \Lambda'$

$$\mathcal{L}'(\Lambda \otimes \Phi') = \mathcal{L}'(\Lambda' \otimes \Phi)$$

But $\mathcal{L}$ and $\mathcal{L}$' - which differ by set of parameters Λ, describe the same physics $\to$ REPARAMETRIZATION INVARIANCE

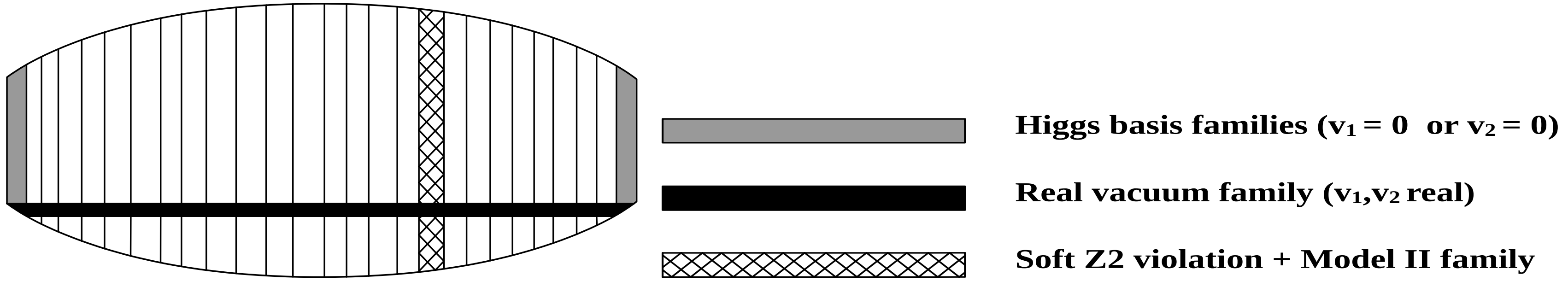
The induced transformation Λ to Λ' - reparametrization transformation - given by $(\rho, \tau, c = \cos\theta, s = \sin\theta)$

Reparametrization group

Reparametrization transformations - the 3–parametrical (ρ, τ, θ) *reparametrization group, operating within the space of Lagrangians* with coordinates given by λ_i, m_{ij}^2.

A set of the physically equivalent Higgs Lagrangians - *the reparametrization equivalent space* (a subspace of the entire space of Lagrangians).

"Family of Lagrangians" with explicit property - eg. a soft Z_2 violating family of Lagrangians



Rephasing group

A particular reparametrization with $\theta = 0$ is equivalent to a change of phase of the *complex* parameters of Lagrangian :

$$\lambda_5 \to \lambda_5\, e^{-2i\rho}, \;\; \lambda_{6,7} \to \lambda_{6,7}\, e^{-i\rho}, m_{12}^2 \to m_{12}^2 e^{-i\rho}.$$

$\to$ **A rephasing invariance.**

Rephasing transformations - the 1−parametrical rephasing group with *the rephasing parameter* ρ **- subgroup of the reparametrization group**

Rephasing equivalent space of Lagrangians

Reparametrization: Lagrangian and Z_2 symmetry

The violation of the Z_2 symmetry allows for the $\phi_1 \leftrightarrow \phi_2$ transitions.

•Exact $\boldsymbol{Z_2}$ symmetry. $\lambda_6 = \lambda_7 = m_{12}^2 = 0$. Only λ_5 can be complex, by rephasing $\to \lambda_5$ real.

•A soft violation of $\boldsymbol{Z_2}$ symmetry. Adding to the Z_2 symmetric Lagrangian the term $m_{12}^2(\phi_1^\dagger \phi_2) + h.c.$, with a generally complex m_{12}^2; as before λ_5 can be made real by rephasing.

•A hard violation of Z_2 symmetry. (Operator dimension 4) with generally complex parameters λ_6, λ_7 are added to V with a softly broken Z_2 symmetry. The *true* hard violation of Z_2 - if V cannot be transformed to the case of Z_2 conservation, nor its weak violation.

Remarks on CP

The complex values of some of parameters in V provide *a necessary condition* for the CP violation in the Higgs sector. If V can be reparametrized so that all parameters became real - no CP violation is present.

The "Real Vacuum"

The minimum of the potential $\rightarrow$ the vacuum expectation values (v.e.v) of the fields ϕ_i: *Typically: In order to get the $U(1)$ symmetry of electromagnetism and using the overall phase freedom of the Lagrangian we choose one vacuum expectation value real (and one complex):*

$$\langle\phi_1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_1 \end{pmatrix} \quad \text{and} \quad \langle\phi_2\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_2 e^{i\xi} \end{pmatrix}, \tag{1}$$

The phase difference ξ between the v.e.v.'s - *a spontaneous CP violation?* Not necessarily the case.. (eg.Branco at al. book: CP Violation). since under the rephasing transformation the ξ changes to:

$$\xi \rightarrow \xi - \rho\,. \tag{2}$$

and we can get "a real vacuum Lagrangian" for $\rho = \xi$.

If in addition $\tau = 0$ (in $\hat{\mathcal{F}}$) angle β ($\tan\beta = v_2/v_1$) changes to $\beta + \theta$ ...$\tan\beta$ - not a physical parameter (as discussed by Howie Haber, and others)

Real vacuum condition

$$\overline{\lambda}_5 \equiv \lambda_5 e^{-2i\xi}, \quad \overline{\lambda}_6 \equiv \lambda_6 e^{-i\xi}, \quad \overline{\lambda}_7 \equiv \lambda_7 e^{-i\xi}, \quad \overline{m}_{12}^2 \equiv m_{12}^2 e^{-i\xi}$$

Introducing

$$\overline{\lambda}_{345} = \overline{\lambda}_3 + \overline{\lambda}_4 + \mathrm{Re}\,\overline{\lambda}_5, \;\; \overline{\lambda}_{67} = \frac{v_1}{v_2}\overline{\lambda}_6 + \frac{v_2}{v_1}\overline{\lambda}_7, \;\; \tilde{\lambda}_{67} = \frac{v_1}{v_2}\overline{\lambda}_6 - \frac{v_2}{v_1}\overline{\lambda}_7,$$

$$\overline{m}_{12}^2 = 2v_1v_2(\nu + i\delta), \;\; \delta = \mathrm{Im}(\underbrace{0}_{Z_2\ sym} + \underbrace{\frac{\overline{\lambda}_5}{2}}_{soft} + \underbrace{\frac{\overline{\lambda}_{67}}{2}}_{hard})\,.$$

The minimum condition does not constrain $\nu = \mathrm{Re}(\overline{m}_{12}^2)/(2v_1v_2)$.

It allows to express $m_{11,22}^2$ via $\overline{\lambda}$'s, v_j and the parameter ν (Pilaftsis ...):

$$\overline{m}_{11}^2 = \underbrace{\overline{\lambda}_1 v_1^2 + \overline{\lambda}_{345} v_2^2}_{Z_2\ sym} \;\; \underbrace{-2\nu v_2^2}_{soft} + \underbrace{\frac{v_2}{v_1}\mathrm{Re}\left(3v_1^2\overline{\lambda}_6 + v_2^2\overline{\lambda}_7\right)}_{hard},$$

$$\overline{m}_{22}^2 = \underbrace{\overline{\lambda}_2 v_2^2 + \overline{\lambda}_{345} v_1^2}_{Z_2\ sym} \;\; \underbrace{-2\nu v_1^2}_{soft} + \underbrace{\frac{v_1}{v_2}\mathrm{Re}\left(v_1^2\overline{\lambda}_6 + 3v_2^2\overline{\lambda}_7\right)}_{hard}.$$

Extremes of potential

The extremes of the potential:

$$\left.\frac{\partial V}{\partial \phi_1}\right|_{\substack{\phi_1=\langle\phi_1\rangle,\\ \phi_2=\langle\phi_2\rangle}} = 0, \qquad \left.\frac{\partial V}{\partial \phi_2}\right|_{\substack{\phi_1=\langle\phi_1\rangle,\\ \phi_2=\langle\phi_2\rangle}} = 0.$$

Trivial electroweak symmetry conserving solution: $\langle\phi_1\rangle = 0$, $\langle\phi_2\rangle = 0$.
Electroweak symmetry breakin solutions - the most general one:

$$\langle\phi_1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \quad \langle\phi_2\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} u \\ v_2 e^{i\xi} \end{pmatrix}.$$

useful : $y_1 = \langle\phi_1^\dagger\rangle\langle\phi_1\rangle, \; y_2 = \langle\phi_2^\dagger\rangle\langle\phi_2\rangle, \; y_3 = \langle\phi_1^\dagger\rangle\langle\phi_2\rangle, w = y_3 y_3^* - y_1 y_2 \,,$

where

$$\partial V/\partial \phi_i = w f(\lambda_j, y_j) = 0$$

solution Ch: $w \neq 0 \to u \neq 0$, charged vacuum

Depending on the parameters of potential, either saddle point or a minimum of the potential, *a charged vacuum*, with a heavy photon, charge nonconservation, etc

Diaz-Cruz, Mendez'1992, Barroso, Ferreira, Santos..'94,'04,'05

solution N: $w = 0 \to u = 0$, physical (neutral) vacuum

The solution has a well known form:

$$\langle\phi_1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_1 \end{pmatrix} \text{ and } \langle\phi_2\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_2 e^{i\xi} \end{pmatrix},$$

$U(1)_{em}$ invariance, massless photon, etc.

The extremum is a minimum if all eigenvalues of mass squared matrix are positive.

Tree-level Vacuum Stability

If a minimum that preserves the U_{em} symmetry exists, then the charge breaking (CB) minima cannot occur... Once a charge-preserving minimum exists, any charge breaking stationary point that might exist lies above the minimum. (Barroso, Ferreira, Santos, hep-ph/0512037*)*

The difference between the values of the potential at the Ch (v_i') and N (v_i) stationary points is given by:

$$V_{Ch} - V_N = M_{H^\pm}^2/2v^2[(v_1'v_2 - v_2'v_1)^2 + u^2v_1^2]$$

with mass of $H^\pm$ calculated for N.

So, if N- minimum, $M_{H^\pm}^2$ positive and $V_{Ch} - V_N$ greater than 0!

(one can show that Ch is a saddle point)

Conclusion

Masslessness of the photon seems to be natural in 2HDM.