

Dynamical breakdown of Abelian gauge chiral symmetry by strong Yukawa interactions

(How to employ massive complex scalar fields)

Tomáš Brauner and Jiří Hošek, Phys.Rev.D72:
045007(2005)

Petr Beneš, Tomáš Brauner and Jiří Hošek, to appear

Lagrangian and its properties
Fermion mass generation and scalar boson mass splitting
Where is the Nambu-Goldstone boson ?
Gauge boson mass generation
Symmetry-breaking loop-generated vertices
SU(2)xU(1) generalization

I. Lagrangian and its properties

$$\begin{aligned}\mathcal{L} = & \bar{\Psi}_1 i \not{D} \Psi_1 + \bar{\Psi}_2 i \not{D} \Psi_2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \\ & (D_\mu \phi)^\dagger D^\mu \phi - M^2 \phi^\dagger \phi - \frac{1}{2} \lambda (\phi^\dagger \phi)^2 + \\ & y_1 [\bar{\Psi}_{1L} \Psi_{1R} \phi + \text{H.c.}] + y_2 [\bar{\Psi}_{2R} \Psi_{2L} \phi + \text{H.c.}]\end{aligned}$$

Symmetry:

$$\underbrace{U(1)_{V_1} \times U(1)_{V_2}}_{\text{global}} \times \underbrace{U(1)_A}_{\text{gauged}}$$

$$\psi_1 \rightarrow e^{i\theta(x)\gamma_5} \psi_1, \quad \psi_2 \rightarrow e^{-i\theta(x)\gamma_5} \psi_2$$

$$\phi \rightarrow e^{-2i\theta(x)} \phi$$

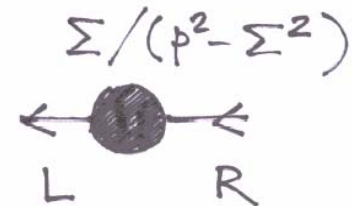
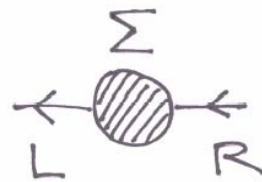
$$A_\mu \rightarrow A_\mu + \frac{1}{g} \partial_\mu \theta$$

- Why two fermion species: no axial anomaly
- With $M^2 > 0$ no symmetry breakdown in scalar sector itself
- Comparison with Higgs mechanism by heart

II. Fermion mass generation and scalar boson mass splitting

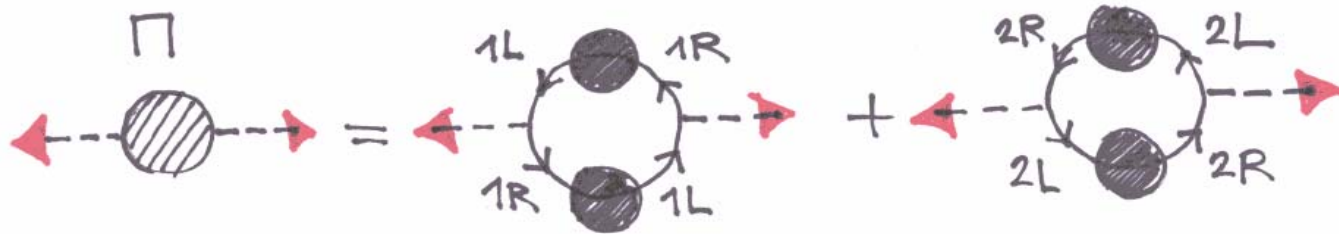
- ASSUME that Yukawa interactions generate the chiral-symmetry breaking fermion proper self-energies Σ :

$$S(p) \equiv \frac{1}{\not{p} - \Sigma(p)} = \frac{\not{p} + \Sigma}{p^2 - \Sigma^2}$$



$$m^2 = \Sigma^2 (p^2 - m^2)$$

THEN Yukawa interactions generate the symmetry-breaking scalar proper self-energy Π

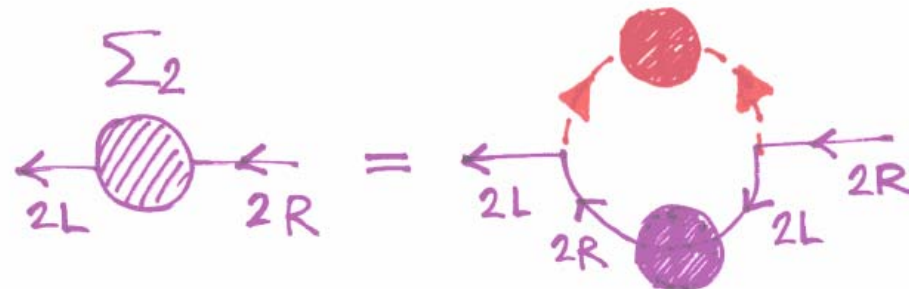
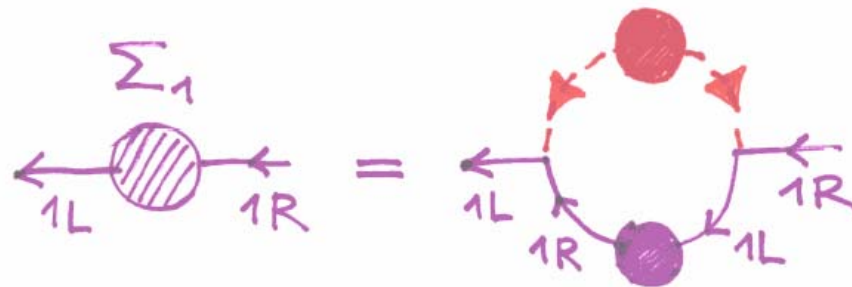


$$\mathcal{L}_{\text{eff}}^{(0)} = \partial_\mu \phi^* \partial^\mu \phi - M^2 \phi^* \phi - \frac{1}{2} \phi^* \Pi \phi^* - \frac{1}{2} \phi \Pi^* \phi$$

$$D^{-1}(p) = \begin{pmatrix} p^2 - M^2 & -\Pi \\ -\Pi^* & p^2 - M^2 \end{pmatrix}$$

$$D(p) = \frac{1}{(p^2 - M^2 - i\Pi)(p^2 - M^2 + i\Pi)} \begin{pmatrix} p^2 - M^2 & \Pi \\ \Pi^* & p^2 - M^2 \end{pmatrix}$$

Yukawa interactions GENERATE Σ :



NICELY CONVERGENT KERNEL (corresponding counter terms prohibited by symmetry)

- By dimensional argument:

$$m_{1,2} = M f_{1,2}(y_1, y_2)$$

- Compare with Higgs:

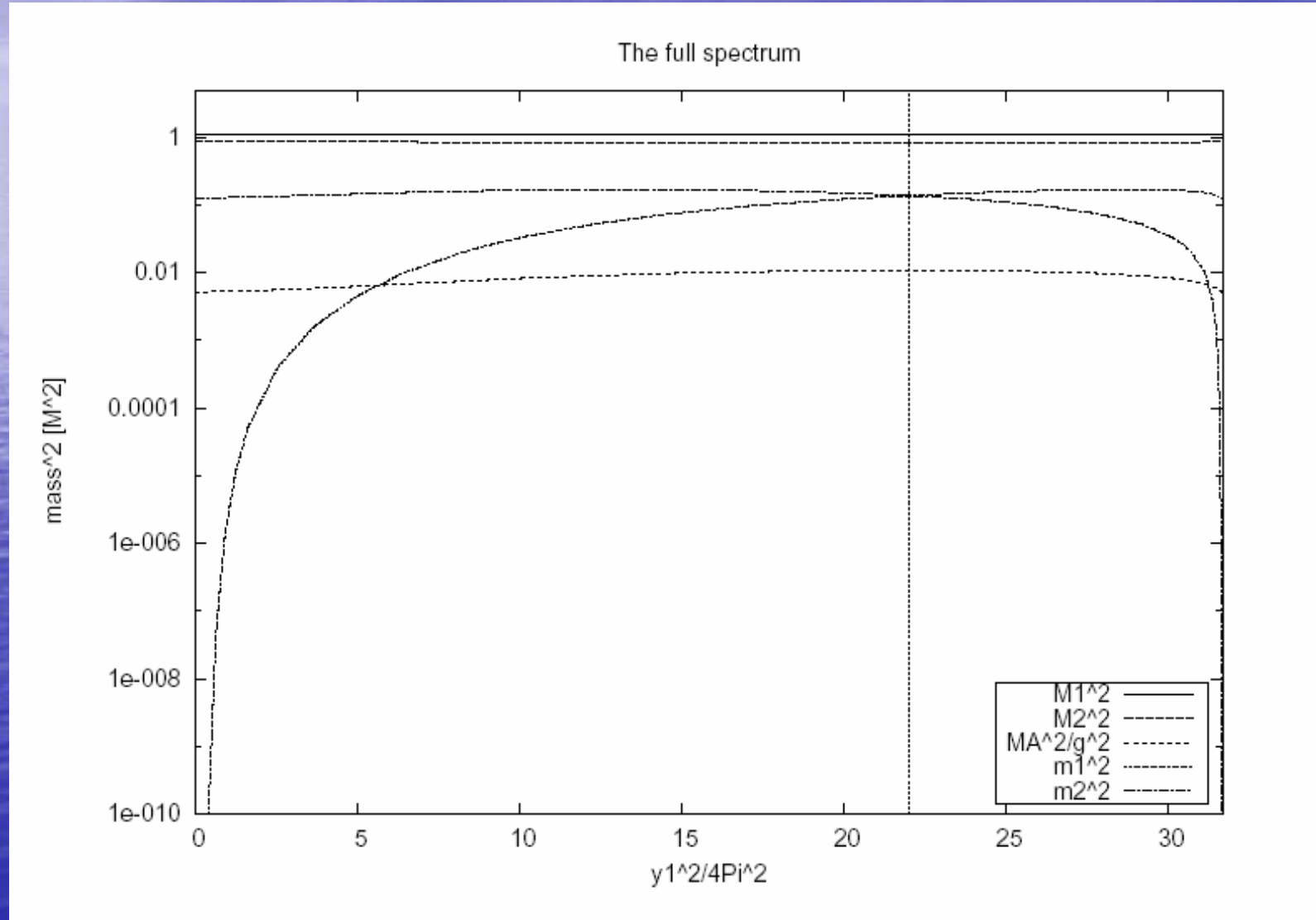
$$m_{1,2} = (-2M^2/\lambda)^{1/2} y_{1,2}$$

Boson mass splitting

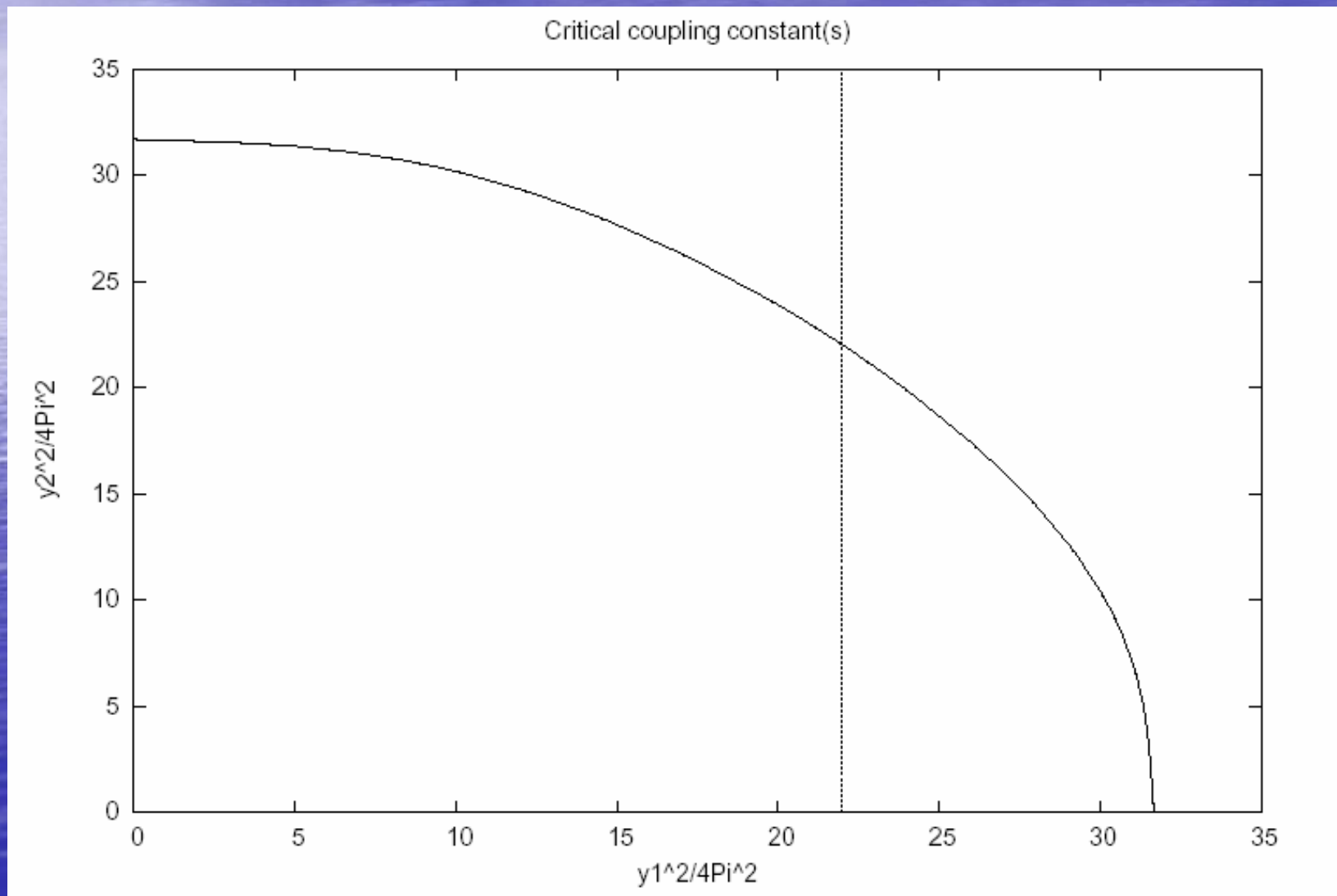
- In particular case of real Π found numerically the real and imaginary parts of Φ are the **mass eigenstates** with masses

$$M_{1,2}^2 = M^2 \pm \Pi(p^2 = M_{1,2}^2)$$

Non-zero UV-finite solutions Σ and Π do exist.
They are found numerically upon Wick rotation.



Solutions found for LARGE YUKAWA COUPLINGS



Large amplification of fermion mass ratios as a response to small changes in Yukawa coupling ratios

Explicit knowledge of non-analytic dependences of masses upon couplings -

ULTIMATE DREAM

Coupling constant λ ignored as unimportant for non-perturbative mass generation

III. Where is the Nambu-Goldstone boson ?

- Axial-vector current

$$j_A^M = \bar{\Psi}_1 \gamma^M \gamma_5 \Psi_1 - \bar{\Psi}_2 \gamma^M \gamma_5 \Psi_2 + 2i [(\partial^M \phi)^\dagger \phi - \phi^\dagger \partial^M \phi]$$

- Axial-vector Ward identities for proper vertices

$$q_\mu \Gamma_{A\Psi_1}^M(p+q, p) = S_1^{-1}(p+q) \gamma_5 + \gamma_5 S_1^{-1}(p)$$

$$q_\mu \Gamma_{A\Psi_2}^M(p+q, p) = -S_2^{-1}(p+q) \gamma_5 - \gamma_5 S_2^{-1}(p)$$

$$q_\mu \Gamma_{A\phi}^M(p+q, p) = -2\bar{D}^{-1}(p+q) \Xi + 2\Xi \bar{D}^{-1}(p)$$

For Σ, Π non-zero the identities imply the massless pole in proper vertices Γ

$$\Gamma_{A\psi_1, \text{pole}}^M = \text{Diagram 1} + \text{Diagram 2}$$

Diagram 1: A vertex with two incoming fermion lines labeled 1 and 1, connected to a loop with two fermion lines labeled 1 and 2, which is then connected to a wavy line labeled X.

Diagram 2: A vertex with two incoming fermion lines labeled 1 and 1, connected to a dashed loop, which is then connected to a wavy line labeled X.

$$\Gamma_{A\psi_2, \text{pole}}^M = \text{Diagram 3} + \text{Diagram 4}$$

Diagram 3: A vertex with two incoming fermion lines labeled 2 and 2, connected to a loop with two fermion lines labeled 1 and 2, which is then connected to a wavy line labeled X.

Diagram 4: A vertex with two incoming fermion lines labeled 2 and 2, connected to a dashed loop, which is then connected to a wavy line labeled X.

$$\Gamma_{A\phi, \text{pole}}^M = \text{Diagram 5} + \text{Diagram 6}$$

Diagram 5: A vertex with two incoming dashed lines, connected to a loop with two fermion lines labeled 1 and 2, which is then connected to a wavy line labeled X.

Diagram 6: A vertex with two incoming dashed lines, connected to a dashed loop, which is then connected to a wavy line labeled X.

Basic quantities to be calculated are the UV finite vectorial tadpoles I:

$$\Gamma_{A\psi_1, \text{pole}}^M \equiv P_{\psi_1}(p+q, p) \frac{i}{q^2} \left[I_{\psi_1}^M(q) + I_{\psi_2}^M(q) + I_{\phi}^M(q) \right]$$

$$\Gamma_{A\psi_2, \text{pole}}^M = P_{\psi_2}(p+q, p) \frac{i}{q^2} \left[I_{\psi_1}^M(q) + I_{\psi_2}^M(q) + I_{\phi}^M(q) \right]$$

$$\Gamma_{A\phi, \text{pole}}^M = P_{\phi}(p+q, p) \frac{i}{q^2} \left[I_{\psi_1}^M(q) + I_{\psi_2}^M(q) + I_{\phi}^M(q) \right]$$

Effective NG couplings are related to Σ and Π :

$$P_{\Psi_1}(p+q, p) \equiv -\frac{1}{N} [\Sigma_1(p+q) + \Sigma_1(p)] \delta_5$$

$$P_{\Psi_2}(p+q, p) \equiv \frac{1}{N} [\Sigma_2(p+q) + \Sigma_2(p)] \delta_5$$

$$P_{\phi}(p+q, p) = -\frac{2}{N} \begin{pmatrix} 0 & \Pi(p+q) + \Pi(p) \\ -\Pi^*(p+q) - \Pi^*(p) & 0 \end{pmatrix}$$

The overall normalization is given by tadpoles I:

$$I_{\psi_{1,2}}(q^2) \equiv \frac{1}{N} J_{\psi_{1,2}}(q^2)$$

$$I_{\phi}(q^2) \equiv \frac{1}{N} J_{\phi}(q^2)$$

$$N = \left(J_{\psi_1}(0) + J_{\psi_2}(0) + J_{\phi}(0) \right)^{1/2}$$

IV. Gauge boson mass generation

- Gauge boson mass squared is the residue at the massless pole of the gauge field polarization tensor (Schwinger):

$$i\Pi^{\mu\nu}(q) \equiv -i(q^2 g^{\mu\nu} - q^\mu q^\nu)\Pi(q^2)$$

- In Higgs (complete polarization tensor):

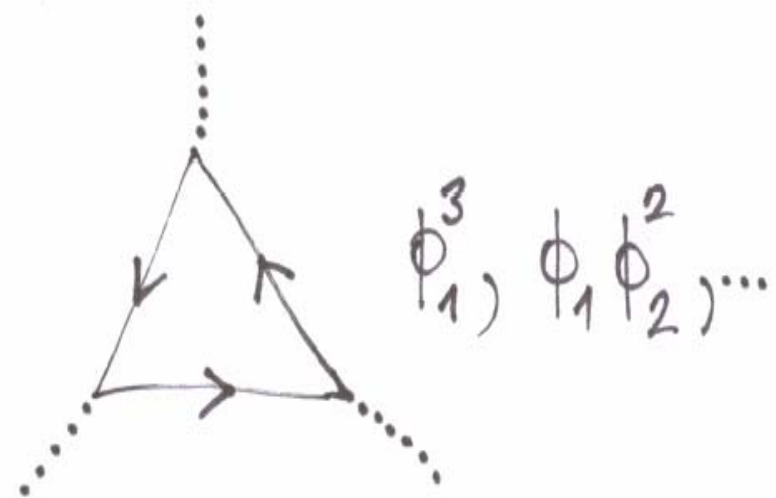
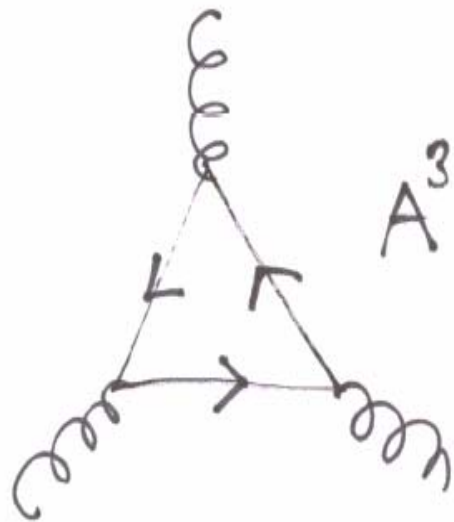
$$i\Pi^{\mu\nu}(q) = \text{diagram} + \frac{i}{q^2} \text{diagram}$$

The equation shows the complete polarization tensor $i\Pi^{\mu\nu}(q)$ as the sum of two terms. The first term is a self-energy loop diagram (a wavy line with a dot) with the label $i(g_V)^2 g^{\mu\nu}$ below it. The second term is a two-point function diagram (two wavy lines connected by a straight line) with the label $[ig_V(iq^\mu)]$ below the left wavy line, $[ig_V(-iq^\nu)]$ below the right wavy line, and $\frac{i}{q^2}$ above the straight line.

In strongly coupled models no control on the bound-state spectrum except NG. Consequently, only the longitudinal part can be computed. By transversality

$$m_A^2 = g^2 [J_{\psi_1}(0) + J_{\psi_2}(0) + J_\phi(0)]$$

V. Symmetry-breaking loop-generated UV-finite vertices: genuine (albeit gedanken) predictions (work in progress)



VI. Outlook

- Common source of fermion and gauge-boson mass generation
- Hope for natural description of wide, sparse and irregular fermion mass spectrum
- $SU(2) \times U(1)$ generalization exists
(T. Brauner, J.H., A model of flavors, hep-ph/0407339)
- Phenomenological viability is to be investigated