Dynamical breakdown of Abelian gauge chiral symmetry by strong Yukawa interactions

(How to employ massive complex scalar fields)

Tomáš Brauner and Jiří Hošek, Phys.Rev.D72: 045007(2005)

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Lagrangian and its properties
Fermion mass generation and scalar boson mass splitting
Where is the Nambu-Goldstone boson ?
Gauge boson mass generation
Symmetry-breaking loop-generated vertices
SU(2)xU(1) generalization

I. Lagrangian and its properties

$$\mathcal{L} = \bar{\psi}_1 i \not{D} \psi_1 + \bar{\psi}_2 i \not{D} \psi_2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} +$$

$$(D_\mu \phi)^\dagger D^\mu \phi - M^2 \phi^\dagger \phi - \frac{1}{2} \lambda (\phi^\dagger \phi)^2 +$$

$$y_1 [\bar{\psi}_{1L} \psi_{1R} \phi + H.c.] + y_2 [\bar{\psi}_{2R} \psi_{2L} \phi + H.c.]$$

Symmetry:

$$\underbrace{U(1)_{V_1} \times U(1)_{V_2}}_{\text{global}} \times \underbrace{U(1)_A}_{\text{gauged}}$$

$$\psi_1 \to e^{i\theta(x)\gamma_5}\psi_1 , \quad \psi_2 \to e^{-i\theta(x)\gamma_5}\psi_2$$

$$\phi \to e^{-2i\theta(x)}\phi$$

$$A_\mu \to A_\mu + \frac{1}{g}\partial_\mu\theta$$

- Why two fermion species: no axial anomaly
- With $M^2>0$ no symmetry breakdown in scalar sector itself
- Comparison with Higgs mechanism by heart

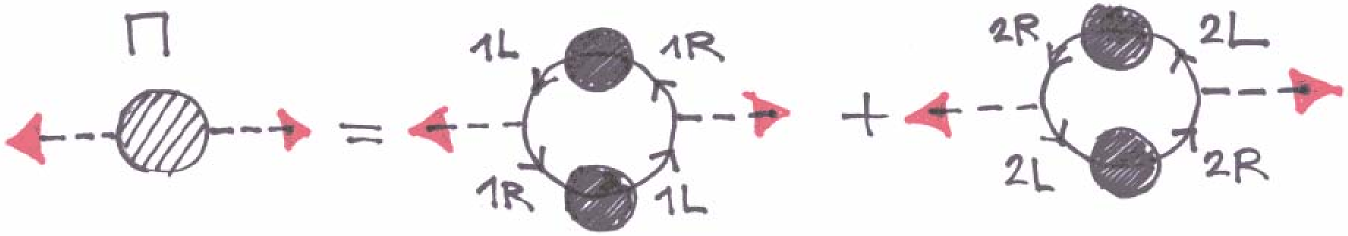
II. Fermion mass generation and scalar boson mass splitting

- ASSUME that Yukawa interactions generate the chiral-symmetry breaking fermion proper self-energies Σ:

$$S(p) \equiv \frac{1}{\not p - \Sigma(p)} = \frac{\not p + \Sigma}{p^2 - \Sigma^2}$$

$$m^2 = \Sigma^2 (p^2 - m^2)$$

THEN Yukawa interactions generate the symmetry-breaking scalar proper self-energy Π

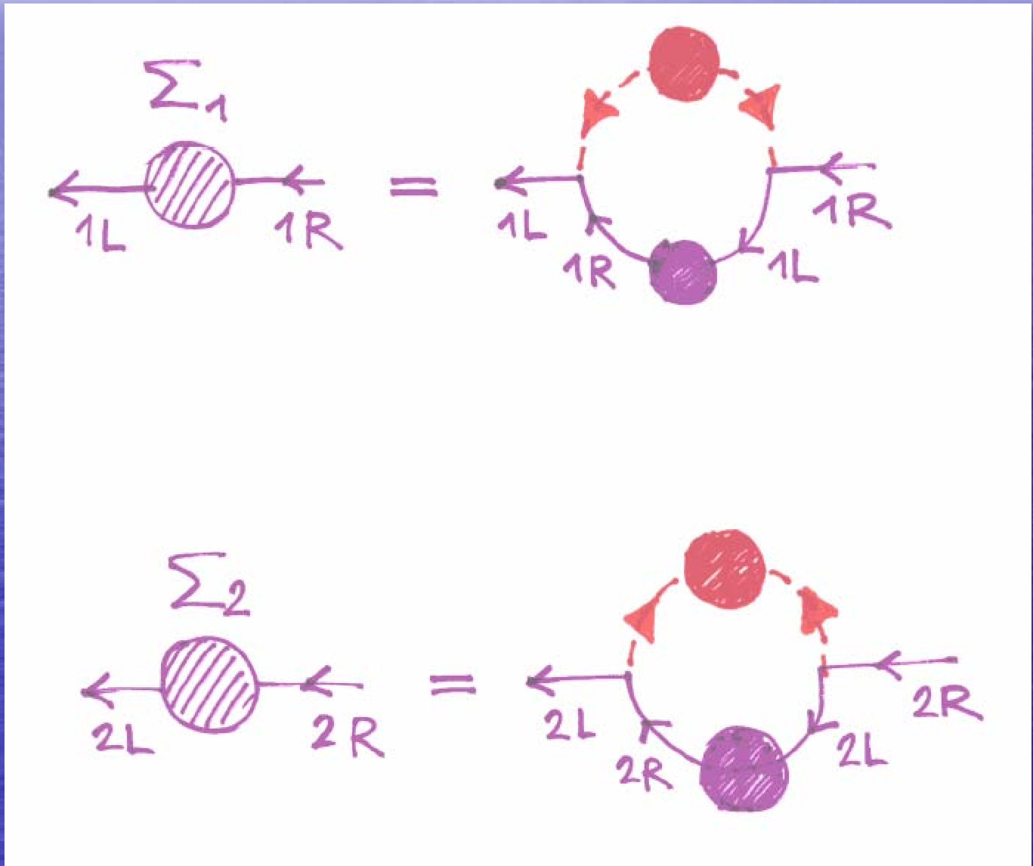


$$\mathcal{L}^{(0)}_{eff} = \partial_\mu \phi^* \partial^\mu \phi - M^2 \phi^* \phi - \frac{1}{2} \phi^* \Pi \phi^* - \frac{1}{2} \phi \Pi^* \phi$$

$$D^{-1}(p) = \begin{pmatrix} p^2 - M^2 & -\Pi \\ -\Pi^* & p^2 - M^2 \end{pmatrix}$$

$$D(p) = \frac{1}{(p^2 - M^2 - |\Pi|)(p^2 - M^2 + |\Pi|)} \begin{pmatrix} p^2 - M^2 & \Pi \\ \Pi^* & p^2 - M^2 \end{pmatrix}$$

Yukawa interactions GENERATE Σ:



NICELY CONVERGENT KERNEL (corresponding counter terms prohibited by symmetry)

- By dimensional argument:

$$m_{1,2} = M f_{1,2}(y_1, y_2)$$

- Compare with Higgs:

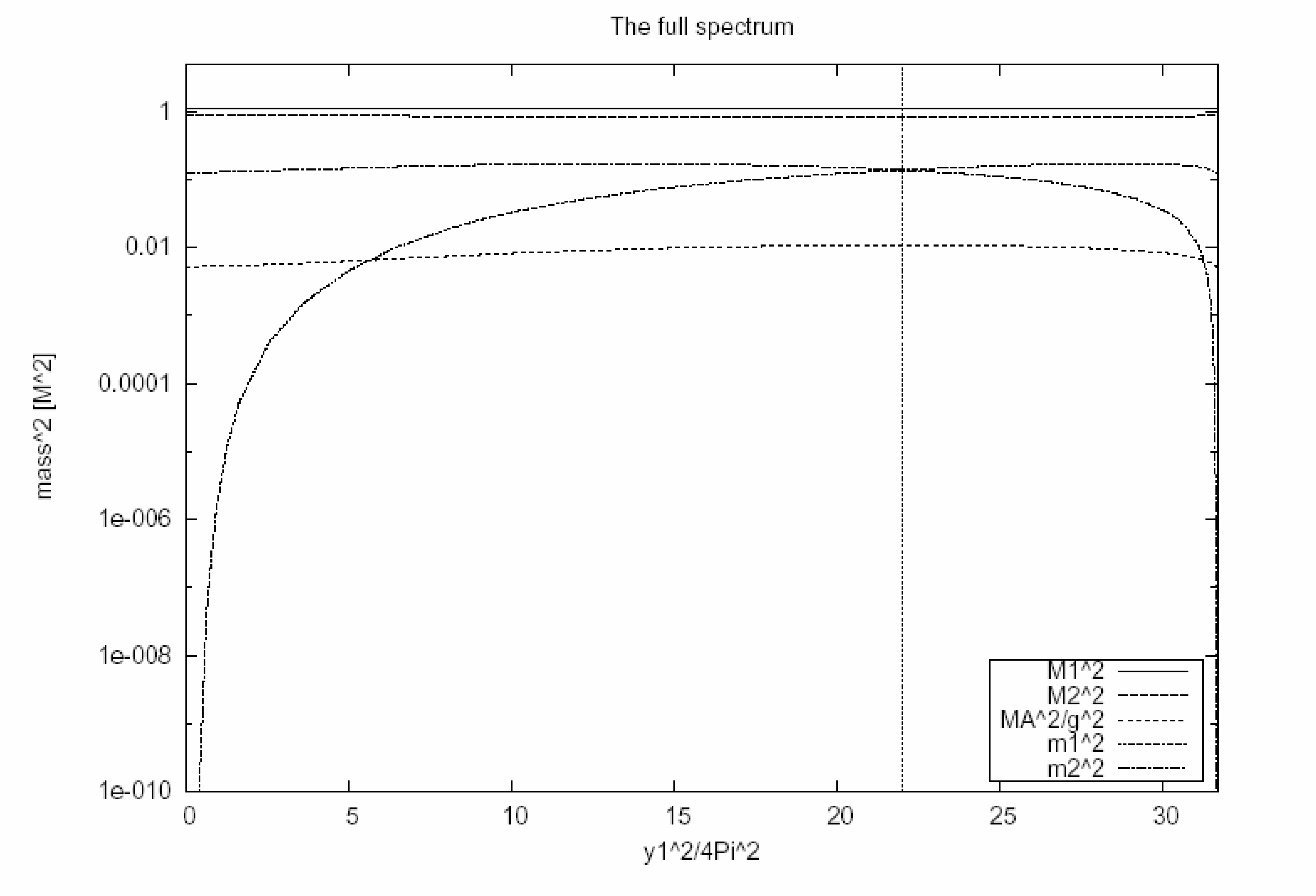
$$m_{1,2} = (-2M^2/\lambda)^{1/2}\, y_{1,2}$$

Boson mass splitting

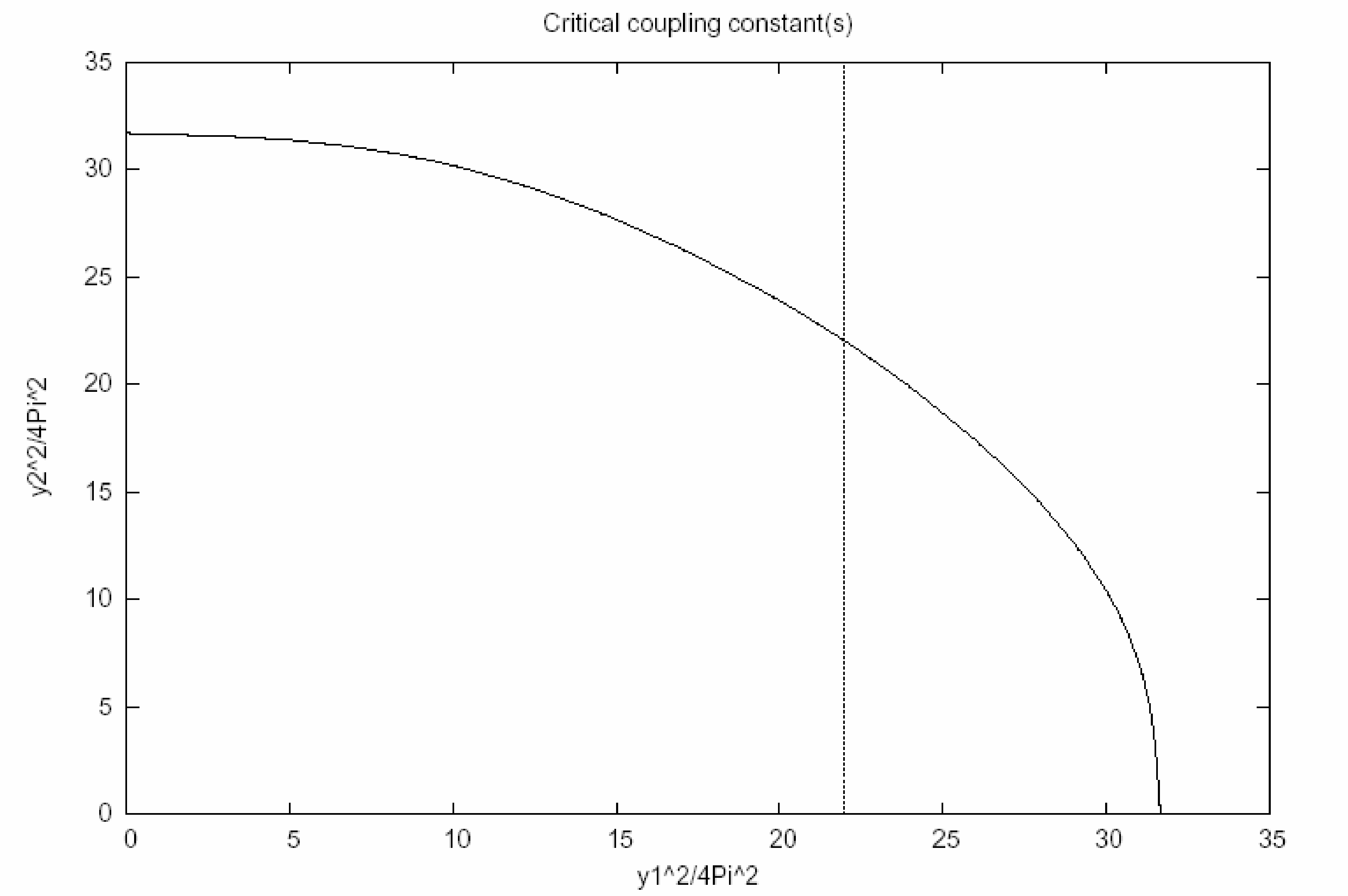
- In particular case of real Π found numerically the real and imaginary parts of Φ are the mass eigenstates with masses

$$M_{1,2}^2 = M^2 \pm \Pi(p^2 = M_{1,2}^2)$$

Non-zero UV-finite solutions Σ and Π do exist. They are found numerically upon Wick rotation.



Solutions found for LARGE YUKAWA COUPLINGS



Large amplification of fermion mass ratios as a response to small changes in Yukawa coupling ratios

Explicit knowledge of non-analytic dependences of masses upon couplings –

ULTIMATE DREAM

Coupling constant λ ignored as unimportant for non-perturbative mass generation

III. Where is the Nambu-Goldstone boson ?

- Axial-vector current

$$j_A^\mu = \bar{\psi}_1 \gamma^\mu \gamma_5 \psi_1 - \bar{\psi}_2 \gamma^\mu \gamma_5 \psi_2 + 2i\left[(\partial^\mu \phi)^\dagger \phi - \phi^\dagger \partial^\mu \phi\right]$$

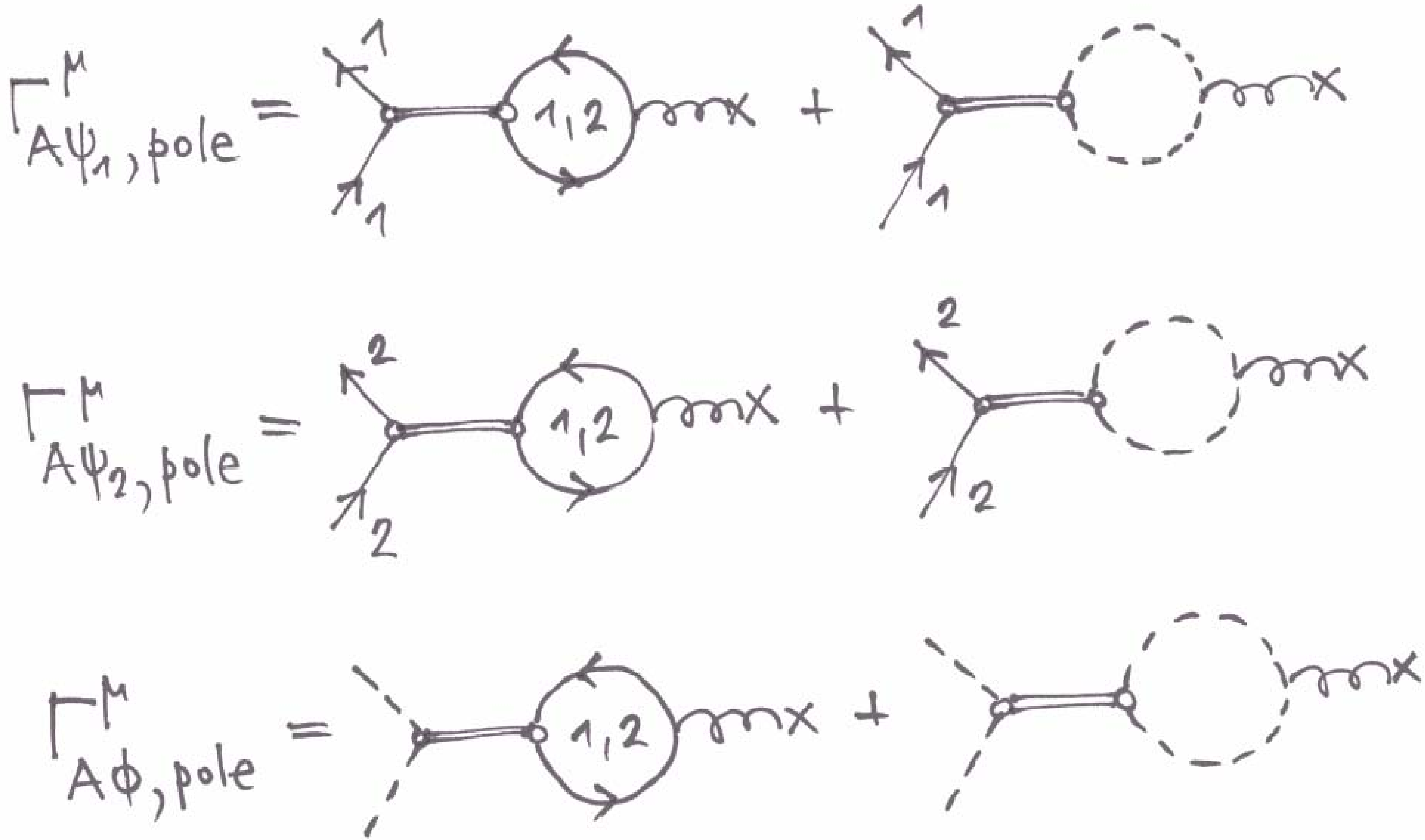
- Axial-vector Ward identities for proper vertices

$$q_\mu \Gamma^\mu_{A\psi_1}(p+q,p) = S_1^{-1}(p+q)\gamma_5 + \gamma_5 S_1^{-1}(p)$$

$$q_\mu \Gamma^\mu_{A\psi_2}(p+q,p) = -S_2^{-1}(p+q)\gamma_5 - \gamma_5 S_2^{-1}(p)$$

$$q_\mu \Gamma^\mu_{A\phi}(p+q,p) = -2D^{-1}(p+q)\Xi + 2\Xi D^{-1}(p)$$

For Σ, Π non-zero the identities imply the massless pole in proper vertices Γ



Basic quantities to be calculated are the UV finite vectorial tadpoles I:

$$\Gamma^{\mu}_{A\psi_1,\,pole} \equiv P_{\psi_1}(p+q,p)\,\frac{i}{q^2}\left[I^{\mu}_{\psi_1}(q)+I^{\mu}_{\psi_2}(q)+I^{\mu}_{\phi}(q)\right]$$

$$\Gamma^{\mu}_{A\psi_2,\,pole} = P_{\psi_2}(p+q,p)\,\frac{i}{q^2}\left[I^{\mu}_{\psi_1}(q)+I^{\mu}_{\psi_2}(q)+I^{\mu}_{\phi}(q)\right]$$

$$\Gamma^{\mu}_{A\phi,\,pole} = P_{\phi}(p+q,p)\,\frac{i}{q^2}\left[I^{\mu}_{\psi_1}(q)+I^{\mu}_{\psi_2}(q)+I^{\mu}_{\phi}(q)\right]$$

Effective NG couplings are related to Σ and Π:

$$P_{\psi_1}(p+q,p) \equiv -\frac{1}{N}\left[\Sigma_1(p+q)+\Sigma_1(p)\right]\gamma_5$$

$$P_{\psi_2}(p+q,p) \equiv \frac{1}{N}\left[\Sigma_2(p+q)+\Sigma_2(p)\right]\gamma_5$$

$$P_{\phi}(p+q,p) \equiv -\frac{2}{N}\begin{pmatrix} 0 & \Pi(p+q)+\Pi(p) \\ -\Pi^*(p+q)-\Pi^*(p) & 0 \end{pmatrix}$$

The overall normalization is given by tadpoles I:

$$I_{\psi_{1,2}}(q^2) \equiv \frac{1}{N} J_{\psi_{1,2}}(q^2)$$

$$I_\phi(q^2) \equiv \frac{1}{N} J_\phi(q^2)$$

$$N = \left(J_{\psi_1}(0) + J_{\psi_2}(0) + J_\phi(0) \right)^{1/2}$$

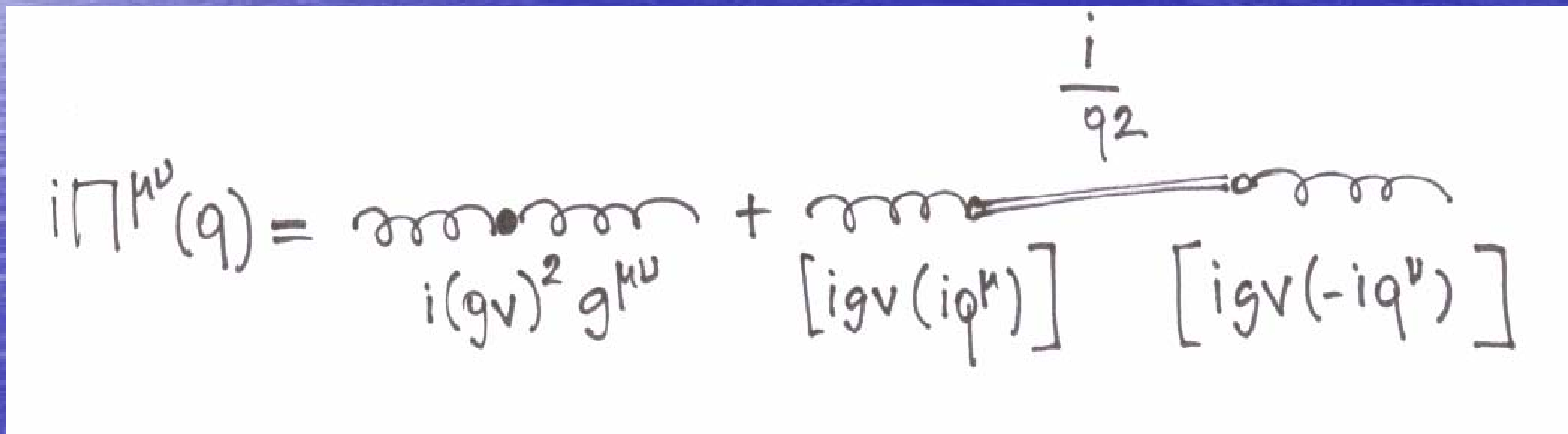
IV. Gauge boson mass generation

- Gauge boson mass squared is the residue at the massless pole of the gauge field polarization tensor

(Schwinger):

$$i\Pi^{\mu\nu}(q) \equiv -i\left(q^2 g^{\mu\nu} - q^\mu q^\nu\right)\Pi(q^2)$$

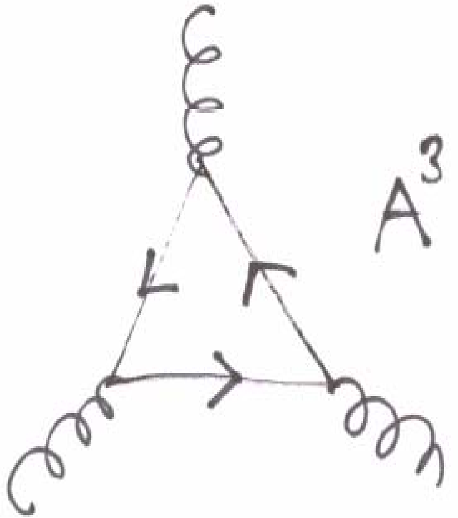
- In Higgs (complete polarization tensor):

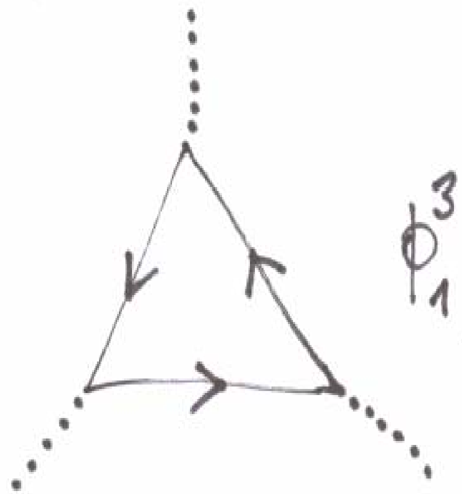


In strongly coupled models no control on the bound-state spectrum except NG. Consequently, only the longitudinal part can be computed. By transversality

$$m_A^2 = g^2\left[J_{\psi_1}(0) + J_{\psi_2}(0) + J_\phi(0)\right]$$

V. Symmetry-breaking loop-generated UV-finite vertices: genuine (albeit gedanken) predictions (work in progress)





VI. Outlook

- Common source of fermion and gauge-boson mass generation
- Hope for natural description of wide, sparse and irregular fermion mass spectrum
- SU(2)xU(1) generalization exists (T. Brauner, J.H., A model of flavors, hep-ph/0407339
- Phenomenological viability is to be investigated