Playing with fermion couplings in Higgsless models

Daniele Dominici
Florence University

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CPNSH, CERN

- **Introduction**
- **A $SU(2)_L \times SU(2)^K \times U(1)$ linear moose**
- **Electroweak precision tests**
- **Unitarity bounds**
- **Delocalizing fermion interactions**
- **Conclusions**

Based on:

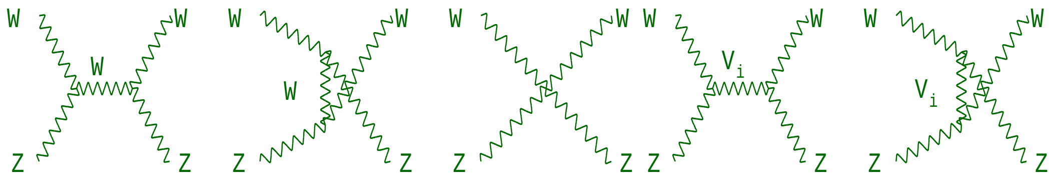
Casalbuoni, De Curtis, D., PRD hep-ph/0405188

Casalbuoni, De Curtis, Dolce, D., PRD hep-ph/05022209

NO Higgs SM

- $W_L W_L$ scattering becomes strong at scales $\sim$ TeV. Partial wave unitarity is violated.
- $W_L W_L$ scattering can remains perturbative at scale $\sim$ TeV through the presence of resonances. Violation of partial wave unitarity is delayed.

(Modern) Higgsless Models



- Diagrams involving new particles (KK modes) cancel the growing terms of $W_L W_L$ scattering amplitudes at high energy. Cancellations are due to symmetries of 5D models. (Chivukula, Dicus, He).

Gauge theory on an interval

Theory with gauge symmetry G in $4+1$ dims and flat metric:

$$
\begin{aligned}
S &= -\frac{1}{2}\int d^4x \int_0^{\pi R} dy \frac{1}{g_5^2}\left[\mathrm{Tr}[F_{MN}F^{MN}]\right], \quad M,N = 0,1\ldots,4 \\
&= -\frac{1}{2}\int d^4x \int_0^{\pi R} dy \frac{1}{g_5^2}\left[\mathrm{Tr}[F_{\mu\nu}F^{\mu\nu}] + 2\mathrm{Tr}[F_{\mu 5}F^{\mu 5}]\right]
\end{aligned}
$$

$$
-\int d^4x \int_0^{\pi R} dy \frac{1}{g_5^2}\mathrm{Tr}[F_{\mu 5}F^{\mu 5}] \ni 2\int d^4x \int_0^{\pi R} dy \mathrm{Tr}[\partial_5 A_\mu \partial^\mu A^5]
$$

Gauge fixing:

$$
-\frac{1}{\xi}\int d^4x \int_0^{\pi R} dy \mathrm{Tr}[\partial^\mu A_\mu - \xi \partial_5 A_5]^2
$$

A_5 Goldstones.
Compactifying without G breaking: KK modes
$A_\mu(x,y) \sim \sum_k A_\mu^{(k)}(x)\cos(ky/R)$ and $M_{A^{(k)}} = k/R$
$k = 0, 1, \ldots$.

Breaking $G = SU(2)$ by boundary conditions:

$$A_\mu^{1,2} = 0|_{y=0} \qquad \partial_y A_\mu^{1,2} = 0|_{y=\pi R}$$

$$A_\mu^{1,2} \sim \sum_k \sin\left(\frac{(2k+1)y}{2R}\right) A_\mu^{1,2(k)}$$

$$M_{A^{1,2(k)}} = \frac{k+1/2}{R}$$

$$\partial_y A_\mu^{3} = 0|_{y=0} \qquad \partial_y A_\mu^{3} = 0|_{y=\pi R}$$

$$A_\mu^{3} \sim \sum_k \cos\left(\frac{ky}{R}\right) A_\mu^{3(k)}$$

$$M_{A^{3(k)}} = \frac{k}{R}$$

Spectrum can be modified by kinetic terms localized on the branes at $y = 0$ and $y = \pi R$, warping the metric

$$ds^2 = \exp\left(-k|y|\right)\eta_{\mu\nu} dx^\mu dx^\nu - dy^2$$

- **Symmetry breaking mechanism of a $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ gauge theory in $4+1$ dims by boundary conditions on the branes** (Csáki, Grojean, Murayama, Pilo, Terning; Nomura).

- **Problem: $\epsilon_3(S)$ electroweak parameter too big, if unitarity is fine** (Barbieri, Pomarol, Rattazzi).

- **Solutions:**

  brane kinetic terms (Cacciapaglia, Csaki, Grojean, Terning; Carena, Tait, Wagner; Carena, Ponton Tait, Wagner; Davoudiasl, Hewett, Lillie, Rizzo).

  fermion delocalization (Cacciapaglia, Csaki, Grojean, Terning; Foadi, Gopalakrishna, Schmidt; Bhattacharya, Csaki, Martin, Shirman, Terning).

Equivalence Theorem for the Kaluza-Klein gauge fields

(Chivukula, Dicus, He)

To compute longitudinal gauge boson or their KK excitation scattering, at energies much larger than their masses, we can simply calculate using their corresponding Goldstones

$$T(A_L^{\mu\,(m)}, A_L^{\mu\,(n)}, \ldots) \simeq T(A^{5(m)}, A^{5(n)}, \ldots) + O(M_{A^{(k)}}/E) \sim const$$

$M_{A^{(k)}}$ **being the biggest one of the gauge boson masses.**

Unitarity limit from coupled channels for $SU(2)$:

$$\Lambda \sim \frac{K}{R} \leq \frac{8\pi^2}{g_5^2} \sim \frac{8\pi^2}{g_4^2}\frac{M_W^2}{M_{W^1}} \sim \frac{\pi^{3/2}}{g}\frac{M_W}{M_{W^1}}\Lambda_{SM}$$

Alternative approach to 5D: dimensional deconstruction and moose models

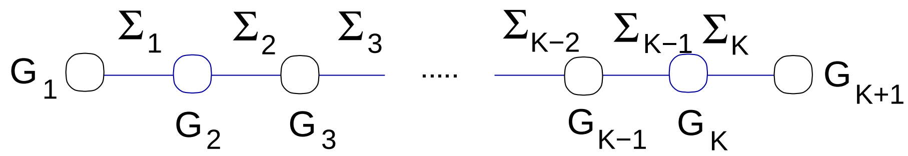
(Arkani-Hamed, Cohen, Georgi)

Theory with gauge symmetry $[G]^{K+1}$ in $3+1$ dims:

$$A^j = A^{ja}T^a, \quad g_c; \quad j = 1, \cdots, K+1$$

Non linear σ-model fields:

$$\Sigma_i = e^{i/(2f_c)\pi^a T^a}, \quad \Sigma_i \to U_i \Sigma_i U_{i+1}^\dagger, \quad U_i \in G_i, \quad i = 1, 2, \cdots, K.$$



$$\mathcal{L}_{moose} = \sum_{i=1}^{K} f_c^2 \mathrm{Tr}[D_\mu \Sigma_i^\dagger D^\mu \Sigma_i] - \frac{1}{2}\sum_{i=1}^{K+1} \mathrm{Tr}[(F_{\mu\nu}^i)^2]$$

$$D_\mu \Sigma_i = \partial_\mu \Sigma_i - i g_c A_\mu^i \Sigma_i + i g_c \Sigma_i A_\mu^{i+1}$$

Mass matrix $\{A_\mu^1, A_\mu^2, \dots, A_\mu^{K+1}\}$:

$$M^2 = g_c^2 f_c^2 \begin{pmatrix} 1 & -1 & 0 & \dots & 0 & 0 \\ -1 & 2 & -1 & \dots & 0 & 0 \\ 0 & -1 & 2 & \ddots & 0 & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 2 & -1 \\ 0 & 0 & 0 & \dots & -1 & 1 \end{pmatrix}$$

Mass eigenvalues:

$$M_k^2 = 4 g_c^2 f_c^2 \sin^2\left(\frac{\pi k}{2(K+1)}\right) \longrightarrow \left(\frac{k}{R}\right)^2, \quad |k| \ll K$$

For $|k| \ll K$ they reproduce the masses of KK excitations for a five dimensional theory with gauge symmetry G, compactification radius R, gauge coupling g_5, lattice spacing a:

$$\pi R = (K+1)a \qquad \frac{a}{g_5^2} = \frac{1}{g_c^2} \qquad a = \frac{1}{g_c f_c}$$

Extra dimension on a lattice

(Hill, Pokorski, Wang; Randall, Shadmi, Weiner; Abe, Kobayashi, Maru, Yoshioka)

Theory with gauge symmetry G in $4+1$ dims and flat metric:

$$\begin{aligned} S &= -\frac{1}{2}\int d^4x \int_0^{\pi R} dy \frac{1}{g_5^2}\left[\mathrm{Tr}[F_{MN}F^{MN}]\right], \quad M,N = 0,1\ldots,4 \\ &= -\frac{1}{2}\int d^4x \int_0^{\pi R} dy \frac{1}{g_5^2}\left[\mathrm{Tr}[F_{\mu\nu}F^{\mu\nu}] + 2\mathrm{Tr}[F_{\mu 5}F^{\mu 5}]\right] \end{aligned}$$

$\downarrow$ **discretization of extra dim with lattice size $a = \pi R/K$**

$$S_{moose} \sim -\frac{1}{2}\int d^4x \frac{a}{g_5^2}\sum_j \left[\mathrm{Tr}[F_{\mu\nu}^j F^{\mu\nu j}] + \frac{2}{a^2}\mathrm{Tr}[(D_\mu\Sigma^j)^\dagger(D^\mu\Sigma^j)]\right]$$

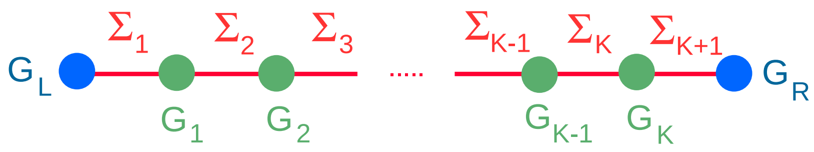
where

$$\Sigma^j \sim e^{i\int_{y_j}^{y_j+a} dt A_5(x,t)} \Rightarrow D_\mu\Sigma^j \sim aF_{\mu 5}^j = ia\partial_\mu A_5^j - i(A_\mu^{j+1} - A_\mu^j).$$

G^{K+1} gauge theory $\Leftrightarrow$ 5D gauge theory compactified

A $SU(2)_L \times SU(2)^K \times U(1)$ linear moose

(Casalbuoni, De Curtis, D., see also Foadi, Gopalakrishna, Schmidt; Hirn, Stern; Chivukula et al; Georgi)



The transformation properties of the fields are

$$\Sigma_1 \to L\Sigma_1 U_1^\dagger, \quad \Sigma_2 \to U_2 \Sigma_3 U_3^\dagger \qquad \ldots \quad \Sigma_{K+1} \to U_K \Sigma_{K+1} R^\dagger$$

$$U_i \in G_i \equiv SU(2)_i, \quad i = 1, 2, \cdots, K \qquad A_\mu^i = A_\mu^{ia}\tau^a/2, \quad g_i,$$

$$L \in G_L \equiv SU(2)_L \quad \tilde{W}_\mu = \tilde{W}_\mu^a \tau^a/2, \quad \tilde{g},$$

$$R \in G_R \equiv SU(2)_R \supset U(1)_Y \qquad \tilde{Y}_\mu = \tilde{\mathcal{Y}}_\mu \tau^3/2, \quad \tilde{g}'$$

$$\mathcal{L} = \sum_{i=1}^{K+1} f_i^2 \mathrm{Tr}[D_\mu \Sigma_i^\dagger D^\mu \Sigma_i] - \frac{1}{2}\sum_{i=1}^{K} \mathrm{Tr}[(F_{\mu\nu}^i)^2] - \frac{1}{2}\mathrm{Tr}[(F_{\mu\nu}(\tilde{W}))^2] - \frac{1}{2}\mathrm{Tr}[(F_{\mu\nu}(\tilde{Y}))^2]$$

$$\begin{aligned} D_\mu \Sigma_1 &= \partial_\mu \Sigma_1 - i\tilde{g}\tilde{W}_\mu \Sigma_1 + i\Sigma_1 g_1 A_\mu^1 \\ D_\mu \Sigma_i &= \partial_\mu \Sigma_i - i g_{i-1} A_\mu^{i-1}\Sigma_i + i\Sigma_i g_i A_\mu^i, \quad i = 2, \cdots, K, \\ D_\mu \Sigma_{K+1} &= \partial_\mu \Sigma_{K+1} - i g_K A_\mu^K \Sigma_{K+1} + i\tilde{g}' \Sigma_{K+1}\tilde{Y}_\mu \end{aligned}$$

For the moment we assume standard fermionic couplings w.r.t. $SU(2)_L \otimes U(1)_Y$: fermions are located at the end of the moose (ψ_L (ψ_R) to the left (right) brane).

Global symmetry: $SU(2)_L \times SU(2)^K \times SU(2)_R$.

$f_i = f_c \ \forall i \Rightarrow$ flat metric in five dims; varying $f_i \Rightarrow$ warped metric.

At the leading order in $O((\tilde{g}/g_i)^2)$

$$\tilde{M}_W^2 \sim \frac{v^2}{4}\tilde{g}^2 \quad \tilde{M}_Z^2 \sim \tilde{M}_W^2/\tilde{c}_\theta^2 \quad \tan\tilde{\theta} = \frac{\tilde{g}}{\tilde{g}'}$$

$$\frac{4}{v^2} \equiv \frac{1}{f^2} = \sum_{i=1}^{K+1} \frac{1}{{f_i}^2}$$

KK masses ($g_i = g_c,\ f_i = f_c$):

$$M_{W^k}^2 \sim g_c^2(K+1)v^2 \left(\sin\frac{k\pi}{2(K+1)}\right)^2 + 2\tilde{M}_W^2 \left(\cos\frac{k\pi}{2(K+1)}\right)^2$$

$$M_{Z^k}^2 \sim g_c^2(K+1)v^2 \left(\sin\frac{k\pi}{2(K+1)}\right)^2 + 2\tilde{M}_Z^2 \left(\cos\frac{k\pi}{2(K+1)}\right)^2$$

Electroweak precision tests

$SU(2)_D$ custodial symmetry: $\epsilon_1 = \epsilon_2 = 0$, at the leading order in $O((\tilde{g}/g_i)^2)$.

The mass matrix of the A_i gauge fields:

$$\mathcal{L}_{\rm mass} = \frac{1}{2}\sum_{i,j=1}^{K} (M_2)_{ij} A^i_\mu A^{\mu j}$$

with

$$(M_2)_{ij} = g_i^2(f_i^2 + f_{i+1}^2)\delta_{i,j} - g_i g_{i+1} f_{i+1}^2 \delta_{i,j-1} - g_j g_{j+1} f_{j+1}^2 \delta_{i,j+1}$$

Calling $\tilde{A}^n_\mu$, $n = 1, \cdots, K$ the mass eigenstates, and m_n^2 the squared mass eigenvalues,

$$A^i_\mu = \sum_{n=1}^{K} S^i_n \tilde{A}^n_\mu$$

$$S^i_m (M_2)_{ij} S^j_n = m_n^2 \delta_{m,n}$$

Dispersive representation (Peskin, Takeuchi):

$$\epsilon_3 \left(= \frac{\tilde{g}^2 S}{16\pi}\right) = -\frac{\tilde{g}^2}{4\pi}\int_0^\infty \frac{ds}{s^2} Im[\Pi_{VV}(s) - \Pi_{AA}(s)]$$

Current current correlator:

$$\int d^4x e^{-iq\cdot x} \left\langle J^\mu_{V(A)} J^\nu_{V(A)} \right\rangle \sim i g^{\mu\nu} \Pi_{VV(AA)}(q^2) + \cdots$$

Vector and axial vector currents associated to the global $SU(2)_L \times SU(2)_R$. The vector meson decay constants

$$\begin{aligned} \langle 0 | J^a_{V\mu} | \tilde{A}^n_b(p,\epsilon)\rangle &= g_{nV}\delta^{ab}\epsilon_\mu \\ \langle 0 | J^a_{A\mu} | \tilde{A}^n_b(p,\epsilon)\rangle &= g_{nA}\delta^{ab}\epsilon_\mu \end{aligned}$$

$$\begin{aligned} J^a_{V(A)\mu}\Big|_{\text{vector mesons}} &= f_1^2 g_1 A^{1a}_\mu + (-) f^2_{K+1} g_K A^{Ka}_\mu \\ g_{nV(A)} &= f_1^2 g_1 S^1_n + (-) f^2_{K+1} g_K S^K_n \end{aligned}$$

Vector meson dominance

$$Im\Pi_{VV(AA)} = -\pi \sum_{nV(nA)} g^2_{nV(nA)} \delta(s - m_n^2)$$

$$\begin{aligned} \epsilon_3 &= \frac{\tilde{g}^2}{4} \sum_n \left(\frac{g^2_{nV}}{m_n^4} - \frac{g^2_{nA}}{m_n^4} \right) = g^2 g_1 g_K f_1^2 f_{K+1}^2 \sum_n \frac{S_n^1 S_n^K}{m_n^4} \\ &= \tilde{g}^2 g_1 g_K f_1^2 f_{K+1}^2 (M_2^{-2})_{1K} = \tilde{g}^2 \sum_{i=1}^{K} \frac{(1-y_i) y_i}{g_i^2} \end{aligned}$$

where

$$y_i = \sum_{j=1}^{i} \frac{f^2}{f_j^2}$$

Since $0 \le y_i \le 1 \Rightarrow \epsilon_3 > 0$. For $f_i = f_c$, $g_i = g_c$

$$\epsilon_3 = \frac{1}{6} \frac{K(K+2)}{K+1} \frac{\tilde{g}^2}{g_c^2}$$

$\epsilon_3 \sim 10^{-3}$: $K = 1 \Rightarrow g_c \sim 16\tilde{g}$. For increasing K, $g_c \sim 10\sqrt{K}$.

Possible solution: Cutting a link

Let us suppose $\exists m : f_m = 0$, then the squared mass matrix

$$(M_2)_{ij} = g_i^2(f_i^2+f_{i+1}^2)\delta_{i,j} - g_i g_{i+1} f_{i+1}^2 \delta_{i,j-1} - g_j g_{j+1} f_{j+1}^2 \delta_{i,j+1}$$

becomes block diagonal, $\Rightarrow (M_2^{-2})_{1K} = 0$

$$\Rightarrow \epsilon_3 = \tilde{g}^2 g_1 g_K f_1^2 f_{K+1}^2 (M_2^{-2})_{1K} = 0$$

However we loose a link (and the corresponding scalar multiplet Σ_m). Add the lagrangian term

$$f_0^2 Tr[\partial_\mu U^\dagger \partial^\mu U], \;\; U = \Sigma_1 \Sigma_2 \cdots \Sigma_{K+1}$$

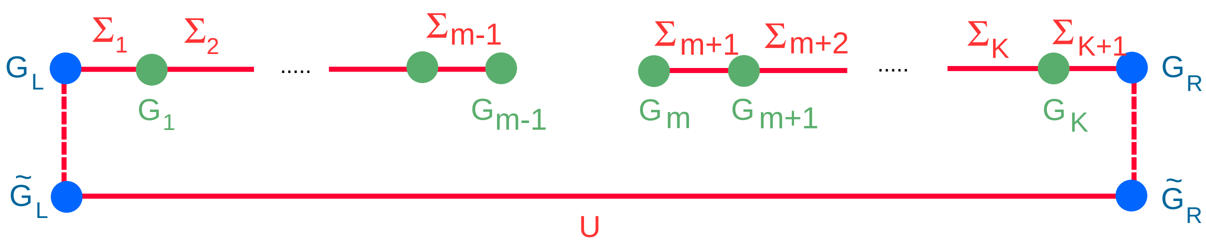
The total lagrangian

$$\begin{aligned} \mathcal{L} &= f_0^2 Tr[\partial_\mu U^\dagger \partial^\mu U] + \sum_{i=1}^{m-1} f_i^2 \mathrm{Tr}[D_\mu \Sigma_i^\dagger D^\mu \Sigma_i] \\ &+ \sum_{i=m+1}^{K+1} f_i^2 \mathrm{Tr}[D_\mu \Sigma_i^\dagger D^\mu \Sigma_i] - \frac{1}{2} \sum_{i=1}^{K} \mathrm{Tr}[(F^i_{\mu\nu})^2] \end{aligned}$$

Enhancement of the symmetry from $G_L \otimes G_R \otimes \prod_{i=1}^{K} G_i$ to $G_L \otimes G_R \otimes \tilde{G}_L \otimes \tilde{G}_R \otimes \prod_{i=1}^{K} G_i$, where $\tilde{G}_{L(R)}$ is a copy of $G_{L(R)}$ and U transforms as

$$U \to \tilde{L} U \tilde{R}^\dagger$$

with $\tilde{L}(\tilde{R}) \in \tilde{G}_{L(R)}$.



For $K = 2$ this model coincides with D-BESS (Casalbuoni et al)

Partial wave unitarity bounds

(see also Chivukula, He; D., De Curtis, Pelaez; Papucci; Muck, Nilse, Pilaftsis, Ruckl)

Equivalence theorem, $\Sigma_i = \exp\left(i f \vec{\pi}\cdot\vec{\tau}/2f_i^2\right)$:

$$\begin{aligned}\mathcal{A}_{W_L^+W_L^-\to W_L^+W_L^-} &\sim \mathcal{A}_{\pi^+\pi^-\to\pi^+\pi^-} \\ &\sim -\frac{1}{4}f^4\sum_{i=1}^{K+1}\frac{u}{f_i^6} + \frac{1}{4}f^4\sum_{i=1}^{K+1} L_{ij}\left(\frac{u-t}{(s-M^2)_{ij}} + (t\to s)\right)\end{aligned}$$

where $L_{ij} = g_i g_j \left(\frac{1}{f_i^2} + \frac{1}{f_{i+1}^2}\right)\left(\frac{1}{f_j^2} + \frac{1}{f_{j+1}^2}\right)$.

High energy limit:

$$\mathcal{A}_{W^+W^-\to W^+W^-} \to -\frac{1}{4}f^4\sum_{i=1}^{K+1}\frac{u}{f_i^6}$$

minimized when $f_i = f_c$, $\forall i$:

$$\mathcal{A}_{W^+W^-\to W^+W^-} \to -\frac{u}{(K+1)^2 v^2}$$

Unitarity condition from $J = 0$ partial wave $|a_0| < 1/2$:

$$a_0 = \frac{s}{16\pi(K+1)^2 v^2} \rightarrow \Lambda = (K+1) 1.2 \text{ TeV}$$

Considering all channels: $\Sigma_i = \exp\left(i\vec{\pi}_i \cdot \vec{\tau}/2f_i\right)$:

$$\mathcal{A}_{\pi_i \pi_i \rightarrow \pi_i \pi_i} \rightarrow -\frac{1}{4}\frac{u}{f_i^2}$$

The unitarity limit is determined by the smallest f_i. Taking all equal

$$\boxed{\Lambda^{TOT} = \sqrt{K+1}\ 1.2 \text{ TeV}}$$

Higgs bosons are not necessary up scales $\sqrt{K+1}$ times the scale of unitarity violation in the Higgsless SM, i.e. 1.2 TeV. Approximately $M_A^{max} < \Lambda^{TOT}$ implies (when $f_i = f_c$) $g_c \lesssim 5$. Increasing K, $M_{W^{(1)}}$ decreases, $\Rightarrow \epsilon_3$ too large.

Delocalizing fermion interactions

(Casalbuoni, De Curtis, Dolce, D.; see also Chivukula, Simmons, He, Kurachi)

Let us build

$$\chi_L^i = \Sigma_i^\dagger \Sigma_{i-1}^\dagger \cdots \Sigma_1^\dagger \psi_L, \quad i = 1, \ldots, K$$

Transformation properties

$$\chi_L^i \to U_i \chi_L^i\,, \quad U_i \in SU(2)_i\,, \quad i = 1, \ldots, K\,.$$

New invariants:

$$\bar{\chi}_L^i i\gamma^\mu(\partial_\mu + i g_i A_\mu^i + \frac{i}{2}\tilde{g}'(B-L)\tilde{\mathcal{Y}}_\mu)\chi_L^i, \quad i = 1, \ldots, K$$

New fermion lagrangian:

$$\begin{aligned}\mathcal{L}_{fermions}^{tot} &= \bar{\psi}_L i\gamma^\mu \left[\partial_\mu + i\tilde{g}\tilde{W}_\mu + \frac{i}{2}\tilde{g}'(B-L)\tilde{\mathcal{Y}}_\mu\right]\psi_L + \\ &+ \sum_{i=1}^K b_i \bar{\chi}_L^i i\gamma^\mu \left[\partial_\mu + i g_i A_\mu^i + \frac{i}{2}\tilde{g}'(B-L)\tilde{\mathcal{Y}}_\mu\right]\chi_L^i +\end{aligned}$$

$$+ \quad \bar{\psi}_R i\gamma^\mu \left[\partial_\mu + i\tilde{g}'\frac{\tau^3}{2}\tilde{\mathcal{Y}}_\mu + \frac{i}{2}\tilde{g}'(B-L)\tilde{\mathcal{Y}}_\mu\right]\psi_R$$

b_i **dimensionless parameters.** **In the unitary gauge ($\Sigma_i \equiv I$) and after a rescaling $\psi_L \rightarrow \frac{1}{\sqrt{1+\sum_{i=1}^K b_i}}\psi_L$:**

$$\begin{aligned}
\mathcal{L}^{tot}_{fermions} &= \bar{\psi}_R i\gamma^\mu \left[\partial_\mu + i\tilde{g}'\frac{\tau^3}{2}\tilde{\mathcal{Y}}_\mu + \frac{i}{2}\tilde{g}'(B-L)\tilde{\mathcal{Y}}_\mu\right]\psi_R + \\
&+ \bar{\psi}_L i\gamma^\mu \Big[\partial_\mu + \frac{1}{1+\sum_{i=1}^K b_i}\Big(i\tilde{g}\tilde{W}_\mu + \sum_{i=1}^K b_i \; i g_i A^i_\mu\Big) \\
&+ \frac{i}{2}\tilde{g}'(B-L)\tilde{\mathcal{Y}}_\mu\Big]\psi_L
\end{aligned}$$

Low-energy limit

Eliminating the A_i fields when $(\tilde{g}/g_i)^2 \ll 1, \quad \forall i$ and after a field redefinition:

$$\mathcal{L}_{eff}^{charg} = \frac{-\tilde{e}}{\sqrt{2}\tilde{s}_\theta}(1 - \frac{b}{2} - \frac{z_w}{2})\overline{\psi}\gamma^\mu \frac{1-\gamma_5}{2}\psi W_\mu^- + \ h.c.$$

$$\mathcal{L}_{eff}^{neutr} = \frac{-\tilde{e}}{\tilde{s}_\theta \tilde{c}_\theta}(1 - \frac{b}{2} - \frac{z_z}{2})\overline{\psi}\gamma^\mu \left[T_L^3 \frac{1-\gamma_5}{2} - Q\tilde{s}_\theta^2 \frac{1 - \frac{\tilde{c}_\theta}{\tilde{s}_\theta} z_{z\gamma}}{1 - \frac{b}{2}}\right]\psi Z_\mu$$

$$- \ \tilde{e}(1 - \frac{z_\gamma}{2})\overline{\psi}\gamma^\mu Q \psi A_\mu$$

where

$$z_\gamma = \tilde{s}_\theta^2 \sum_{i=1}^{K} \left(\frac{\tilde{g}}{g_i}\right)^2 \qquad z_w = \sum_{i=1}^{K} \left(\frac{\tilde{g}}{g_i}\right)^2 (1 - y_i)^2$$

$$z_z = \frac{1}{\tilde{c}_\theta^2} \sum_{i=1}^{K} \left(\frac{\tilde{g}}{g_i}\right)^2 \left(\tilde{c}_\theta^2 - y_i\right)^2 \qquad z_{z\gamma} = -\frac{\tilde{s}_\theta}{\tilde{c}_\theta} \sum_{i=1}^{K} \left(\frac{\tilde{g}}{g_i}\right)^2 \left(\tilde{c}_\theta^2 - y_i\right)$$

$$b = \frac{2\sum_{i=1}^{K} y_i b_i}{1+\sum_{j=1}^{K} b_j}$$

Current current interaction terms are also generated

$$\mathcal{L}_{eff}^{quart} = \beta \sum_{a=1}^{3} \left(\bar{\psi}_L \gamma^\mu \frac{\tau^a}{2} \psi_L\right)^2$$

with

$$\beta = \frac{1}{8f^2}\left(\bar{b}_K - b\right)^2 - \frac{1}{8f^2}\sum_{i=1}^{K} x_{i+1}\bar{b}_i^2$$

$x_i = f^2/f_i^2$ **and** $\bar{b}_i = 2\frac{\sum_{j=1}^{i} b_j}{1+\sum_{j=1}^{K} b_j} \quad (i = 1, \cdots, K)$

Physical quantities:

$$M_Z^2 = \tilde{M}_Z^2(1 - z_z) \quad M_W^2 = \tilde{M}_W^2(1 - z_w)$$

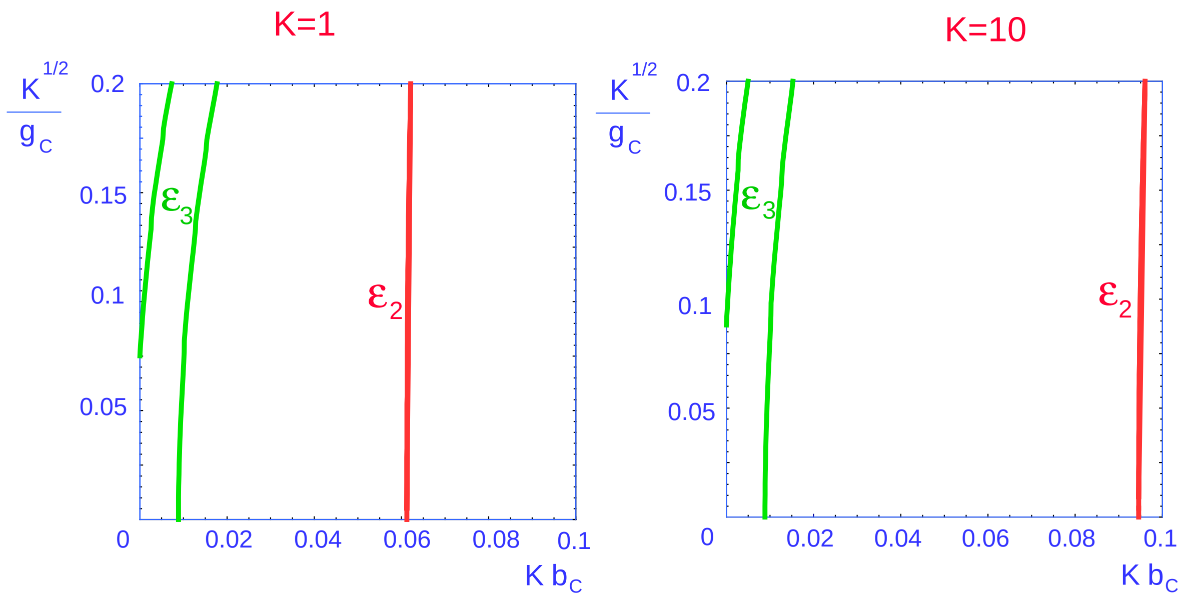
$$e = \tilde{e}(1 - \frac{z_\gamma}{2}) \quad \frac{G_F}{\sqrt{2}} = \frac{1}{8}\tilde{g}^2\Big(1 - \frac{b}{2}\Big)^2\frac{1 - z_w}{M_W^2} + \frac{1}{4\beta}$$

$$\epsilon_1 \simeq 0, \quad \epsilon_2 \simeq 0 \quad \epsilon_3 \simeq \sum_{i=1}^K y_i(\frac{\tilde{g}^2}{g_i^2}(1 - y_i) - b_i)\,.$$

where $y_i = \sum_{j=1}^i \frac{f^2}{f_j^2}$.

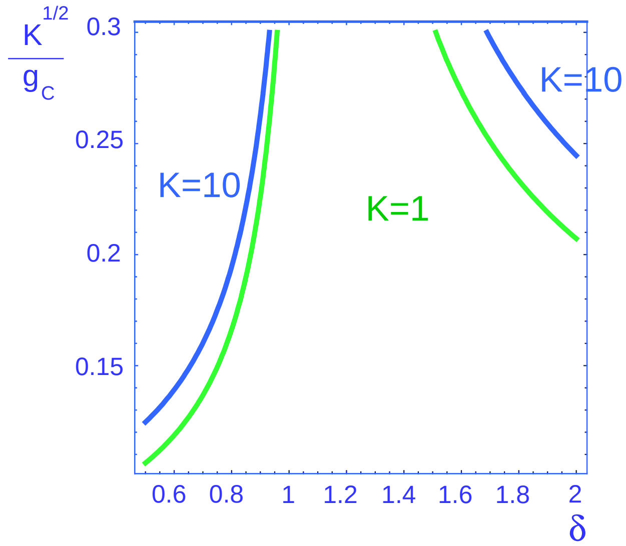
Assume: $g_i \equiv g_c$, $b_i \equiv b_c$, $f_i \equiv f_c$, $\forall i$.

95% CL bounds from ϵ_1: independent on g_c, $b_c < 0.14$ for $K = 1$, $b_c < 0.025$ for $K = 10$.



95% CL bounds on the parameter space $(Kb_c, \sqrt{K}/g_c)$ from the experimental values of ϵ_2 and ϵ_3 for $K = 1$ (left) $K = 10$ (right). The allowed parameter space from ϵ_2 is the region to the left of the dashed line and from ϵ_3 the region between the continuous lines.

Assuming $b_i = \delta \frac{\tilde{g}^2}{g_i^2}(1 - y_i)$, $(g_i \equiv g_c, f_i \equiv f_c, \quad \forall i)$:



95% CL bounds on the parameter space $(\delta, \sqrt{K}/g_c)$ from the experimental value of ϵ_3 for $K = 1$ (continuous line), $(K = 10$ dash line). The allowed parameter space is the region between the corresponding lines.

In conclusion with some fine tuning (fermion delocalization) a portion of parameter space is allowed

Continuum limit ($K \to \infty$)

Let $K \to \infty$ with $Ka \to \pi R$, R being the length of the segment.

$$\lim_{a\to 0} a f_i^2 = f^2(y) \qquad \lim_{a\to 0} a g_i^2 = g_5^2(y) \qquad \lim_{a\to 0} \frac{b_i}{a} = b(y).$$

The lagrangian, $g_5(y) = g_5$, and $f(y) = \bar{f}$ $\quad\to$ flat metric

$$S = -\frac{1}{4}\int d^4x \int_0^{\pi R} dy [\frac{1}{g_5^2}(F^a_{MN})^2 + \frac{1}{\tilde{g}^2}(F^a_{\mu\nu})^2\delta(y) + \frac{1}{\tilde{g}'^2}(F^3_{\mu\nu})^2\delta(y-\pi R)] + S_{ferm}$$

$SU(2) \to U(1)_{em}$ via boundary conditions:

$$\partial_y A_\mu^{1,2} + \frac{g_5^2}{\tilde{g}^2} m_n^2 A_\mu^{1,2}|_{y=0} = 0 = A_\mu^{1,2}(y=\pi R) = 0$$

$$\partial_y A_\mu^3 + \frac{g_5^2}{\tilde{g}^2} m_n^2 A_\mu^3|_{y=0} = 0 \qquad \partial_y A_\mu^3 - \frac{g_5^2}{\tilde{g}'^2} m_n^2 A_\mu^3|_{y=\pi R} = 0$$

The continuum limit of the fermion action ($b = 0$)

$$S_{ferm} = \int d^4x \int_0^{\pi R} dy \left[\delta(y) i\bar{\psi}_L \not{D} \psi_L + \delta(\pi R - y) i\bar{\psi}_R \not{D} \psi_R\right]$$

where

$$\not{D}\psi_L = \left(\not{\partial} + i\frac{\tau^a}{2}\not{A}^a(y) + iY_L \not{A}^3(\pi R)\right)\psi_L$$

$$\not{D}\psi_R = \left(\not{\partial} + iY_R \not{A}^3(y)\right)\psi_R$$

$$\Sigma^j = e^{i\int_{y_j}^{y_j+a} dz A_5(x,z)} \quad : \qquad \Sigma_1 \cdots \Sigma_i \to P\Big[\exp(i\int_0^y dz A_5(x,z))\Big]$$

$$\chi_L^i(x) = \Sigma_i^\dagger \cdots \Sigma_1^\dagger \psi_L(x) \to \chi_L(x,y) = P\Big[\exp(i\int_0^y dz A_5(x,z))\Big]^\dagger \psi_L(x)$$

Fermions leave on the branes but interact with bulk gauge bosons via Wilson lines.

$$S^b_{ferm} \;=\; \int d^4x \int_0^{\pi R} dy\, [b(y) i\bar{\chi}_L \not{D} \chi_L]$$

$$\not{D}\chi_L = \left(\not{\partial} + i\frac{\tau^a}{2}\not{A}^a(y) + iY_L \not{A}^3(\pi R)\right)\chi_L$$

Mass terms for the fermions

$$\lambda^{ij}\bar{\psi}_L^i \Sigma_1 \Sigma_2 \cdots \Sigma_{K+1} \psi_R^j \to \lambda^{ij}\bar{\psi}_L^i P\Big[(\exp(i\int_0^{\pi R} dz A_5(x,z))\Big]\psi_R^j.$$

Assuming $g_5(y) = g_5$

$$\epsilon_3 \equiv 0 \longrightarrow b(y) = \frac{\tilde{g}^2}{g_5^2}\int_y^{\pi R} dz \frac{f^2}{f^2(z)} \qquad \frac{1}{f^2} = \int_0^{\pi R}\frac{dy}{f^2(y)}$$

Flat metric $f(y) = \bar{f}$

$$b(y) = \frac{\tilde{g}^2}{g_5^2}\left(1 - \frac{y}{\pi R}\right)$$

Randall-Sundrum metric $f(y) = \bar{f}e^{ky}$

$$b(y) = \frac{\tilde{g}^2}{g_5^2}\frac{e^{-2\pi kR} - e^{-2ky}}{e^{-2\pi kR} - 1}$$

In general: $b(0) = \frac{\tilde{g}^2}{g_5^2}, \qquad b(\pi R) = 0.$

Signatures: Triple-gauge-boson vertices

(Chivukula, Simmons, He, Kurachi, Tanabashi)

5-dim $SU(2)_A \otimes SU(2)_B$ gauge theory broken to the $U(1)_{em}$ by BC's.

The fermion probability distribution is related to the W boson wave function to minimize the deviations in EW parameters (*ideal delocalization*)

The main characteristics are contained in our deconstructed moose model: important property $\longrightarrow$ the KK resonances of the $W^{\pm}, Z$ gauge bosons are

fermiophobic

Consequences:

- very narrow KK resonances ($\Gamma/M \sim 10^{-4}, 10^{-3}$)
- loose constraints by direct collider search for new gauge bosons

measurements of triple-gauge-boson vertices can provide bounds on KK masses

The actual lower bound on Δg_1^Z from LEPII leads
$M_{W^1} > 500$ **GeV for the *ideal delocalized* model in flat metric.**

@LHC with 30 fb^{-1} from WZ production $\Delta g_1^Z < 0.11 \rightarrow$
$M_{W^1} > 800$ **GeV**

@ 500(800) GeV LC with polarized beams
$\Delta g_1^Z < 0.0048(0.0027)$ **and** $\Delta k_Z < 9.8(4.2)10^{-4} \rightarrow$
$M_{W^1} > 2.6(4.0)$ **TeV**

Collider Phenomenology

(Birkedal, Matchev, Perelstein)

Common feature of the Higgless models: the scale of perturbative unitarity violation is raised by new massive vector bosons whose masses and couplings are constrained by *unitarity sum rules*.

A good test $\longrightarrow$ analysis of the vector boson fusion at future colliders

(the most promising channel for Higgsless models with fermion delocalization since the KK resonances are fermiophobic)

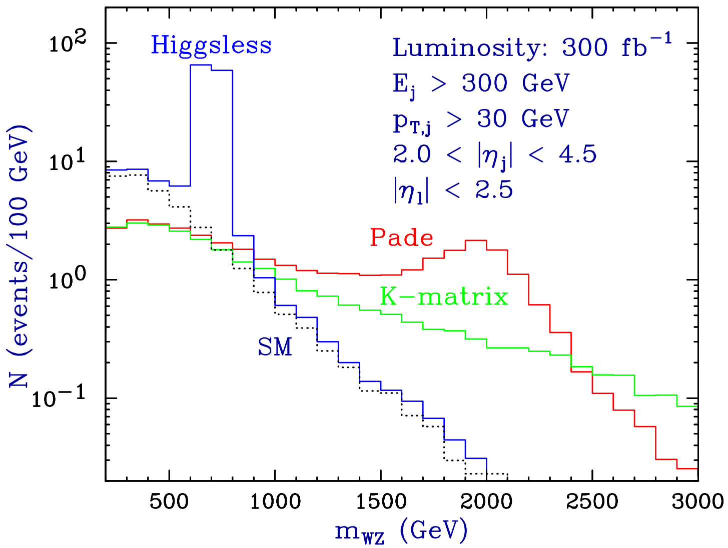
Simplifying assumption: the sum rules are saturated by the first KK resonance V^1

$$g_{WV^1Z} \lesssim \frac{g_{WWZ} M_Z^2}{\sqrt{3} M_{V^1} M_W}, \qquad \Gamma(V^1) = \frac{\alpha (M_{V^1})^3}{144 s_W^2 M_W^2}$$

a very narrow and light resonance in WZ scattering

Higgsless Models at LHC

(Birkedal, Matchev, Perelstein)



Typical final state includes two forward jets + a pair of vector bosons

Cuts to suppress the SM BCKGND and possible signal from Drell-Yan:

$$2 < |\eta| \leq 4.5,\ E > 300 \text{ GeV},\ p_T > 30 \text{ GeV}$$

The gold-plated final state is $2j + 3l$+missing E_T

Discovery reach @ LHC (10 events)

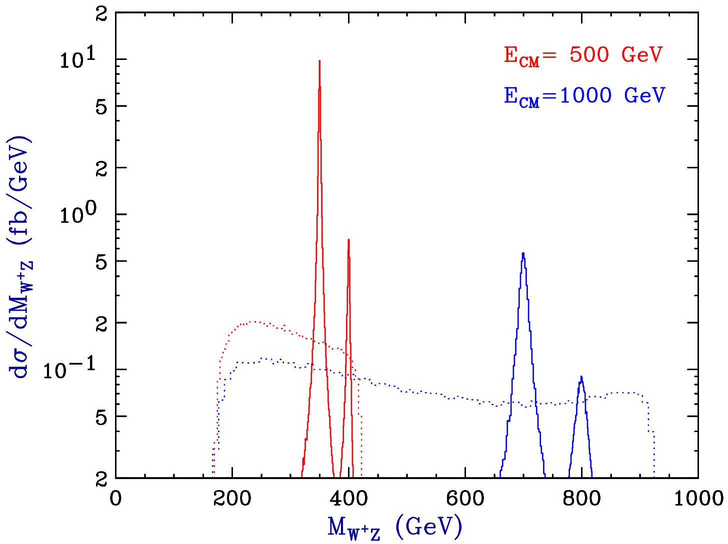
$M_{V^1} \leq 550(1000)$ GeV **with 10(60)** fb^{-1}

To identify the resonance as a part of a Higgsless model $\longrightarrow$ test the *unitarity sum rules*: measure of the mass and couplings $\longrightarrow$ a task for the ILC

Higgsless Models at ILC

(Birkedal, Matchev, Perelstein)

The first KK excitations of the Higgsless models are expected to be below $1\ \mathrm{TeV}$ and can be produced at ILC via $e^+e^- \longrightarrow V^{1,\pm}e^{\mp}\nu_e$ and $e^+e^- \longrightarrow V^{1,0}e^+e^-$.



Higgsless signal (solid) and SM bkgrd (dotted) for $E_{CM} = 500$ GeV $M_{V^1} = 350, 400$ GeV (red), $E_{CM} = 1000$ GeV $M_{V^1} = 700, 800$ GeV (blue). Hadronic decays of W and Z can be used to reconstruct the invariant V^1 mass.

Conclusions

- Moose models appear deconstructing Higgsless models from five to four dimensions

- Hope: the scale where partial wave unitarity is violated is higher w.r.t. the Higgsless SM due to the exchange of KK excitations in the four gauge boson amplitude scattering

- Problem: compatibility between precision electroweak data and unitarity requirement

- Possible solution with fine tuning: delocalize the fermion interactions

- Signature: new gauge bosons at future colliders (Birkedal, Matchev, Perelstein)

Planar moose

Only possible diagrams are the ones with zero loops.
A moose diagram is like a Feynman diagram with lines corresponding to links and vertices corresponding to gauge groups.

$$\begin{aligned} E &= \text{number of external links,} \\ I &= \text{number of internal links,} \\ V_\ell &= \text{number of gauge groups with } \ell \text{ links,} \\ L &= \text{number of loops,} \\ S &= \text{number of remaining Goldstone multiplets,} \end{aligned}$$

we have

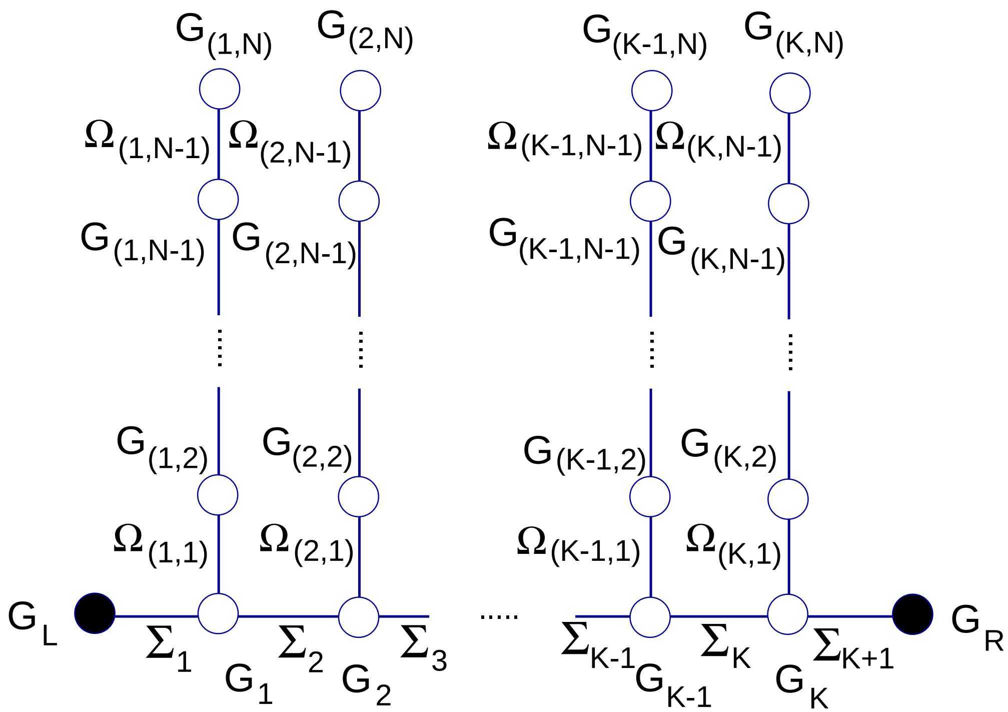
$$L = I - (\sum_\ell V_\ell - 1) \quad S = I + E - \sum_\ell V_\ell$$

implying

$$L = S - (E - 1)$$

We need at least two external links ($E = 2$) in order to get the right weak phenomenology together with the requirement of one scalar multiplet ($S = 1$), $\rightarrow L = 0$.

Planar moose



$$\epsilon_3 = g^2 \sum_{i=1}^{K} \frac{y_i(1-y_i)}{\tilde{g}_i^2}, \qquad \frac{1}{\tilde{g}_i^2} = \sum_{j=1}^{N} \frac{1}{g^2_{(i,j)}}$$