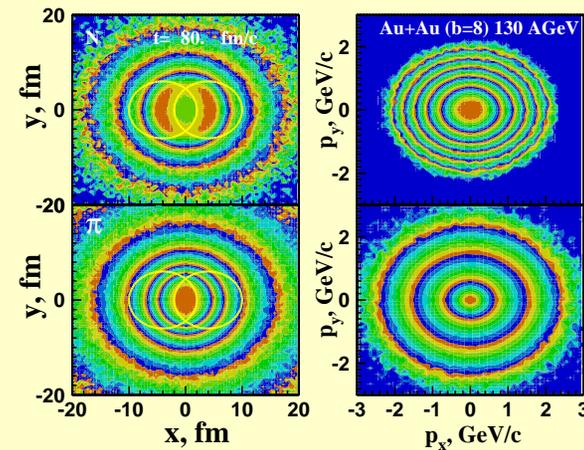


# Collective flow and freeze-out in relativistic HICs

E.Z., L. Bravina, K. Tywoniuk, J. Bleibel, G. Burau, C. Fuchs, A. Faessler

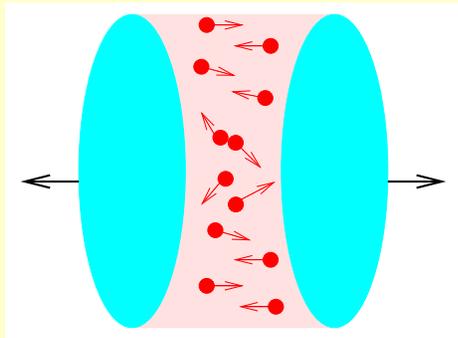
September 30, 2005  
QCD at Cosmic Energies -II

- ❖ Motivation
- ❖ Basic principles of microscopic models
- ❖ Directed flow of hadrons at SPS and RHIC
- ❖ Elliptic flow
- ❖ Influence of particle freeze-out on the development of elliptic flow
- ❖ Conclusions



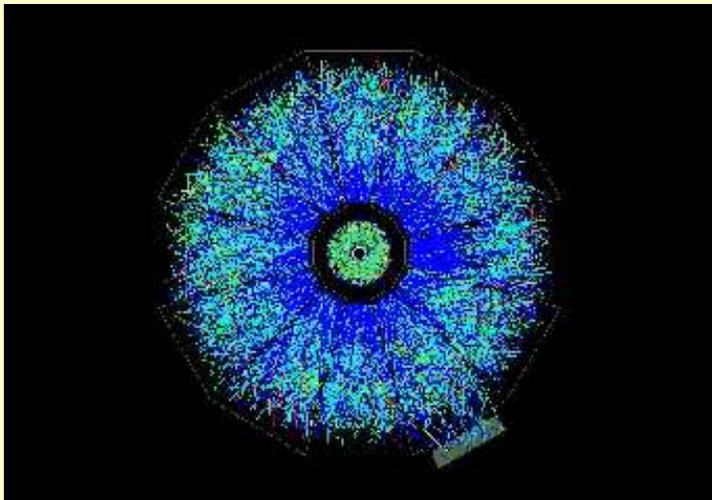
# Introduction

K. Werner, hep-ph/0206111



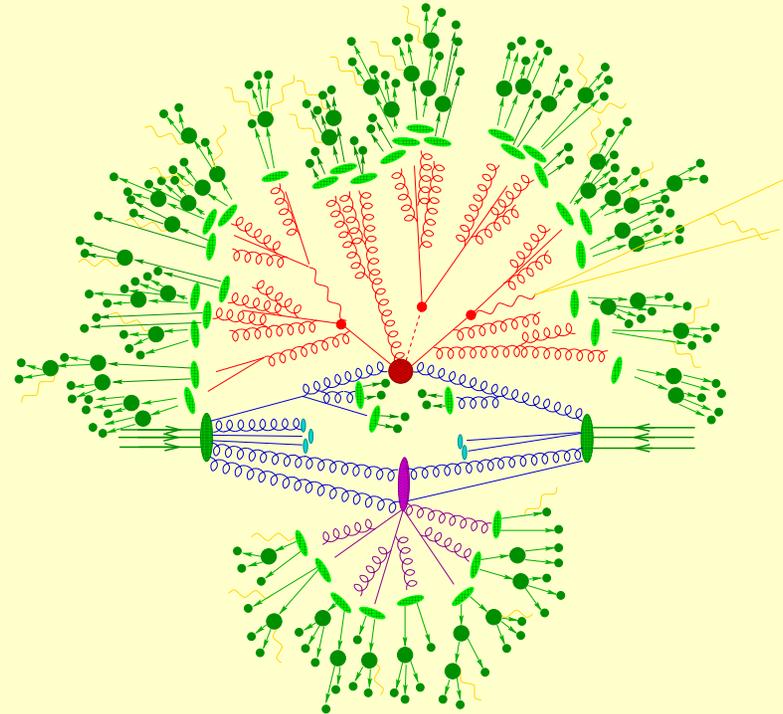
Schematic picture of **A + A** collision

J.S. Lange, hep-ph/0403104



STAR: **Au+Au** collision at  $\sqrt{s}=200$  GeV

A. Schaelicke et al., hep-ph/0311270



Event generator view of **pp** interaction at high energy

## Basic principles of microscopic models

### Transport equation

$$\underbrace{\mathcal{L}_1^{(0)} f_1(\mathbf{x}, \mathbf{p}, t)} = \underbrace{C(\mathbf{x}, \mathbf{p})} + \underbrace{S(\mathbf{x}, \mathbf{p})}$$

streaming term

collision term

source term

### Model:

### Input:

❖ Initialization of projectile and target

❖ Propagation (with given  $H$ )

$$\dot{\mathbf{p}}_i = -\frac{\partial H}{\partial \mathbf{r}_i}$$

$$\dot{\mathbf{r}}_i = \frac{\partial H}{\partial \mathbf{p}_i}$$

❖ Scattering and particle production

❖ ( next time-step)

❖ Fermi-gas model; liquid drop model

❖ Potentials: Coulomb, Yukawa, Skyrme, etc.

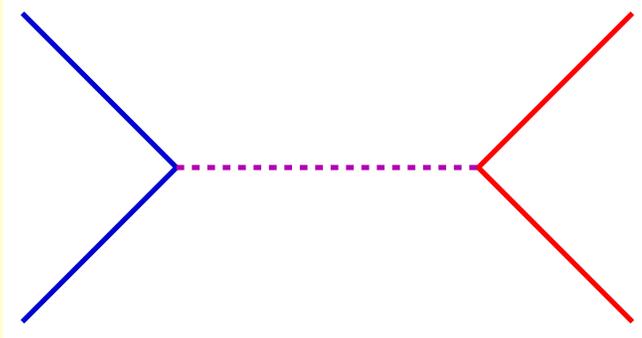
❖ Cross sections (tabulated); resonances (widths and branching ratios);

❖ Partons (distribution and clusterization) or **Strings**

# The string picture

K. Werner, hep-ph/0206111

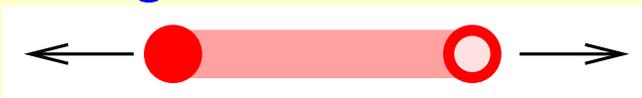
Let us consider  $e^+e^-$  annihilation. Electron and positron annihilate and form a virtual photon, then the virtual photon decays into a quark-antiquark pair.



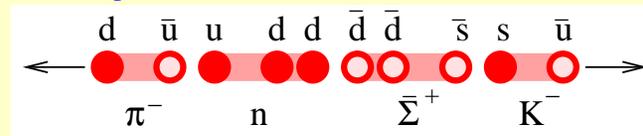
The quark and antiquark move apart from each other



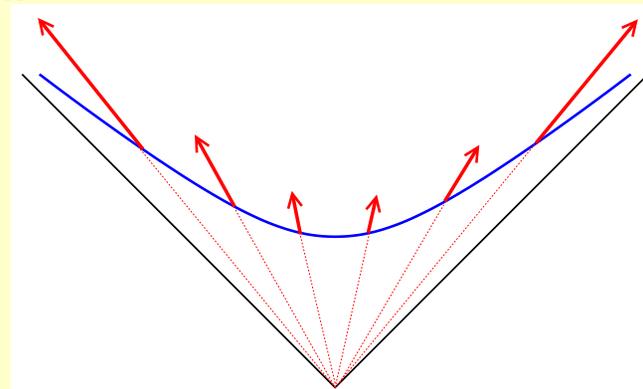
But there is a gluon field acting between the two, whose energy is proportional to the separation distance. This object is called string.



The string breaks via quark-antiquark production, and these new string pieces are finally hadrons or resonances.

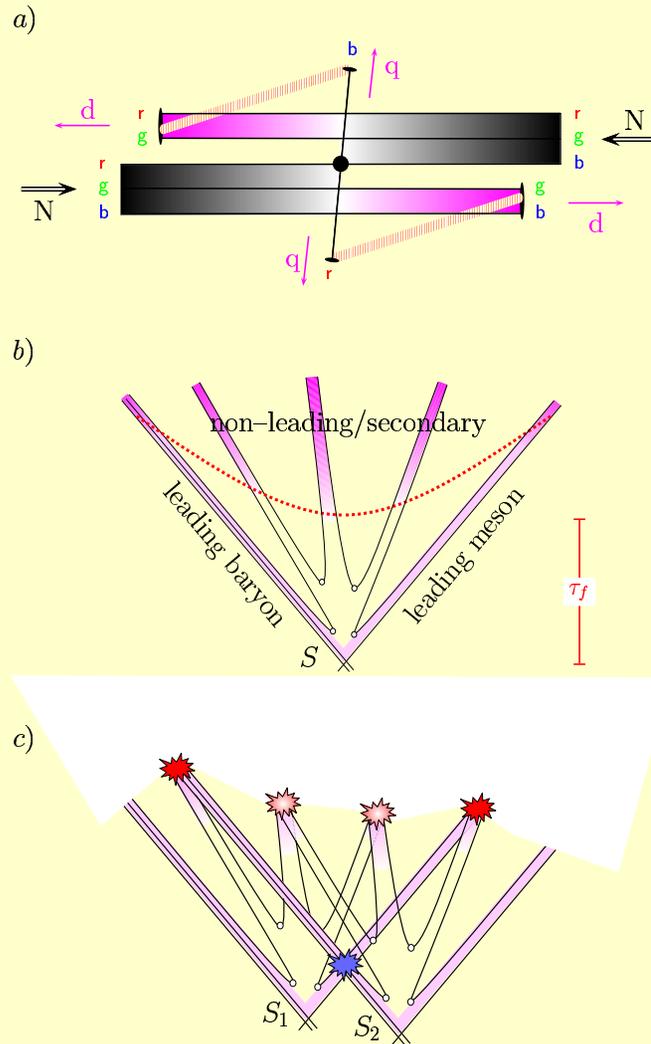


The space-time picture of the string dynamics: at given  $\tau$  (hyperbola) the velocities of the string pieces (arrows) point all back to the origin and are larger towards the edges. This string decay provides a flat rapidity distribution.



# Hadron-String-Dynamics (HSD) approach

W. Cassing, E. Bratkovskaya, Phys. Rep. 308 (1999) 65  
W. Cassing et al., hep-ph/0311358



## Model features

- ❖ **baryons:**  
N,  $\Delta$ ,  $N^*(1440)$ ,  $N^*(1535)$ ,  $\Lambda$ ,  $\Sigma$ ,  $\Sigma^*$ ,  
 $\Xi$ ,  $\Xi^*$ ,  $\Omega$  + anti-states;
- ❖ **mesons:**  $0^-$ ,  $1^-$  octet states;
- ❖  **$hh$  @  $\sqrt{s} \geq 2.6 \text{ GeV}$ :** FRITIOF +  
PYTHIA(5.5) + JETSET(7.3);
- ❖ **lower energies :** in line with exper-  
imental cross sections;
- ❖ **very high momentum tail :**  
PYTHIA(6.2);
- ❖ **formation time  $\tau_f \approx 0.8 \text{ fm}$ ;**
- ❖ **o explicit parton cascading**

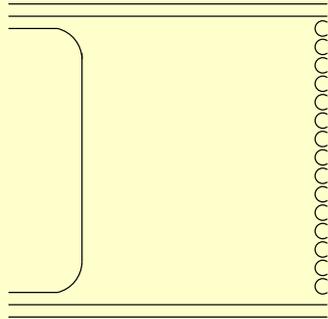
Schematic picture of **N + N** collision

# Dual Topological Unitarization Models

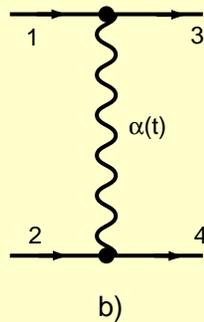
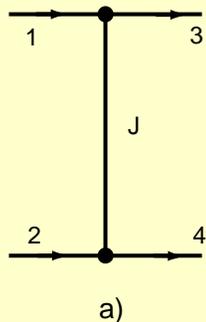
These models are based on Gribov-Regge field theory (color exchange)

## Planar digram

In  $hh$  interactions a one-string mechanism (with quark-antiquark annihilation) is possible in  $p\bar{p}$  collisions but not in  $pp$

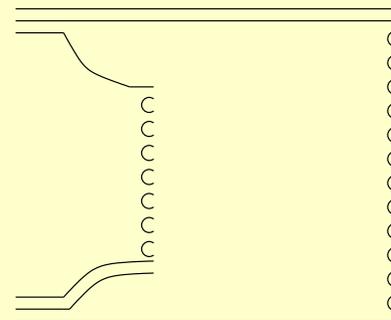


Such a diagram corresponds to the **Reggeon exchange** in the GRT (weight  $\propto 1/N$ ; contribution  $\propto s^{-1/2}$ )

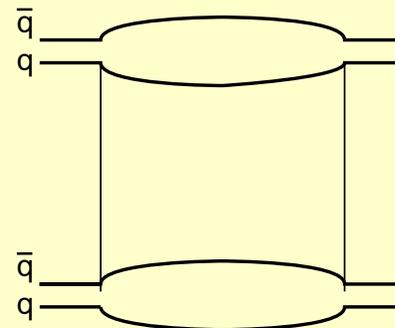


## Cylinder digram

The simplest topology which contribution does not vanish at  $s \rightarrow \infty$  is a two-string diagram



Such a diagram corresponds to the **Pomeron exchange** in the GRT (weight  $\propto 1/N^2$ ). Its square has the topology of a cylinder



## Hard processes and multipomeron exchanges

The inelastic  $hh$  cross section  $\sigma_{\text{in}}(s)$  can be calculated via the real part of the eikonal  $u(s, b)$

$$\sigma_{\text{in}}(s) = 2\pi \int_0^{\infty} \{1 - \exp[-2u^{\text{R}}(s, b)]\} b db$$

The eikonal can be presented as a sum of three terms corresponding to soft and hard Pomeron exchange, and triple Pomeron exchange, which is responsible for the single diffraction process,

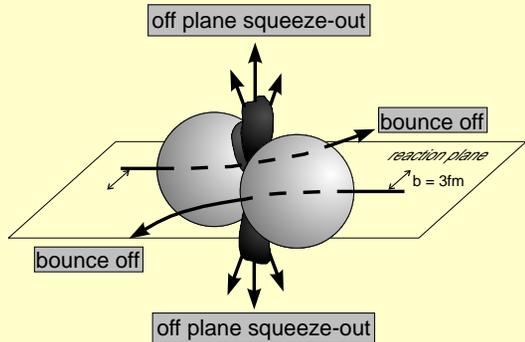
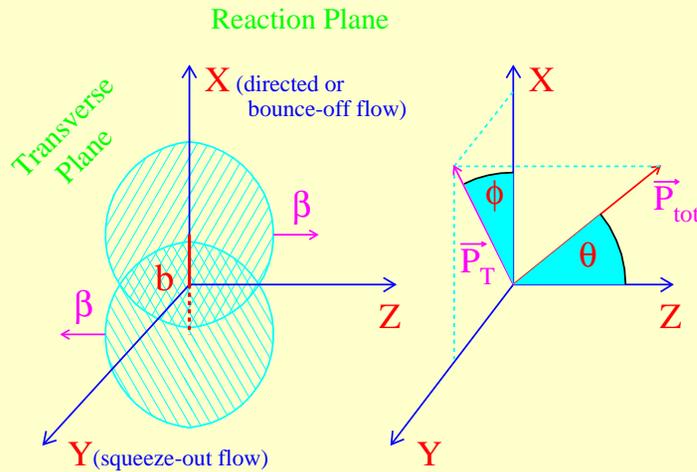
$$u^{\text{R}}(s, b) = u_{\text{soft}}^{\text{R}}(s, b) + u_{\text{hard}}^{\text{R}}(s, b) + u_{\text{triple}}^{\text{R}}(s, b)$$

Using the Abramovskii-Gribov-Kancheli (AGK) cutting rules we get

$$\begin{aligned} \sigma_{\text{in}}(s) &= \sum_{i,j,k=0; i+j+k \geq 1} \sigma_{ijk}(s) , \\ \sigma_{ijk}(s) &= 2\pi \int_0^{\infty} b db \exp[-2u^{\text{R}}(s, b)] \\ &\times \frac{[2u_{\text{soft}}^{\text{R}}(s, b)]^i}{i!} \frac{[2u_{\text{hard}}^{\text{R}}(s, b)]^j}{j!} \frac{[2u_{\text{triple}}^{\text{R}}(s, b)]^k}{k!} . \end{aligned}$$

The last equation enables one to determine the number of strings and hard jets.

# Transverse Collective Flow of Particles



**Directed flow:**

$$\mathbf{v}_1 = \left\langle \frac{\mathbf{p}_x}{p_T} \right\rangle \equiv \langle \cos(\phi') \rangle$$

**Flow Decomposition:**

**Transverse flow = Radial**  
**+ Bounce-off + Squeeze-out**

S. Voloshin and Y. Zhang, ZPC 70 (1996) 665

**Modern analysis:**

**Transverse flow =**  
**Radial + Directed + Elliptic + ...**  
**{isotropic} {anisotropic}**

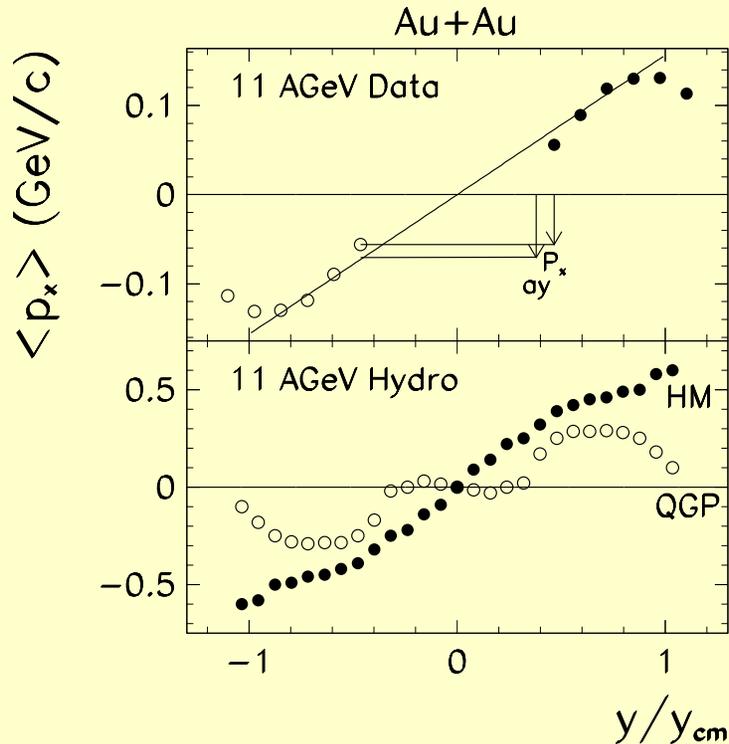
$$E \frac{d^3N}{d^3p} = \frac{1}{2\pi} \frac{d^2N}{p_T dp_T dy} \left( 1 + \sum_{n=1}^{\infty} 2v_n \cos(n\phi') \right)$$

**Elliptic flow:**

$$v_2 = \left\langle \left( \frac{p_x}{p_T} \right)^2 - \left( \frac{p_y}{p_T} \right)^2 \right\rangle \equiv \langle \cos(2\phi') \rangle$$

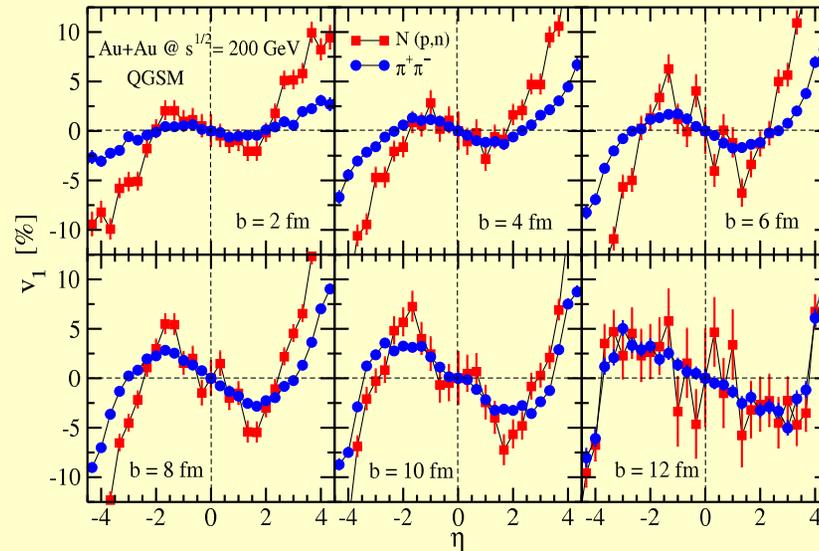
# Softening of Directed Flow

L.P. Csernai, D. Röhrich, PLB 458 (1999) 454



Transition to the **Quark-Gluon Plasma**  
 → decrease in pressure → softening  
 of the directed flow

L. Bravina, PLB 334 (1995) 49  
 H. Liu, S. Panitkin, N. Xu, PRC 59 (1999) 348  
 R.J.M. Snellings *et al.*, PRL 84 (2000) 2803  
 G. Burau *et al.*, PRC 71 (2005) 054905



**Wiggle structure:** The effect is more pronounced in peripheral and light-ion collisions, therefore, it cannot be explained by the softening of the **EOS** because of the formation of strings  
**QGS**M calculations for **Au+Au** at  $\sqrt{s} = 200$  **AGeV** )

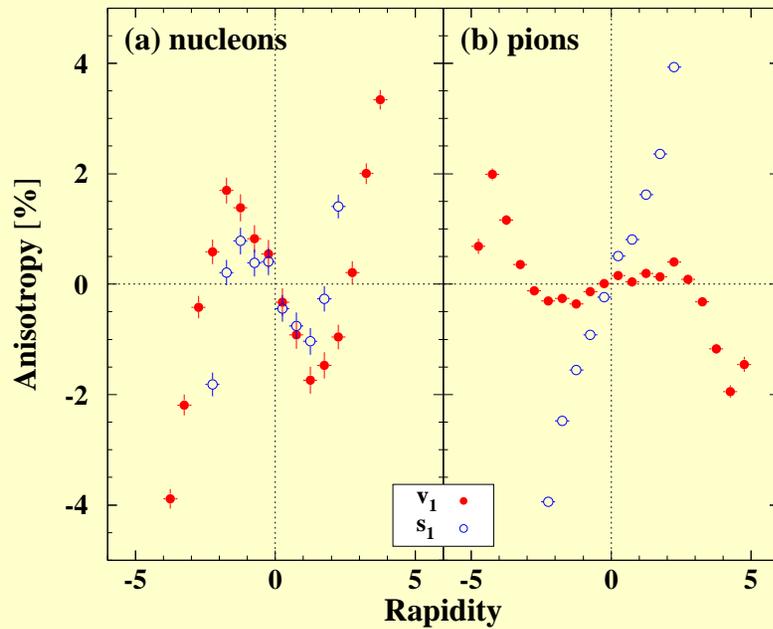
# Directed Flow at RHIC

R.J.M. Snellings *et al.*, PRL 84 (2000) 2803

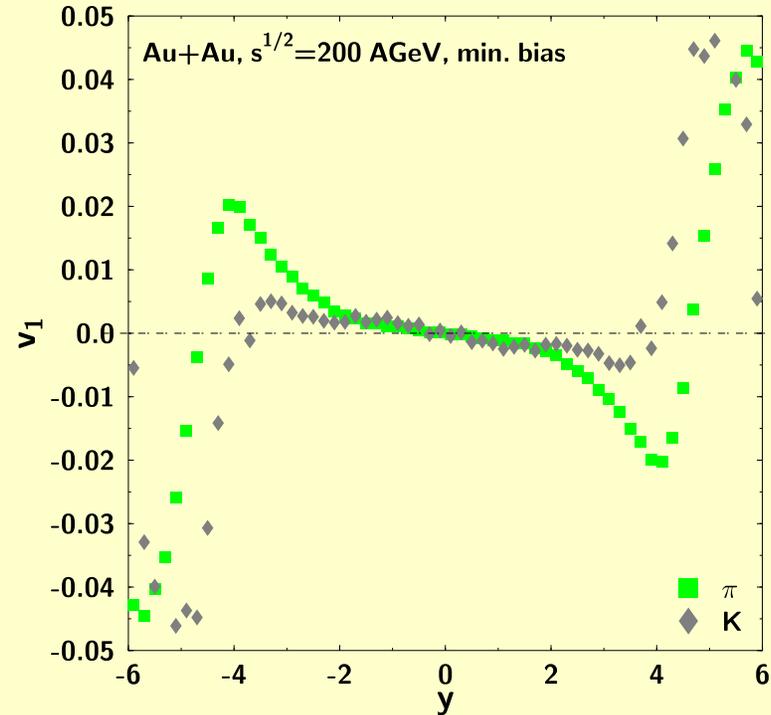
M. Bleicher and H. Stöcker, PLB 526 (2002) 309

Rapidity dependence of the  $v_1$  of  $\pi, N$

Rapidity dependence of the  $v_1$  of  $\pi, K$



RQMD calculations for Au+Au at  $\sqrt{s} = 200$  AGeV ( $5 fm \leq b \leq 10 fm$ )



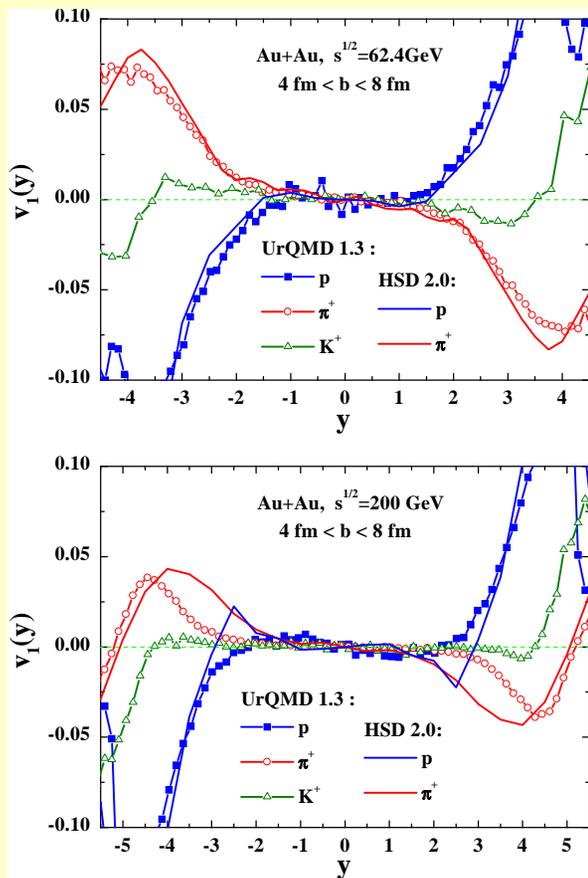
UrQMD calculations for Au+Au at  $\sqrt{s} = 200$  AGeV (minimum bias events)

**Antiflow / vanishing of flow at midrapidity**

# Directed Flow at RHIC

H. Stöcker *et al.*, nucl-th/0412022

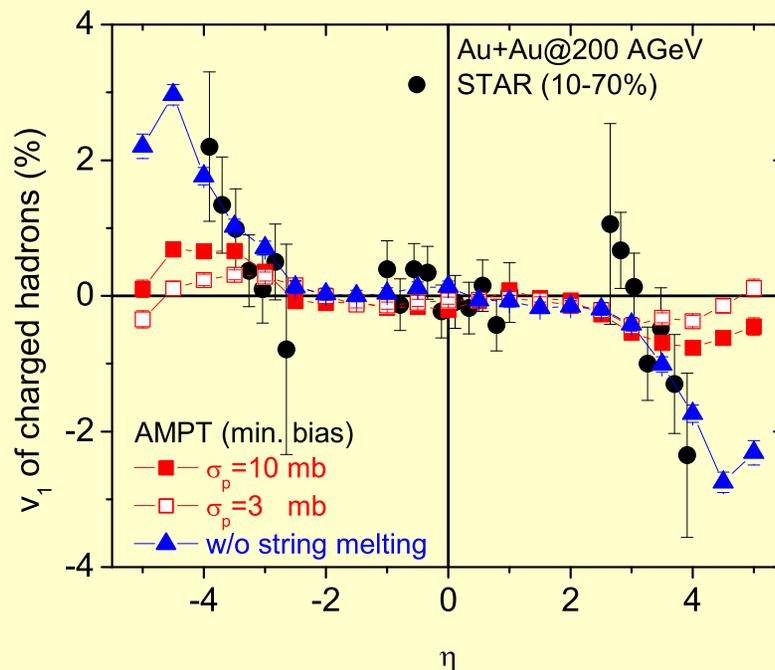
## Rapidity dependence of the $v_1$



UrQMD and HSD calculations for Au+Au  
 at  $\sqrt{s} = 200\text{ AGeV}$  ( $4\text{ fm} \leq b \leq 8\text{ fm}$ )

L.-W. Chen, V. Greco, C.M. Ko, P.F. Kolb, PLB 605,  
 95 (2005)

## Rapidity dependence of the $v_1$ of charged hadrons

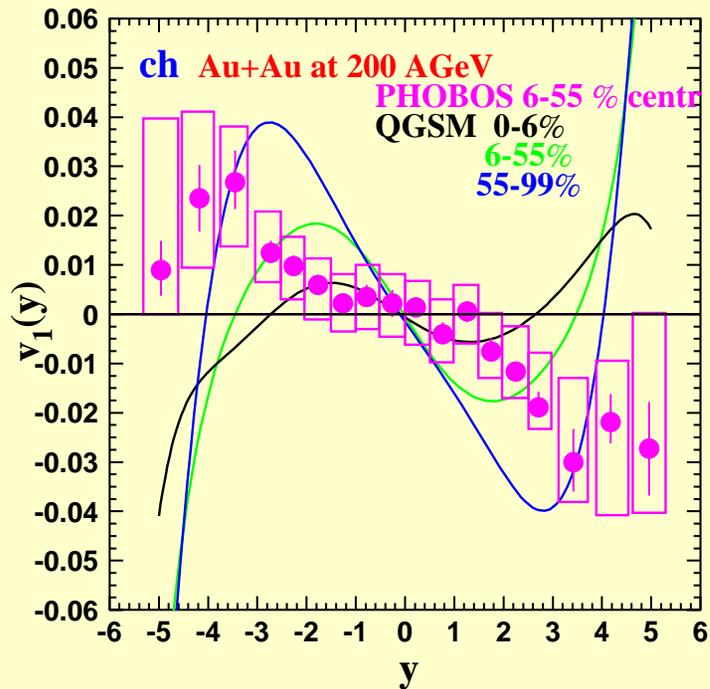


AMPT calculations for Au+Au at  
 $\sqrt{s} = 200\text{ AGeV}$  (minimum bias events)

# Directed Flow at RHIC

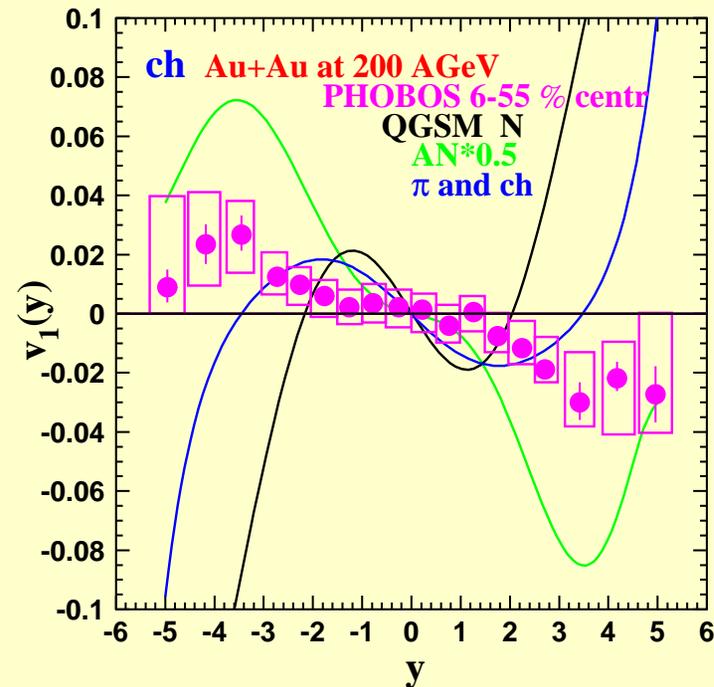
M.B. Tonjes *et al.* (PHOBOS Collab.), JPG 30 (2004) 1243

## Centrality dependence of the DF of charged particles in QGSM



- (1) DF increases with rising  $b$
- (2) Exp.: antiflow increases up to fragmentation regions

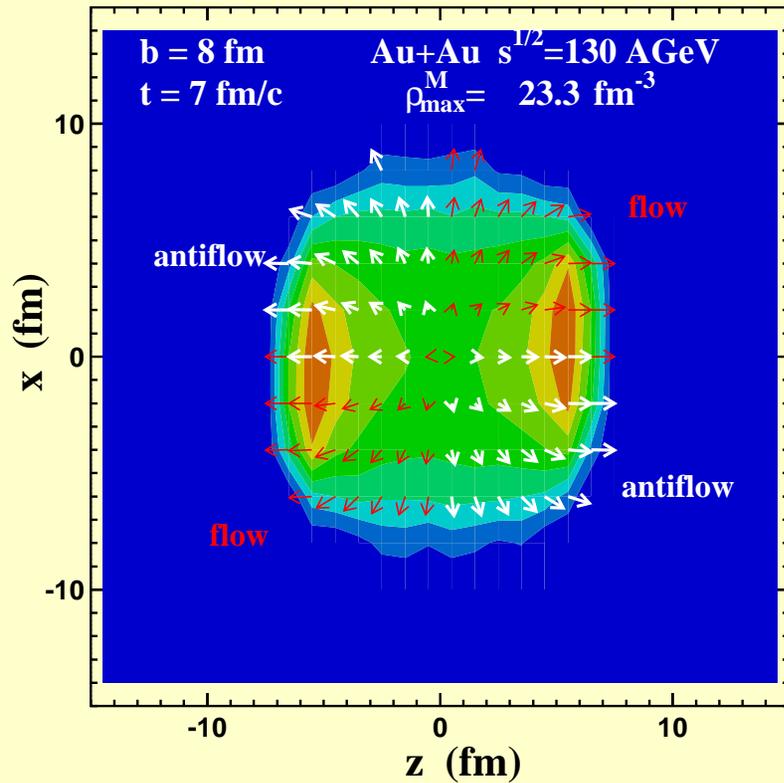
## Directed flow of different species



Antibaryons have strongest antiflow because of annihilation

## Development of Directed Flow

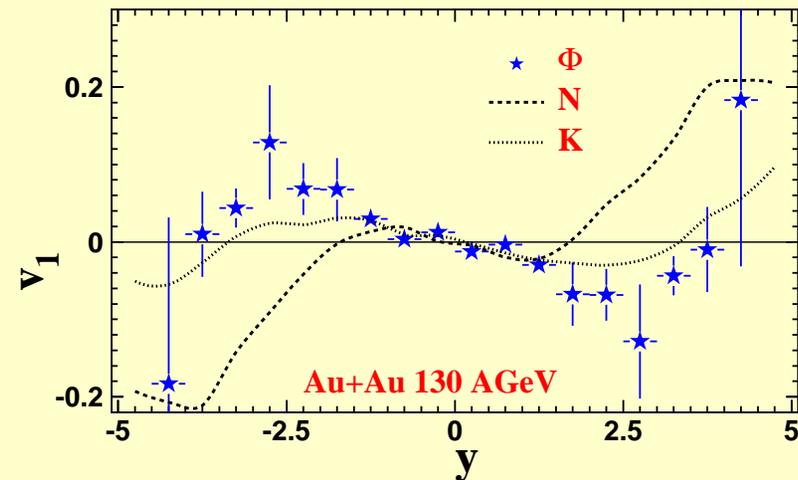
**Resulting flow** = normal flow - antiflow



Although the normal flow component is always slightly larger than the antiflow one, in central rapidity window the anti-flow can overshadow its normal counterpart

L. Bravina *et al.*, NPA 715 (2003) 665c

**Directed flow  $v_1(y, all p_t)$  of  $\phi, N, K$  in minimum bias Au+Au events at  $\sqrt{s} = 130$  AGeV**



Directed flow of  $\phi$  mesons  $v_1(y)$  has negative slope (antiflow) at  $|y| \leq 2$ . This distribution is similar to those of other hadrons at  $|y| \leq 2$  in *Au + Au* at  $\sqrt{s} = 130$  AGeV because of similarities of their production and dynamics

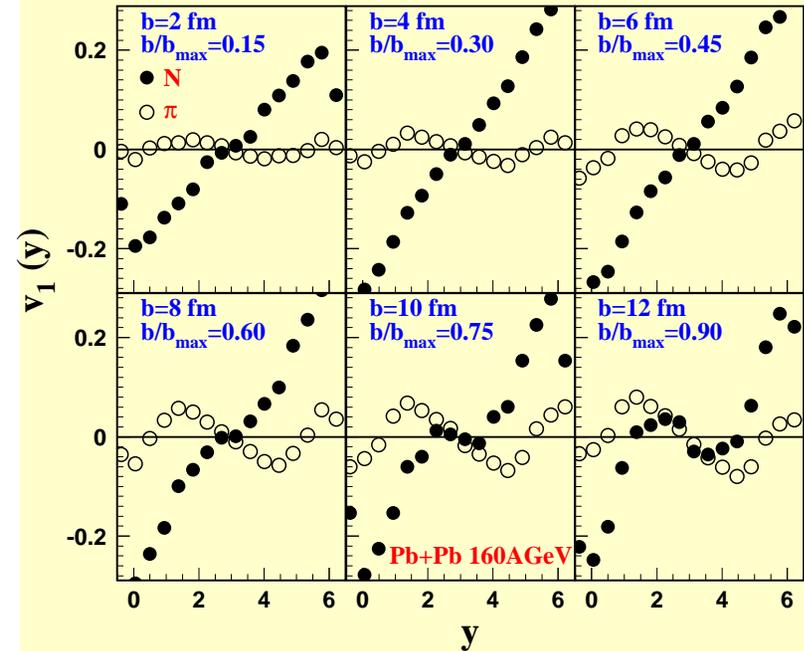
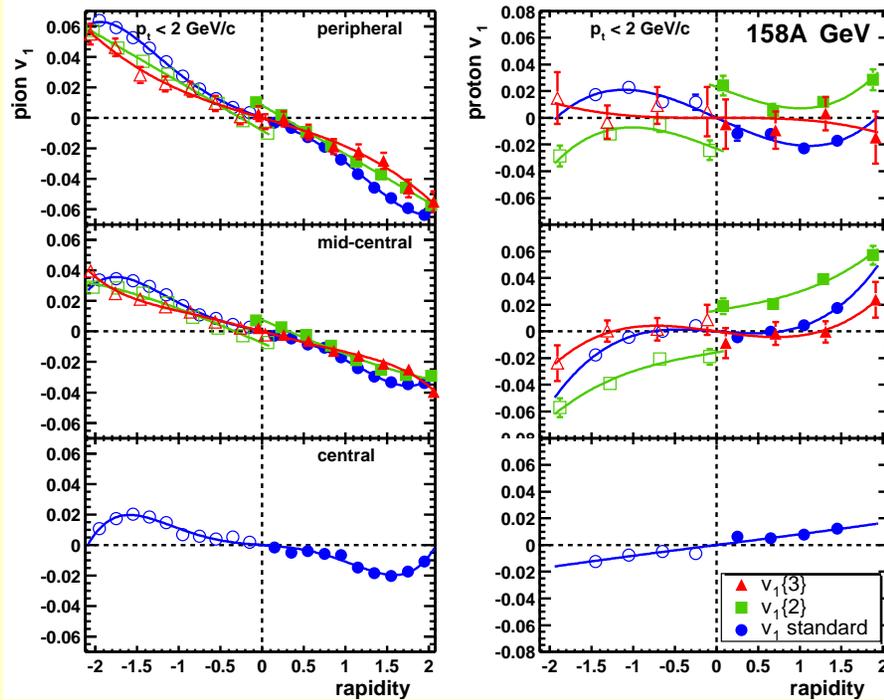
# Directed Flow at SPS

C. Alt *et al.* (NA49 Collab.), PRC 68 (2003) 034903

Rapidity dependence of the  $v_1$  of  $\pi, p$

L. Bravina *et al.*, PRC 61 (2000) 064902

Rapidity dependence of the  $v_1$  of  $\pi, N$



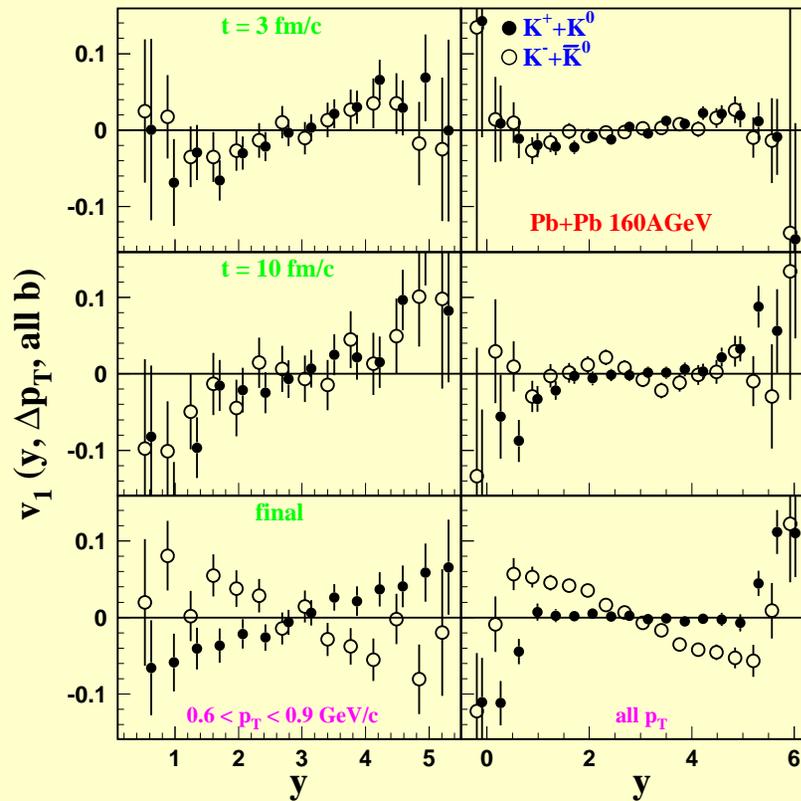
QGSM calculations for Pb+Pb at 158 AGeV

NA49 Collaboration results for Pb+Pb at 158 AGeV ( $p_t \leq 2$  GeV)

# $P_T$ Dependence of Directed Flow

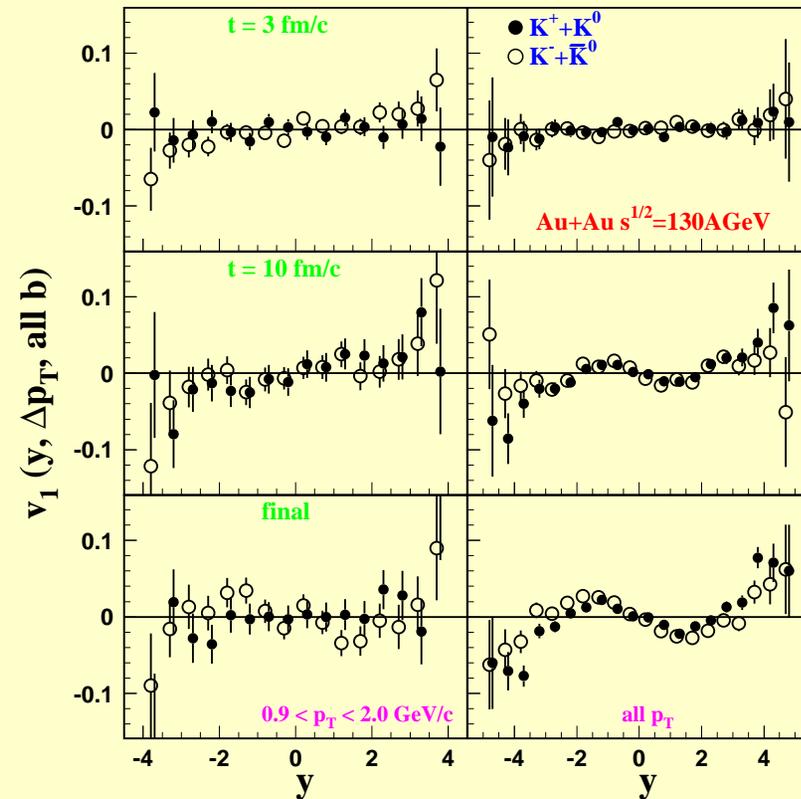
L. Bravina, L. Csernai, A. Faessler, C. Fuchs, E. Z., PLB 543 (2002) 217

## Kaon flow at SPS



**Strong difference between kaons and antikaons**

## Kaon flow at RHIC



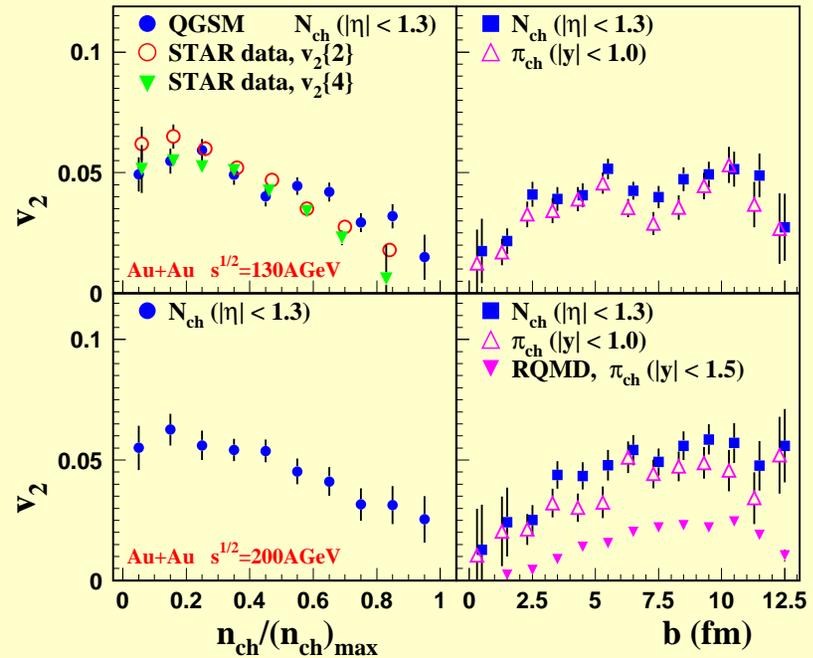
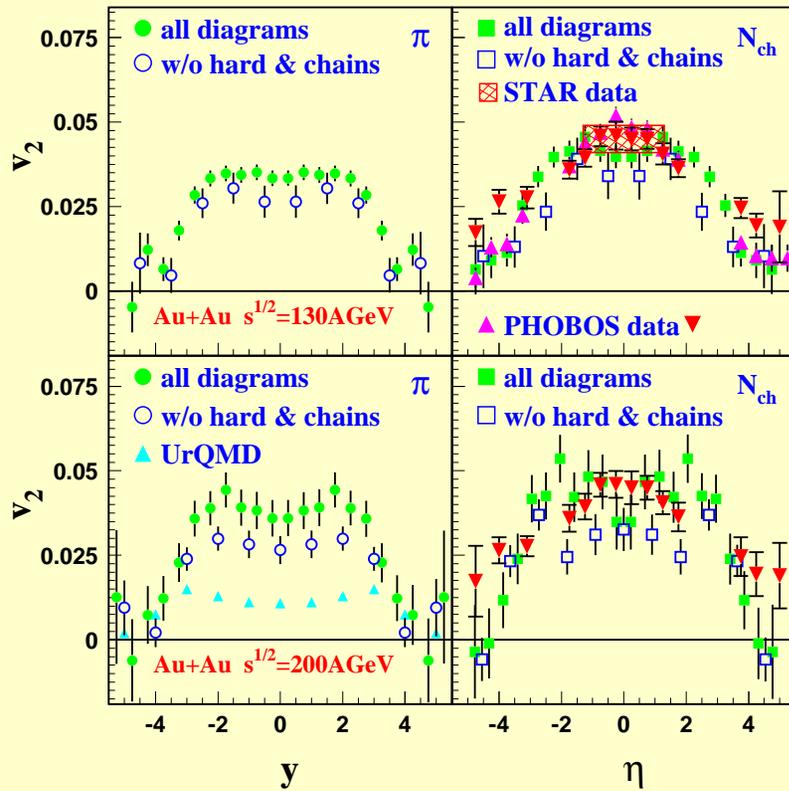
**No difference between kaons and antikaons**

## Directed Flow (Conclusions):

- ❖ Microscopic models reproduce, at least qualitatively, basic features of the  $v_1$  development
- ❖ **Directed Flow = Normal Flow – Antiflow**  
**Normal Flow  $\geq$  Antiflow** (except of the midrapidity range)
- ❖ The softening of the flow may be misinterpreted as the softening of EOS due to formation of the **QGP**, but:  
**QGP**  $\rightarrow$  the effect is stronger for semi-central collisions  
**Cascade**  $\rightarrow$  the effect is stronger for semi-peripheral and peripheral ones
- ❖ **At RHIC:** The directed flow of both mesons and baryons is zero or antiflow-oriented at  $|y| \leq 2$
- ❖ The directed flow of high- $p_t$  hadrons is elongated in normal direction

# Elliptic Flow at RHIC

E. Z., L. Bravina, A. Faessler, C. Fuchs, PLB 508 (2001) 184  
 PPNP 53 (2004) 183  
 M. Bleicher and H. Stöcker, PLB 526 (2002) 309  
 S. Manly *et al.* (PHOBOS Collab.), NPA 715 (2003) 614c



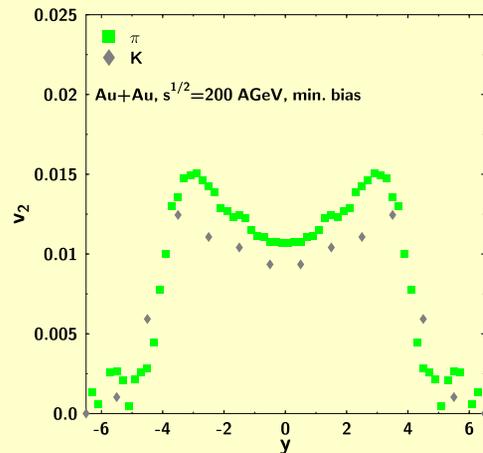
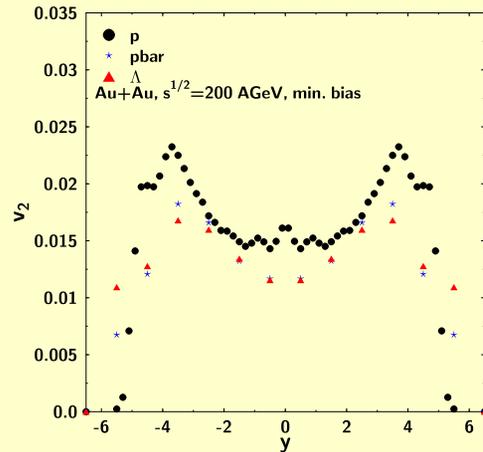
(Pseudo)rapidity dependencies of the elliptic flow of charged particles in the whole  $\eta$  range at both energies were obtained *before* the experimental data became available

# Elliptic Flow at RHIC

M. Bleicher and H. Stöcker, PLB 526 (2002) 309

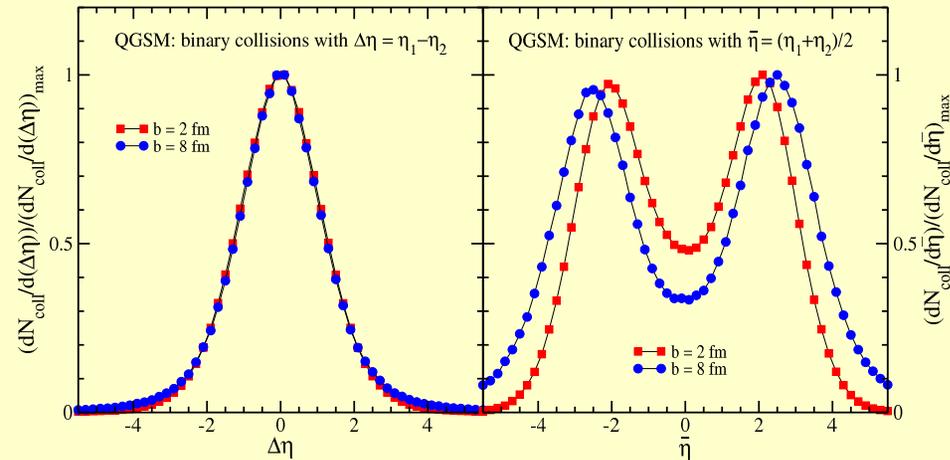
## Origin of the two-hump structure

G. Burau *et al.*, PRC 71 (2005) 054905



UrQMD calculations for Au+Au at  $\sqrt{s} = 200$  AGeV (minimum bias events)

The number of binary particle collisions per pseudorapidity interval normalized to its maximum as function of the pseudorapidity difference  $\Delta\eta = \eta_1 - \eta_2$  (left) and the mean pseudorapidity  $\bar{\eta} = (\eta_1 + \eta_2)/2$  (right) of the two colliding particles



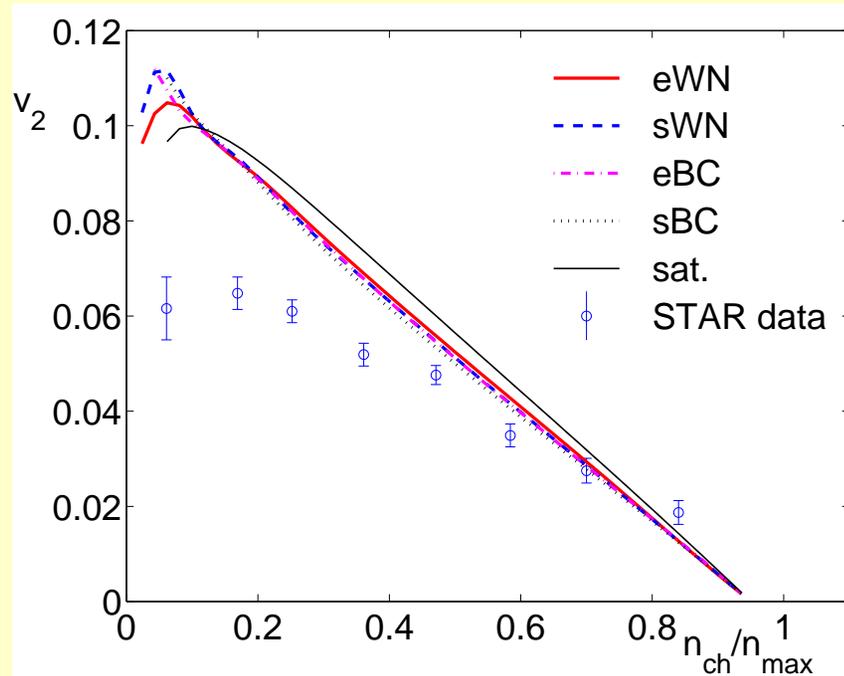
**Left:** particles with similar rapidities interact most likely

**Right:** a double peak structure appears in the same rapidity region as seen for the  $v_2(\eta)$

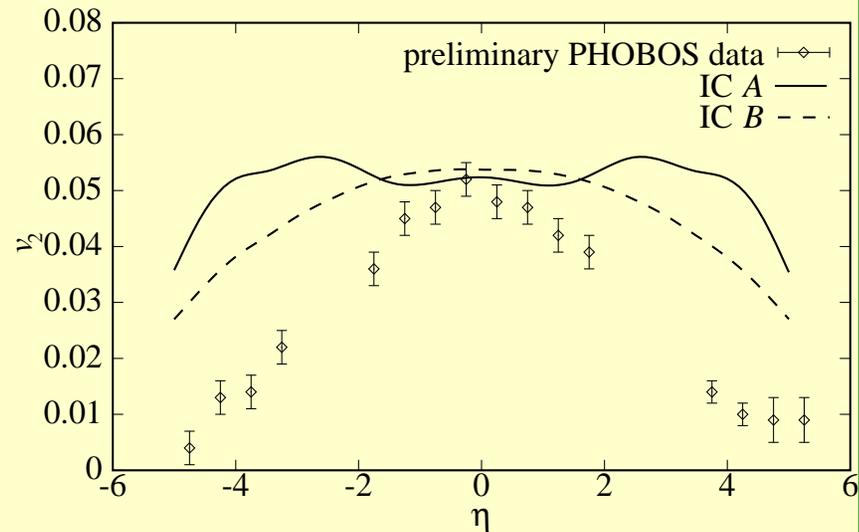
# Elliptic Flow at RHIC (Hydro)

P. Huovinen *et al.*, PLB 503 (2001) 58  
T. Hirano and K. Tsuda, NPA 715 (2003) 821c

## Centrality Dependence



## Pseudorapidity Distribution

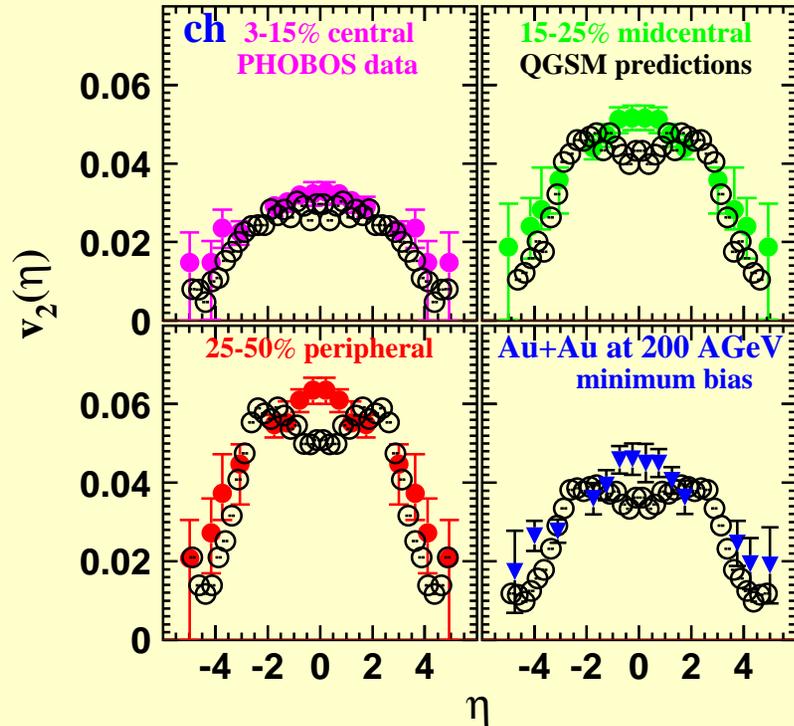


**The agreement between the experimental distributions and model simulations is not good.**

# Elliptic Flow at RHIC

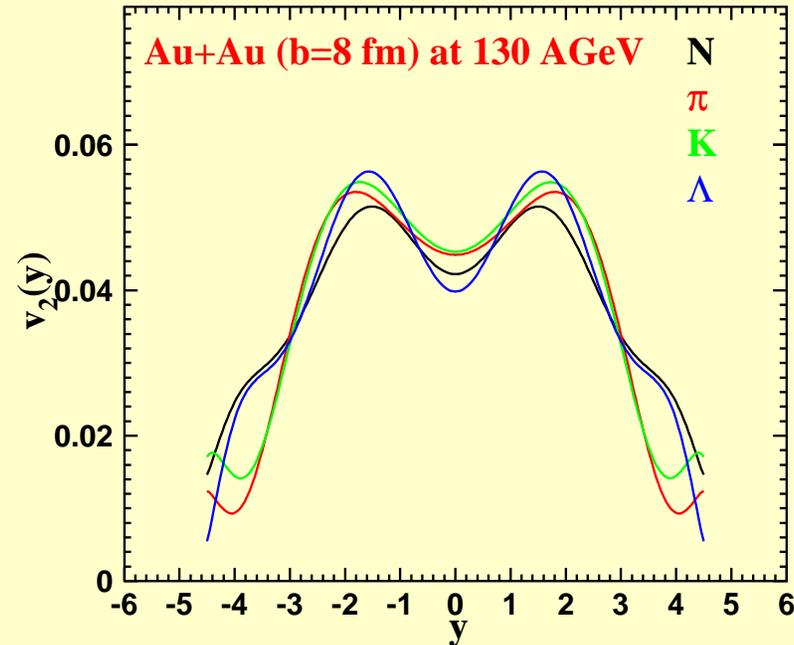
B. Back *et al.* (PHOBOS Collab.), nucl-ex/0407012

L. Bravina *et al.*, hep-ph/0412343



$v_2(\eta)$  of charged hadrons in **Au+Au** at  $\sqrt{s} = 200$  **AGeV** for (a)  $\sigma/\sigma_{\text{geo}} = 0 - 15\%$ , (b)  $\sigma/\sigma_{\text{geo}} = 15 - 25\%$ , (c)  $\sigma/\sigma_{\text{geo}} = 25 - 50\%$ , and (d) minimum bias events.

## Rapidity dependence of the $v_2$

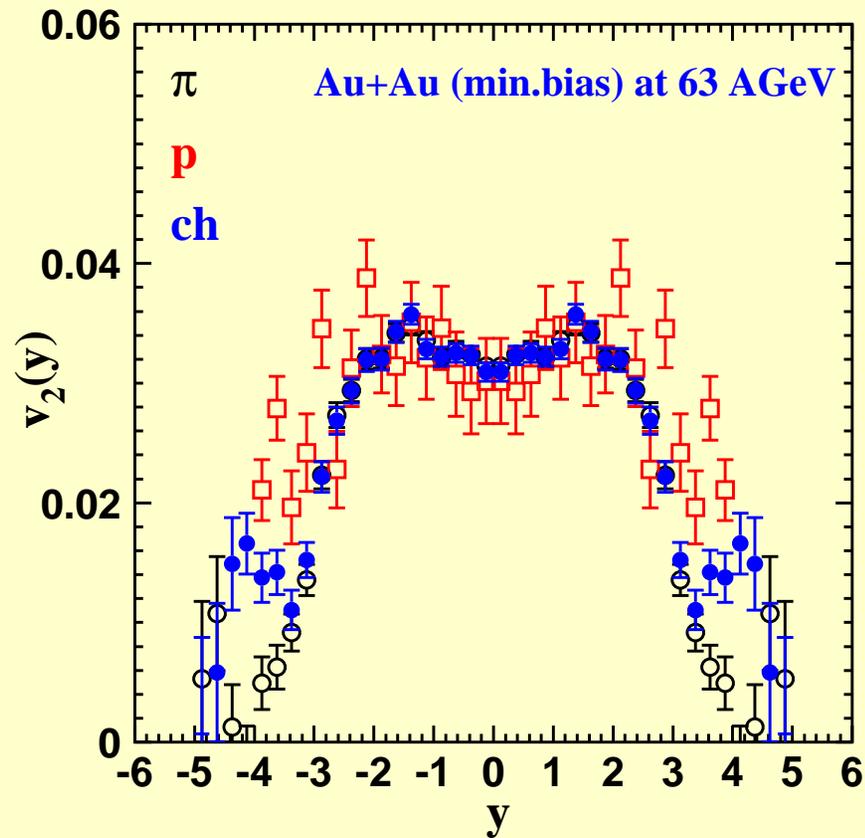


$v_2(\eta)$  distribution of  $\pi, N, K, \Lambda$  in **Au+Au** at  $\sqrt{s} = 200$  **AGeV**

No difference for different species

## Elliptic Flow at RHIC

Rapidity dependence of  $v_2$  in Au+Au at  $\sqrt{s} = 63$  AGeV

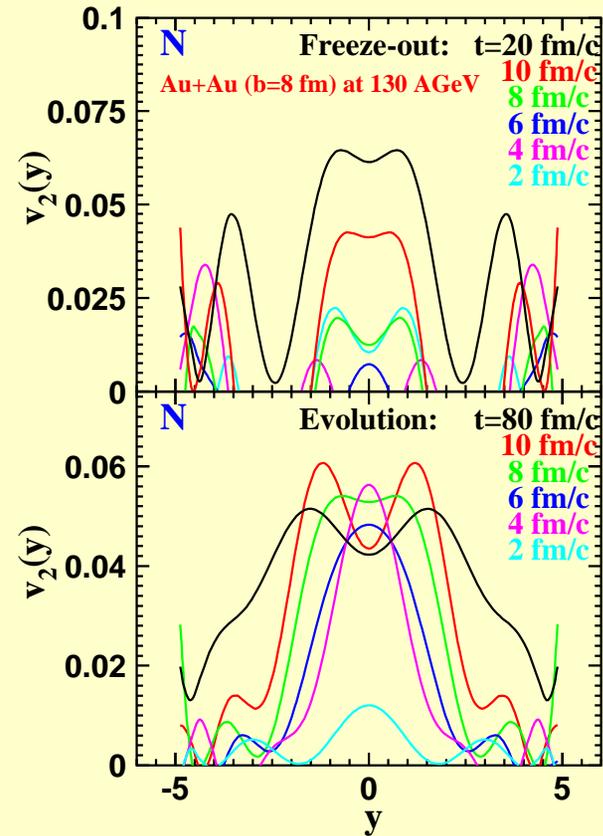
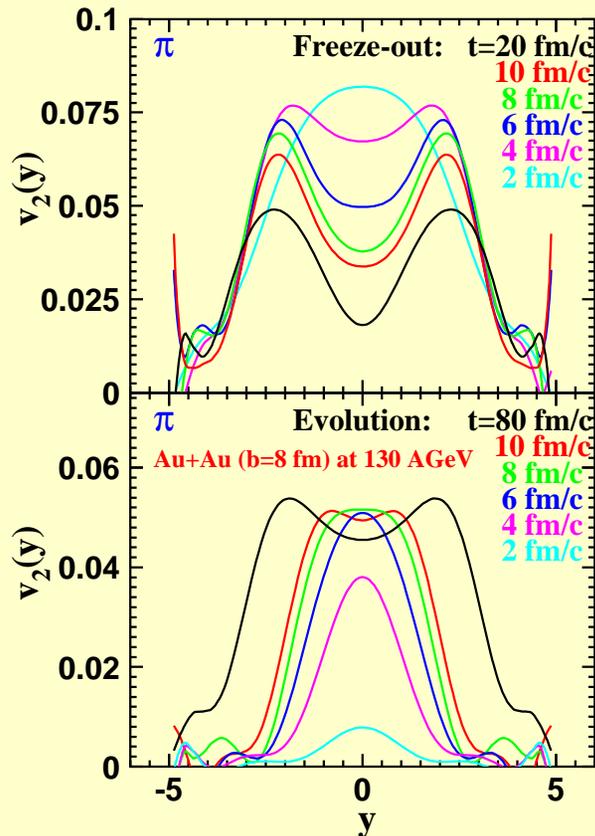


- (1) No difference for different species
- (2) The flow is only slightly weaker than that at 200 AGeV

# Time Evolution of the Elliptic Flow

**PIONS (Au+Au, 130 AGeV, b=8 fm)**

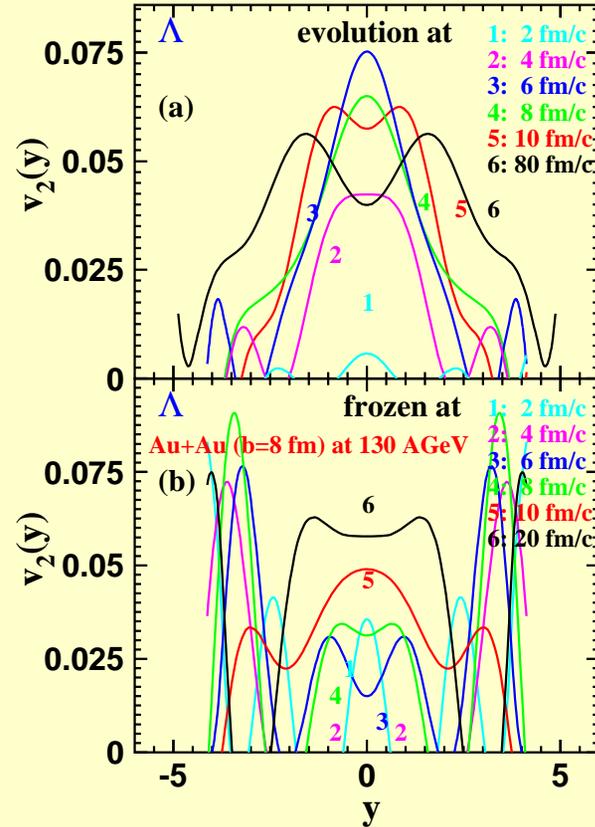
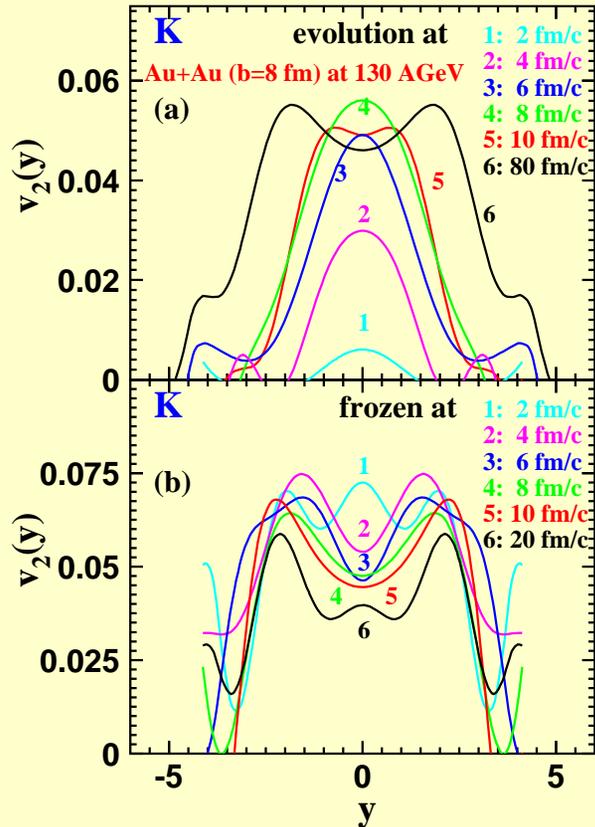
**NUCLEONS**



- (1) The **earlier** the freeze-out of **pions**, the stronger their elliptic flow
- (2) The **later** the freeze-out of **nucleons**, the stronger their elliptic flow
- (3) The flow formation is not over e.g. at  $t = 6 \text{ fm}/c$  due to continuous freeze-out of particles

# Time Evolution of the Elliptic Flow

**KAONS (Au+Au, 130 AGeV, b=8 fm)      LAMBDA S**



- (1) The **earlier** the freeze-out of **kaons**, the stronger their elliptic flow
- (2) The **later** the freeze-out of **lambdas**, the stronger their elliptic flow

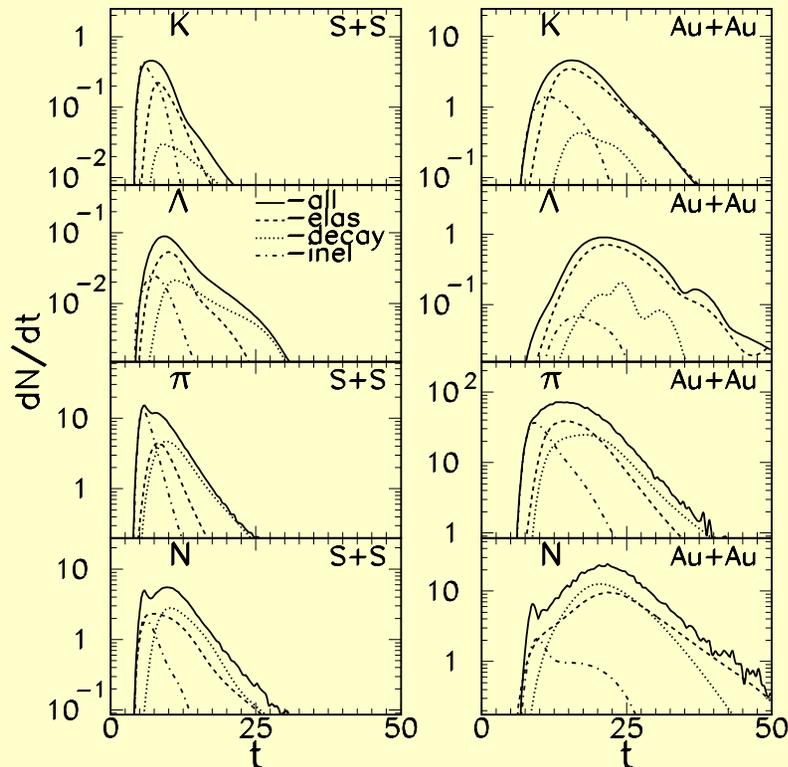
This is the main difference in the formation of the elliptic flow of mesons and baryons

# Sequential Freeze-Out

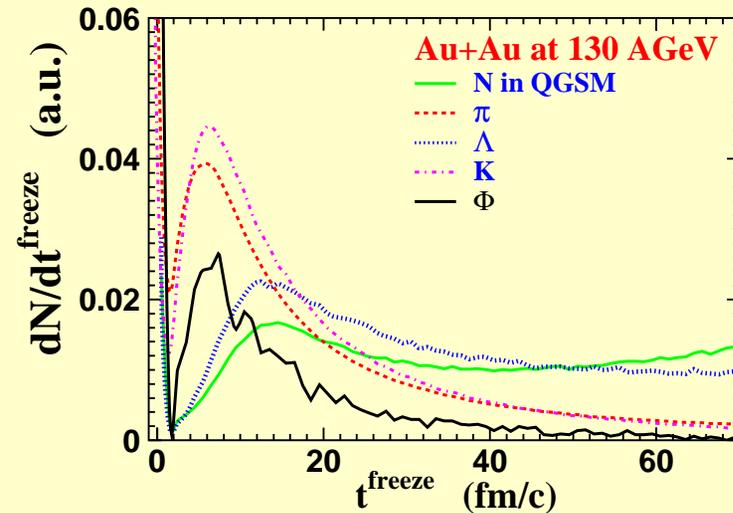
L. Bravina *et al.*, NPA 715 (2003) 665

L. Bravina, L. Csernai, *et al.*, PLB 354 (1995) 196

AGS



RHIC

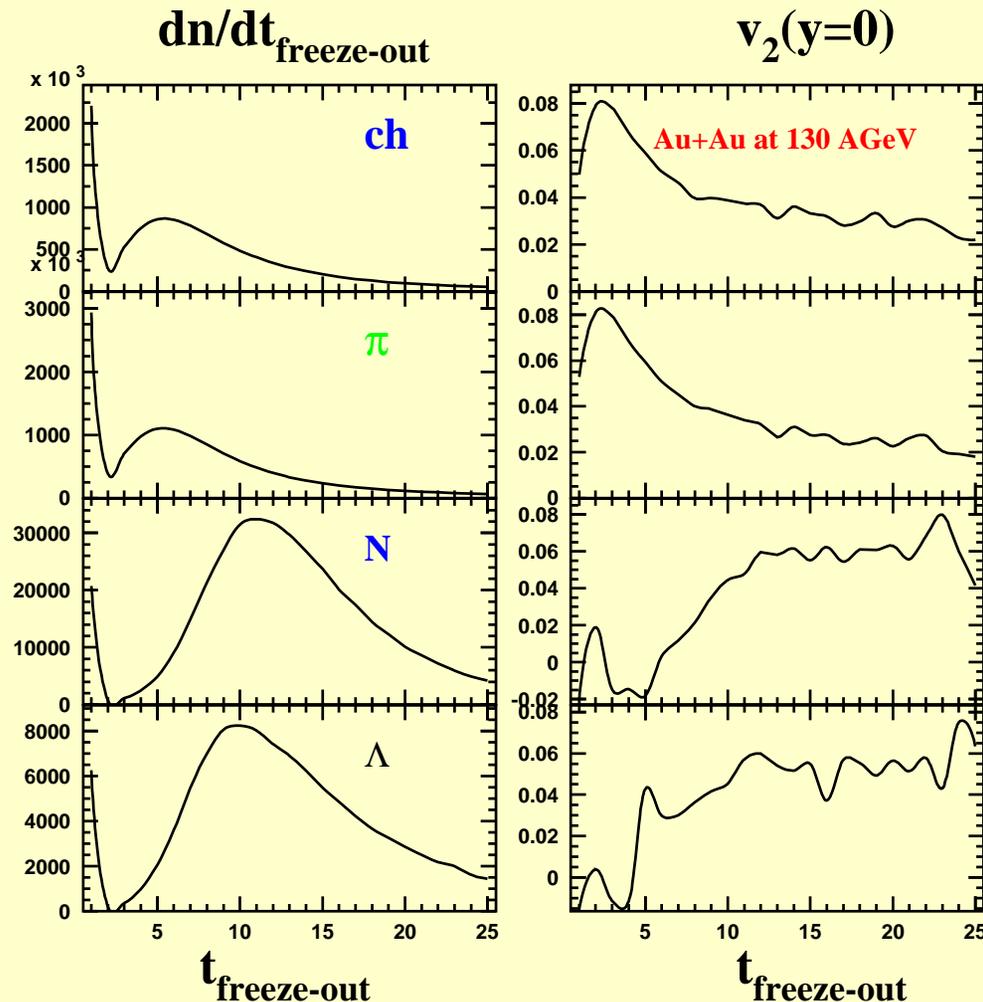


Freeze-out order: **1 - pions**; **2 - kaons**;  
**3 - lambdas**; **4 - nucleons**

- (1) compared to lower energies, many hadrons leave the system immediately after their production
- (2) mesonic distributions are peaked at  $t = 8 - 10 \text{ fm}/c$
- (3) distributions of baryons are wider due to the large number of rescatterings

## Freeze-Out and Elliptic Flow

Au+Au (b=8 fm) at  $\sqrt{s} = 130$  AGeV



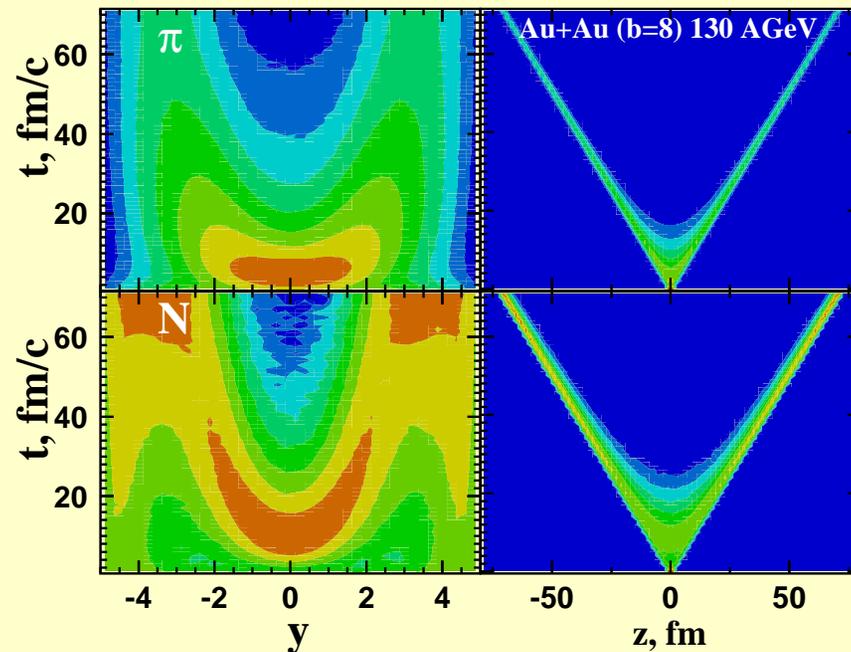
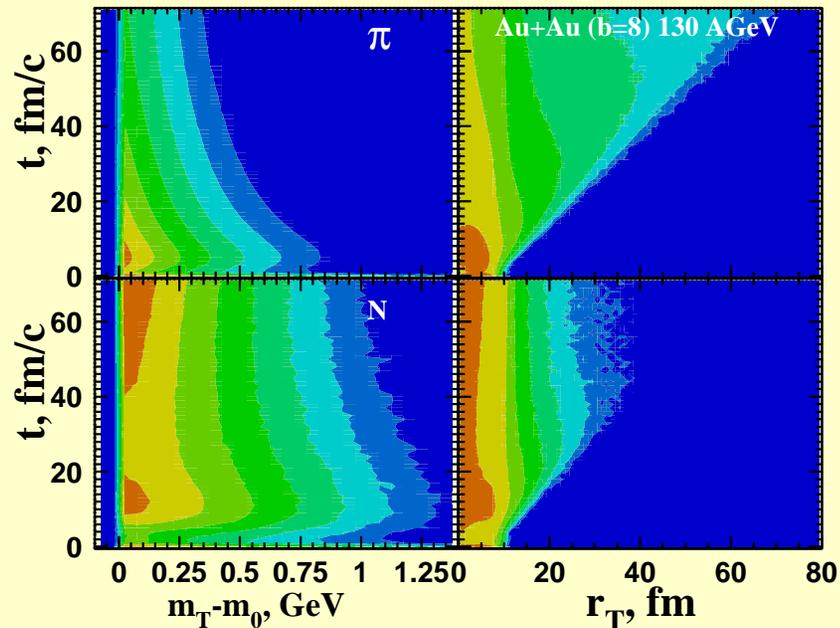
- (1) Substantial part of hadrons leaves the system immediately after their production within the first two fm/c.
- (2) Baryons and mesons are completely different: pions emitted within the first few fm/c carry the strongest flow. In contrast to pions, the baryon fraction acquires stronger elliptic flow during the subsequent rescatterings, developing the hydro-like flow.

# Freeze-Out of Hadrons at RHIC

L. Bravina *et al.*, hep-ph/0412343

Time evolution of  $m_T$  and  $r_T$  spectra

Rapidity and  $z$ -distributions

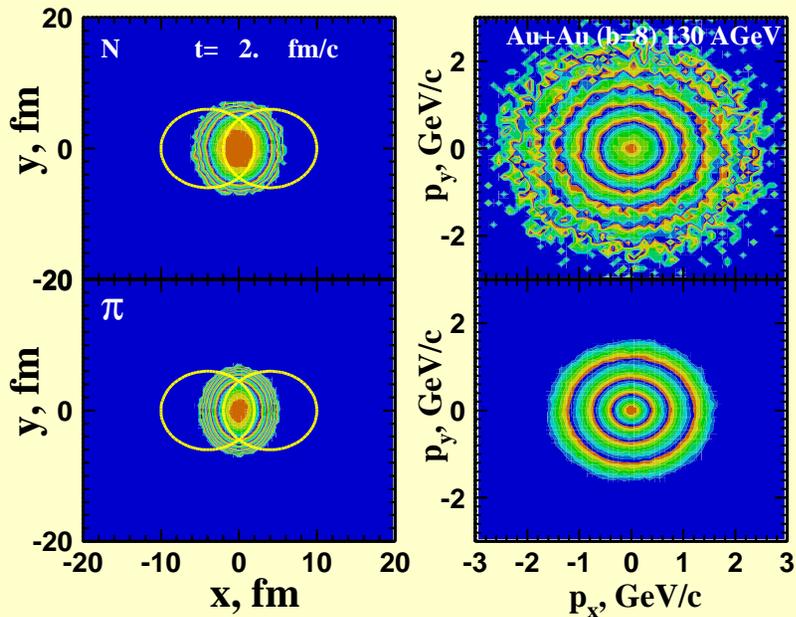


**Pions** are emitted mainly from the overlapping almond-shaped zone in  $(x, y)$ -plane of nuclear collision. As the radial symmetry of this region is restored, the elliptic flow of pions becomes weaker. **Nucleons** are coming both from the overlapping zone and from the spectator regions. This circumstance explains why the  $v_2^N$  maintains its strength after  $t \approx 12$  fm/c.

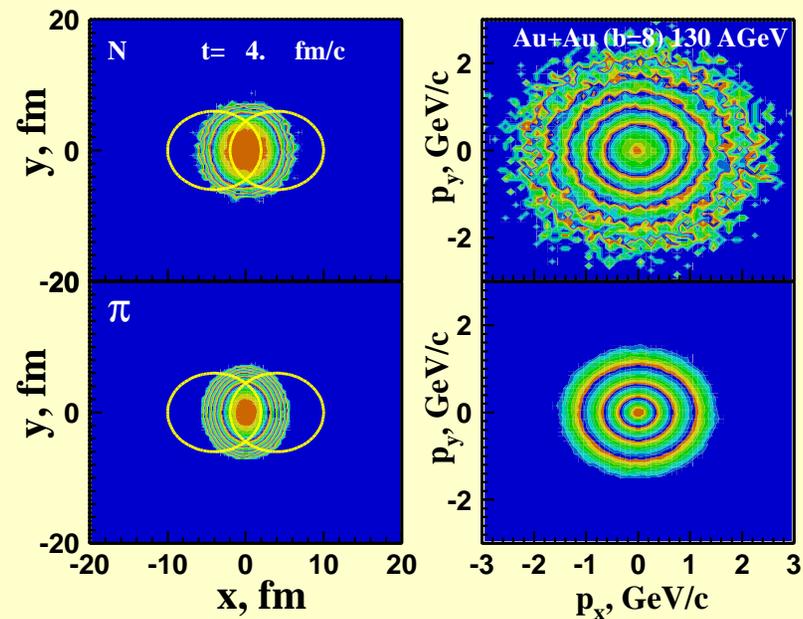
# Freeze-out and Elliptic Flow of $\pi, N$ at RHIC

Anisotropy in coordinate space and elliptic flow of nucleons and pions in **Au+Au** collisions at  $\sqrt{s} = 130$  **AGeV** with the impact parameter  $b = 8$  **fm**.

$t = 2$  **fm/c**



$t = 4$  **fm/c**

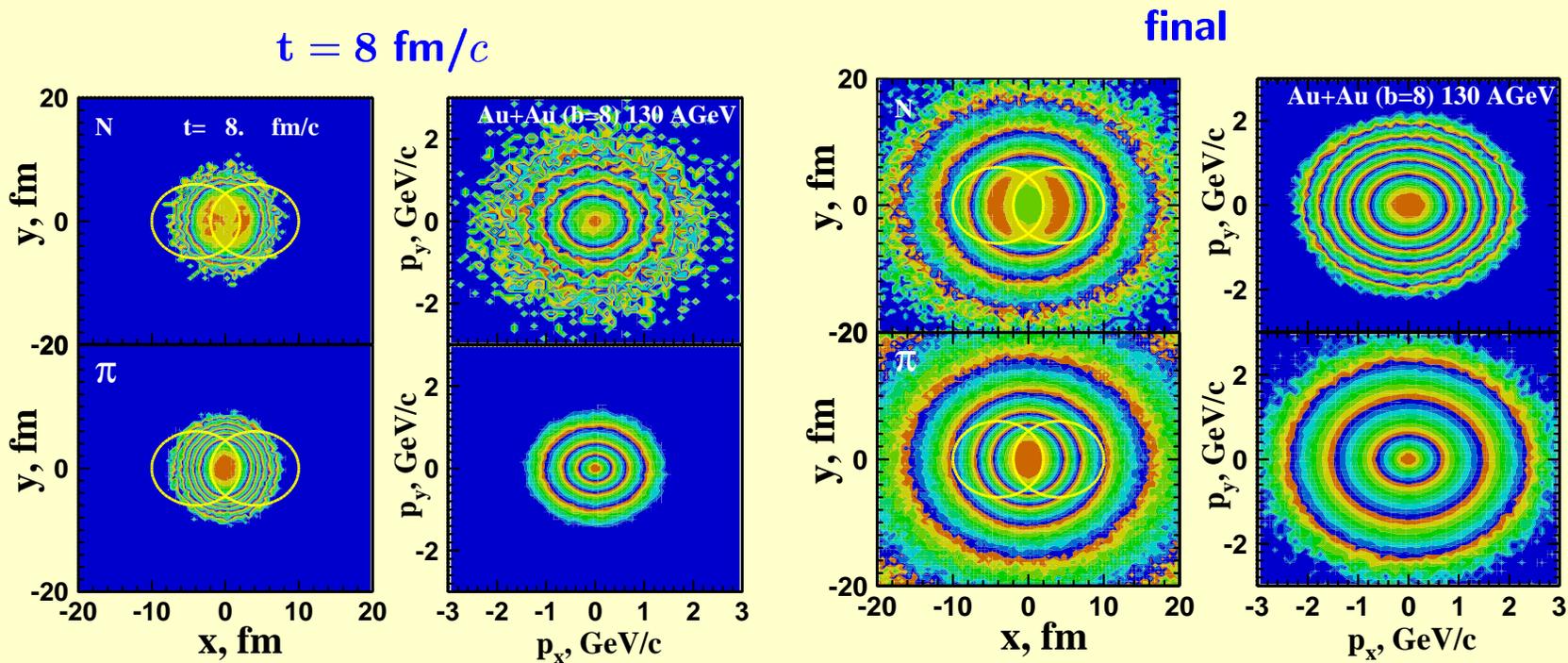


Strong anisotropy in coordinate space, but weak anisotropy in the momentum space

Anisotropy starts to develop in the momentum space for low momenta

## Freeze-out and Elliptic Flow of $\pi, N$ at RHIC

Anisotropy in coordinate space and elliptic flow of nucleons and pions in **Au+Au** collisions at  $\sqrt{s} = 130$  **AGeV** with the impact parameter  $b = 8$  **fm**.



**Nucleons** leave the overlapping region; at the end of the reaction we see the pronounced maxima at centers of nuclei. Most of the **pions** is staying within the overlapping area till the end of reaction

The anisotropy in coordinate space (almond-shaped region) is transformed into the anisotropy in the momentum space.

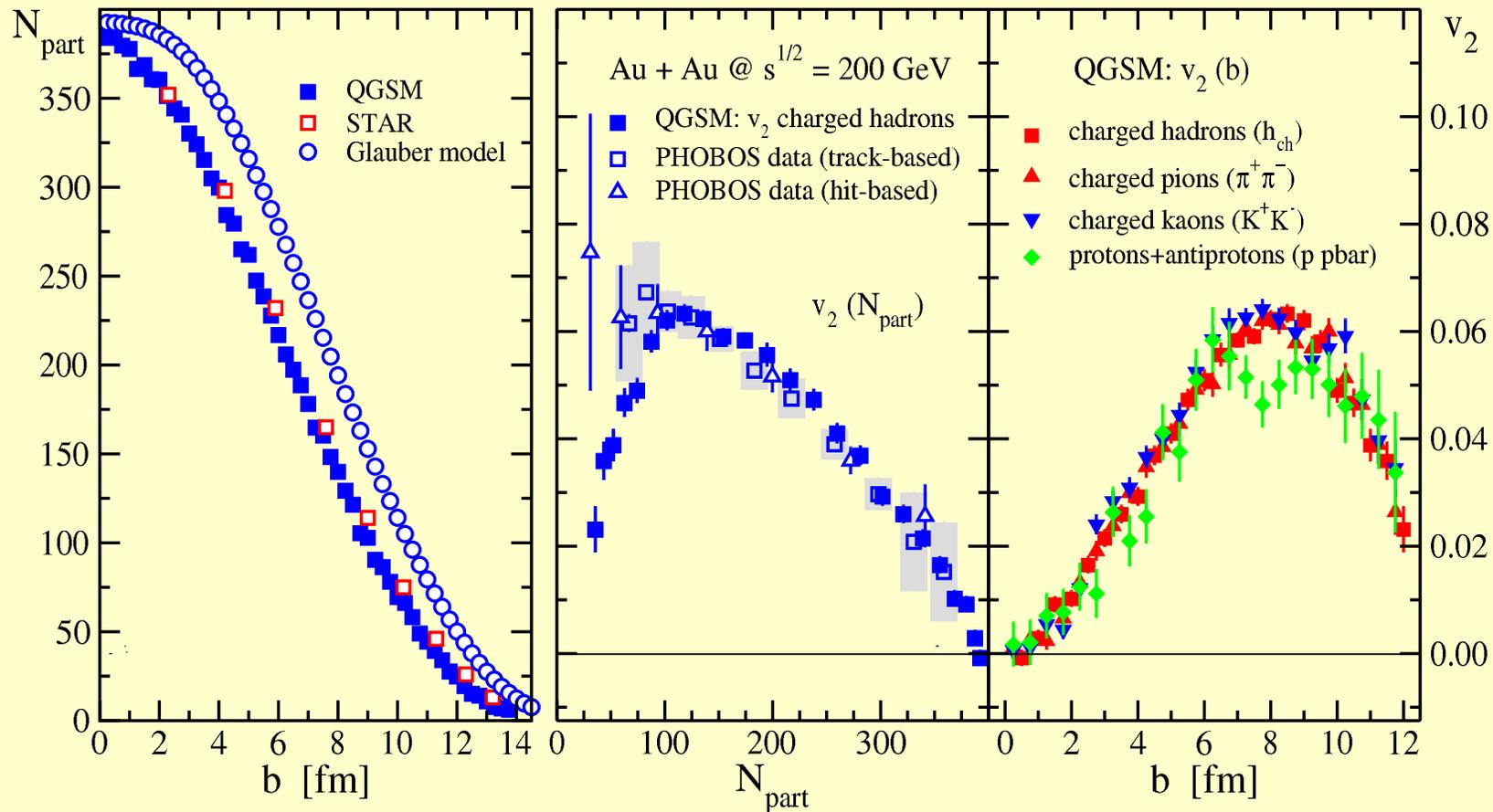
# Elliptic Flow at RHIC

G. Burau *et al.*, PRC 71 (2005) 054905

B.B. Back *et al.* (PHOBOS Collab.), nucl-ex/0407012

J. Adams *et al.* (STAR Collab.), nucl-ex/0409033

## Centrality dependence of $v_2$ in Au+Au at $\sqrt{s} = 200$ AGeV



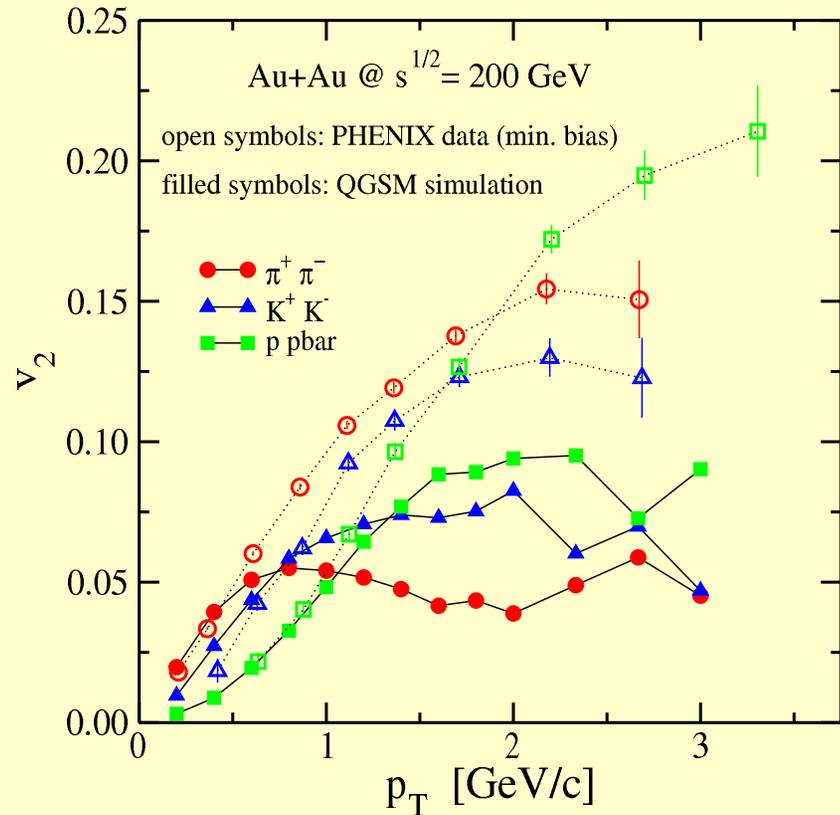
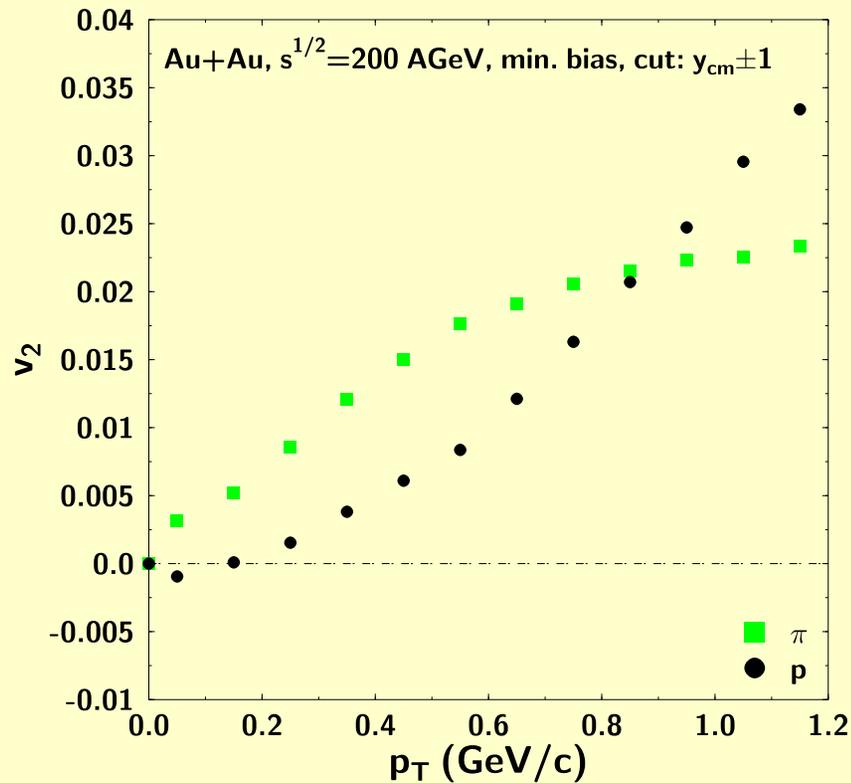
**Good agreement between model simulations and experimental data**

# Elliptic Flow at RHIC

## Transverse momentum dependence of $v_2$

M. Bleicher and H. Stöcker, PLB 526 (2002) 309

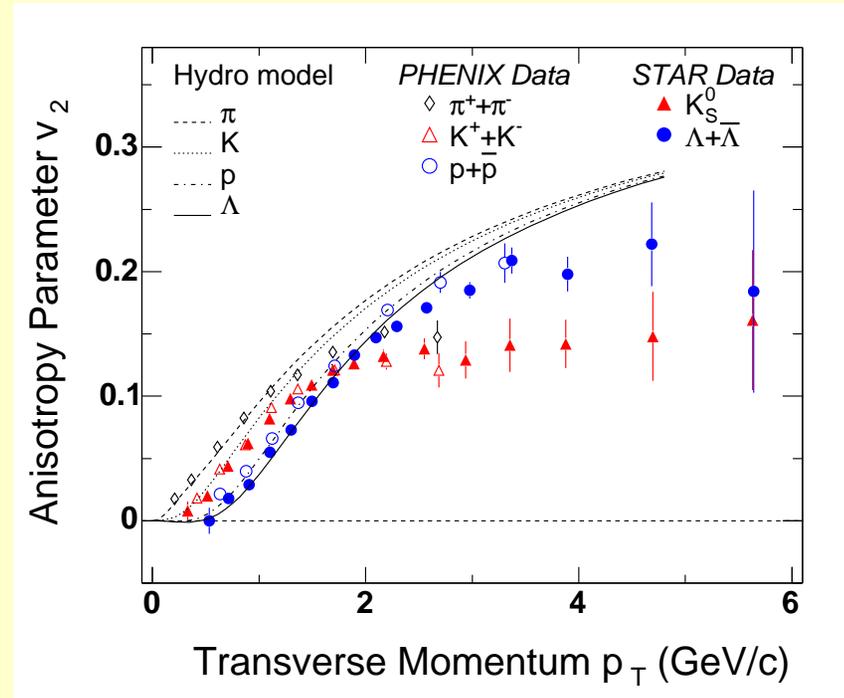
G. Burau *et al.*, PRC 71 (2005) 054905



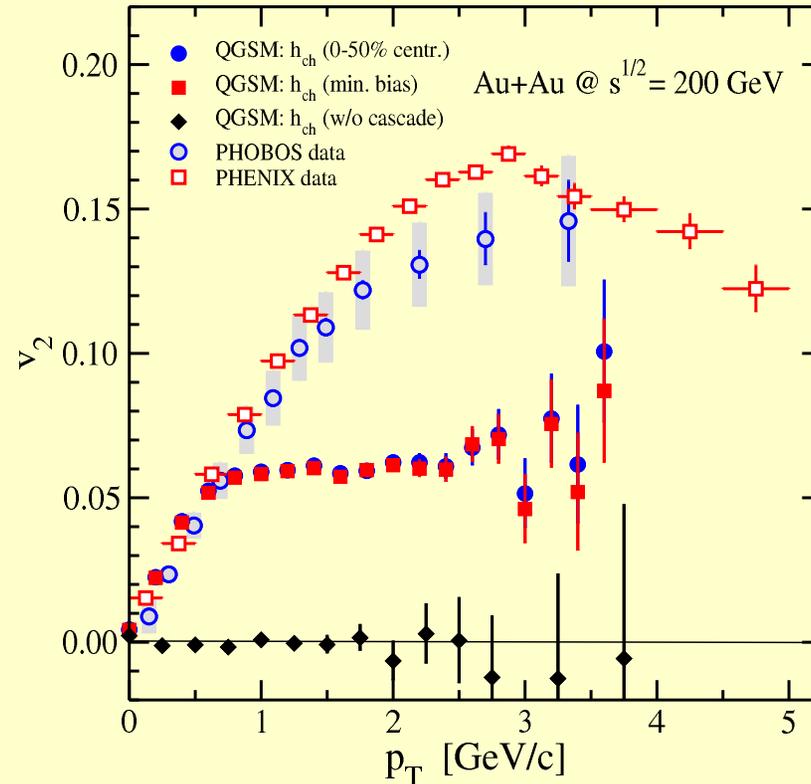
**Both UrQMD and QGSM show crossing of the elliptic flow for mesons and baryons. This agrees with the experimental data**

# Elliptic Flow at RHIC

## Hydrodynamics:



## QGSM:

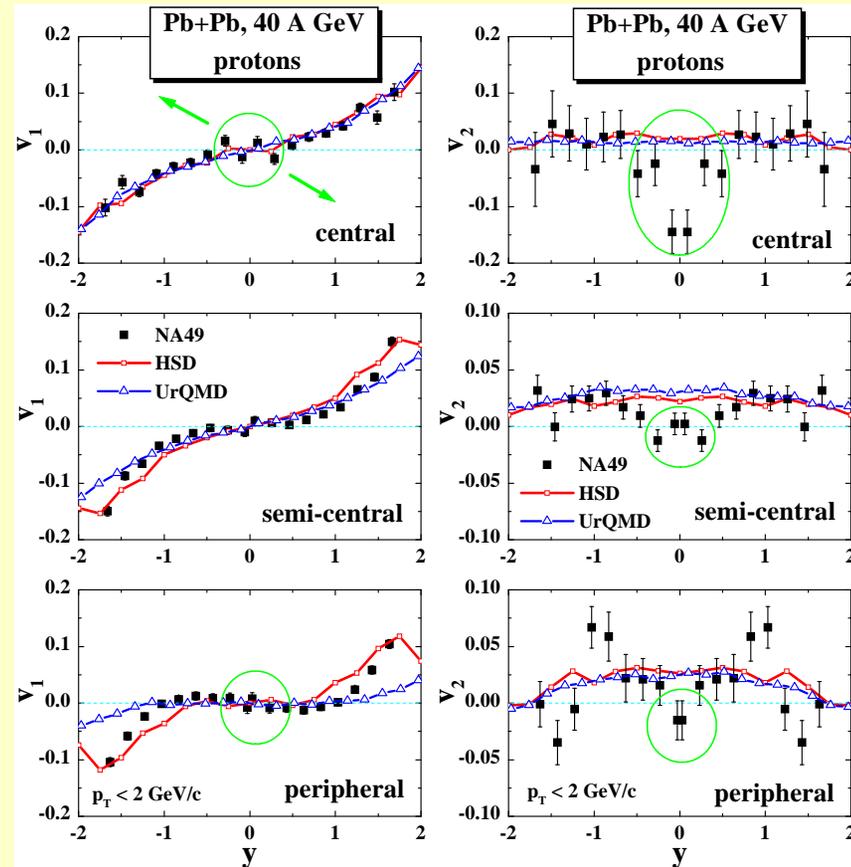
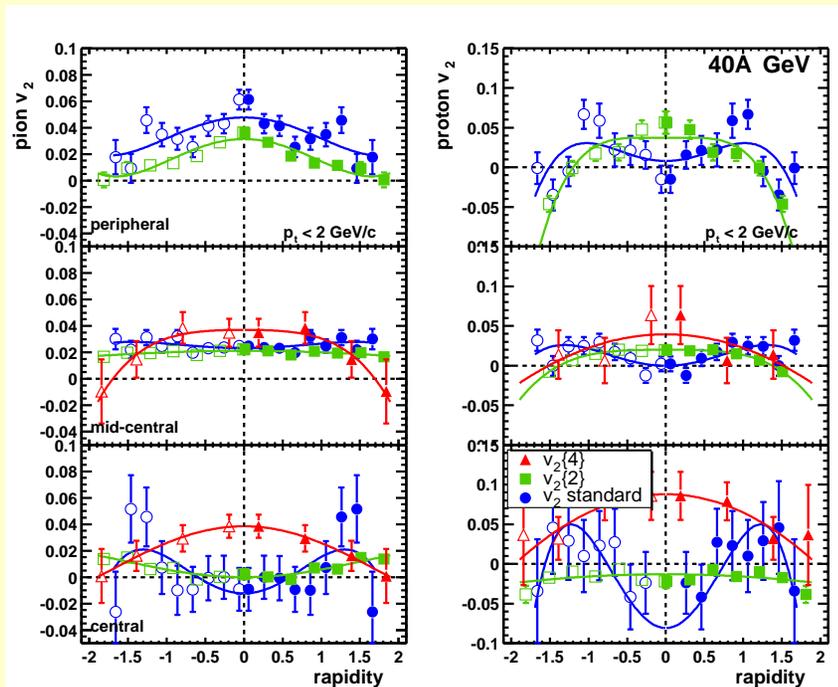


The magnitude of the  $v_2(p_t)$  distribution is underestimated  
Possible explanation: **jets**

# Elliptic Flow at 40 AGeV

H. Stöcker *et al.*, nucl-th/0412022

C. Alt *et al.* (NA49 Collab.), PRC 68 (2003) 034903



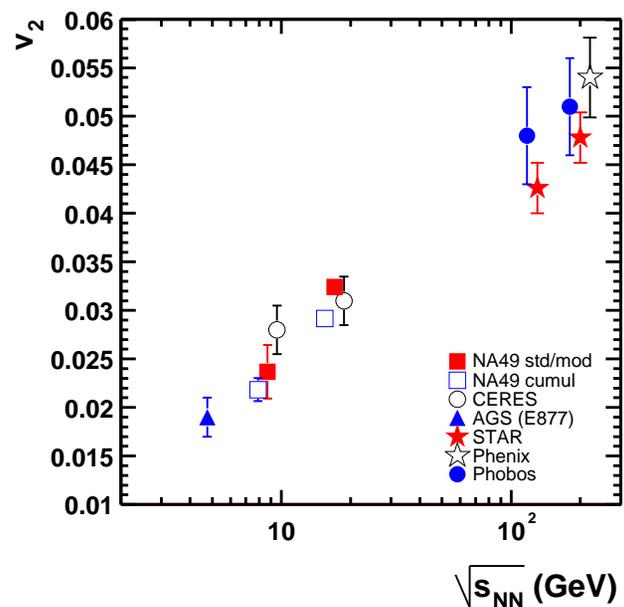
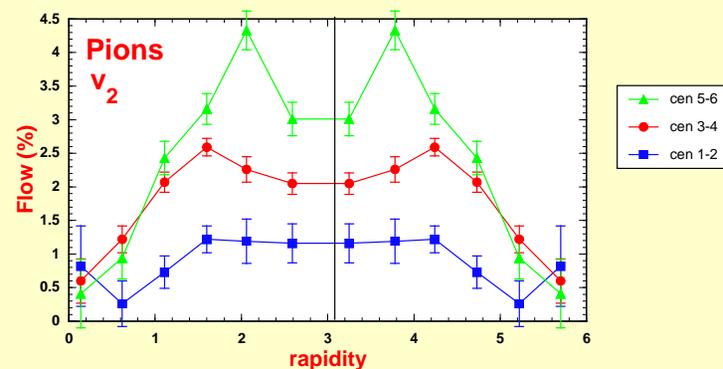
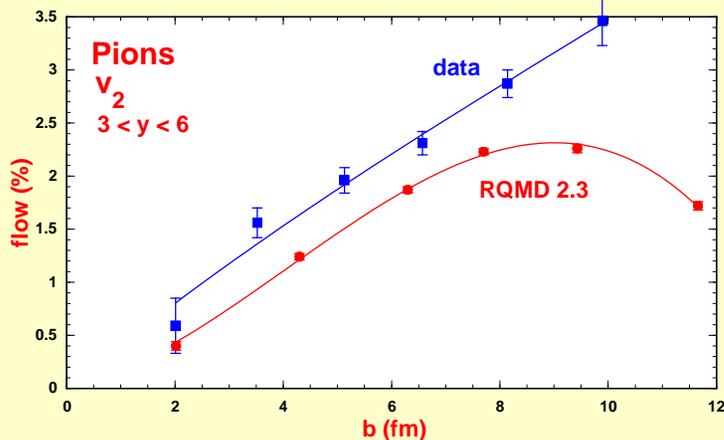
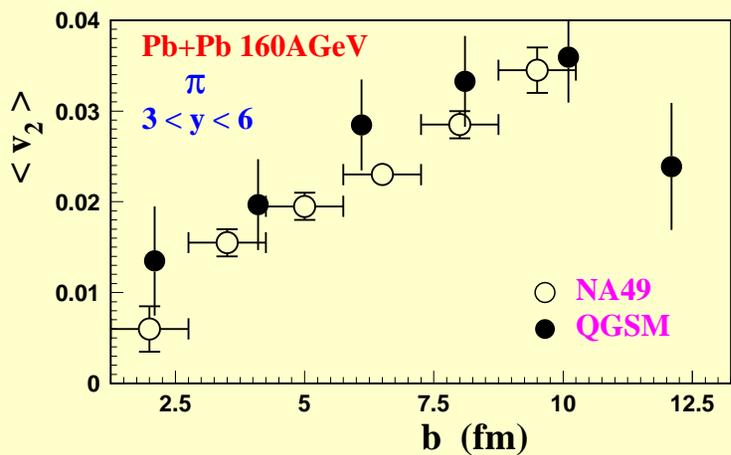
**Collapse of proton elliptic flow:  
evidence for a first order phase transition  
or for non-flow two-particle correlations?**

## Elliptic Flow and Freeze-Out (Conclusions):

- ❖ Microscopic models reproduce, at least qualitatively, basic features of the  $v_2$  development, i.e. centrality, rapidity, and  $p_t$ - dependences
- ❖ EF of hadrons increases with rising  $P_T$
- ❖ EF at  $y = 0$  reaches maximum at  $t \approx 8$  fm at RHIC
- ❖ The magnitude of the  $v_2(p_t)$  distribution is underestimated at  $p_t \geq 1\text{GeV}$
- ❖ Pions are emitted mainly from the overlapping almond-shaped zone of the reaction. As the symmetry of the almond is restored, their elliptic flow decreases. Nucleons are coming both from the overlapping area and from the spectator domains
- ❖ Freeze-out dynamics for baryons and mesons is different and, therefore, development of particle collective flow should be studied together with the freeze-out picture
- ❖ **General trend:** The earlier the freeze-out of mesons, the stronger the  $v_2^M(y = 0)$ , while the  $v_2$  of baryons frozen earlier is weaker than the  $v_2$  of baryons frozen later on

# more distributions...

SPS



# more distributions...

RHIC

