Rare B-Decays

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Rare B decays

Two inclusive rare B-decays of current experimental interest

$$\bar{B} \to X_s \gamma \quad \text{and} \quad \bar{B} \to X_s l^+ l^-$$

$X_s =$ any hadronic state with $S = -1$, containing no charmed particles

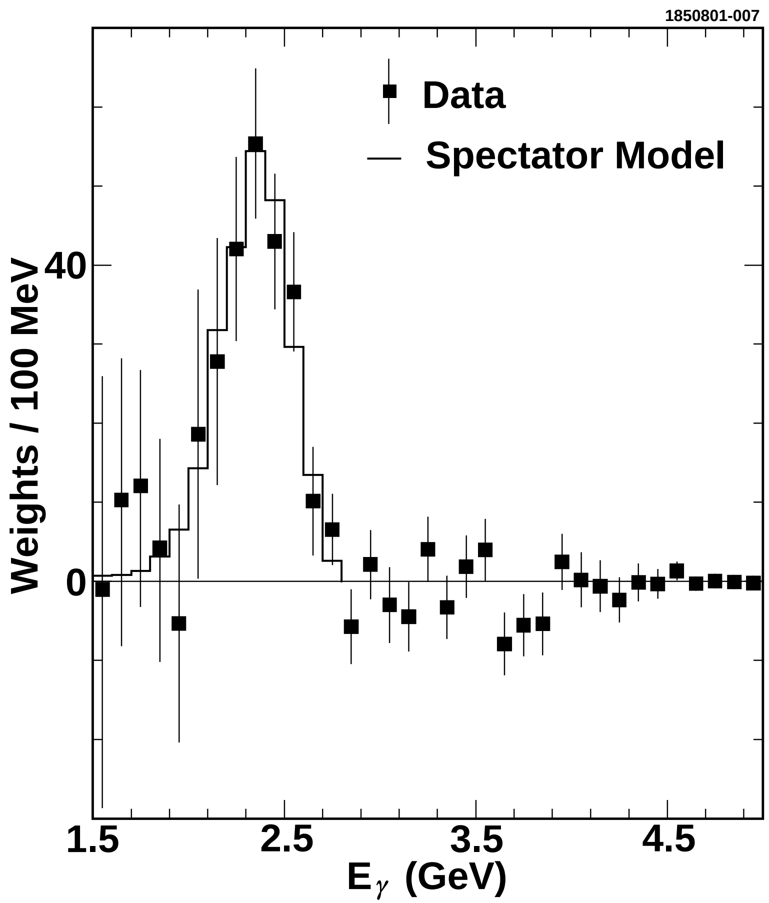
Theoretical Interest:

- Accurate measurements anticipated in near future
- Non-perturbative effects under control
- Sensitivity to new physics

Status of the NNLO perturbative calculations:

- $\bar{B} \to X_s l^+ l^-$: completed
- $\bar{B} \to X_s \gamma$: $\sim \frac{1}{3}$ way through [Misiak, Steinhauser, Greub, Haisch, Gorbahn, Schröder, Czakon,...]

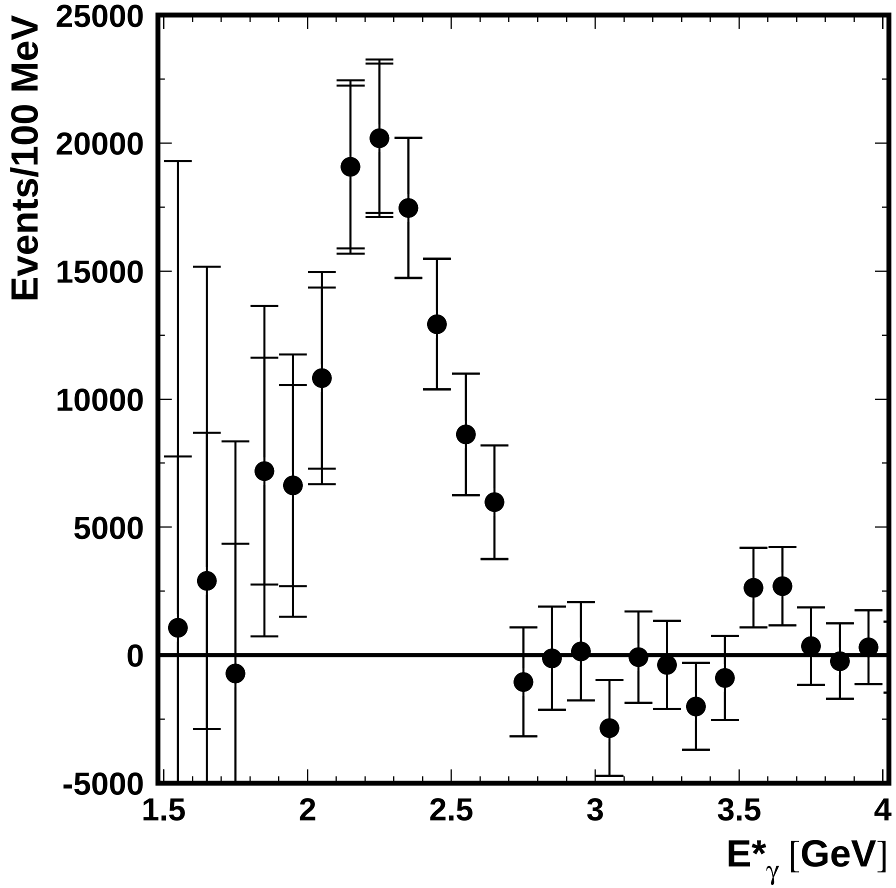
Measurement of $\bar{B} \to X_s \gamma$



CLEO

hep-ex/0108032

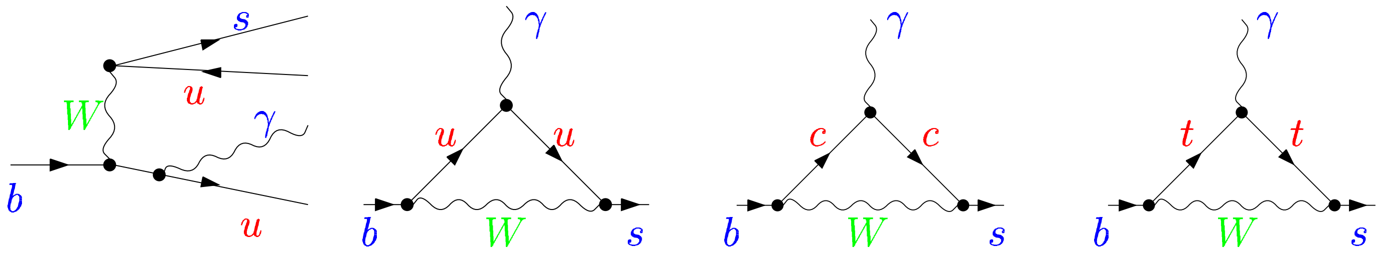
PRL 87 (2001) 251807



BELLE

hep-ex/0403004

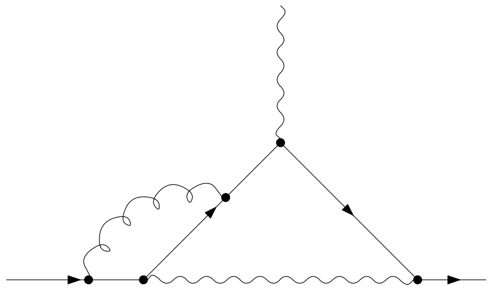
Examples of the leading electroweak diagrams for $\bar{B} \to X_s \gamma$:



$$\left|\frac{V_{ub}V_{us}}{V_{cb}}\right| \simeq \left|\frac{V_{ub}V_{us}}{V_{ts}}\right| \simeq 2\%$$

$\sim +200\%$ $\sim -100\%$

In the amplitude, after including LO QCD effects.



QCD logarithms $\alpha_s \ln \frac{M_W^2}{m_b^2}$ enhance BR($\bar{B} \to X_s\gamma$) more than twice.

Effective field theory method is the most convenient for resummation of such large logarithms.

The effective Lagrangian:

$$\mathcal{L} = \mathcal{L}_{QCD\times QED}(q,l) + \frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_{i=1}^{10} C_i(\mu) O_i$$

$(q = u, d, s, c, b, \;\; l = e, \mu)$

$$O_i = \begin{cases} (\bar{s}\Gamma_i c)(\bar{c}\Gamma_i' b), & i = 1,2, \quad |C_i(m_b)| \sim 1 \\ (\bar{s}\Gamma_i b)\Sigma_q(\bar{q}\Gamma_i' q), & i = 3,4,5,6, \quad |C_i(m_b)| < 0.07 \\ \frac{e m_b}{16\pi^2}\bar{s}_L \sigma^{\mu\nu} b_R F_{\mu\nu}, & i = 7, \quad C_7(m_b) \sim -0.3 \\ \frac{g m_b}{16\pi^2}\bar{s}_L \sigma^{\mu\nu} T^a b_R G^a_{\mu\nu}, & i = 8, \quad C_8(m_b) \sim -0.15 \\ \frac{e^2}{16\pi^2}(\bar{s}_L \gamma_\mu b_L)(\bar{l}\gamma^\mu \gamma_5 l), & i = 9, 10 \quad |C_i(m_b)| \sim 4 \end{cases}$$

Three steps of the calculation:

Matching: Evaluating $C_i(\mu_0)$ at $\mu_0 \sim M_W$ by requiring equality of the SM and the effective theory Green functions

Mixing: Deriving the effective theory RGE and evolving $C_i(\mu)$ from μ_0 to $\mu_b \sim m_b$

Matrix elements: Evaluating the on-shell amplitudes at $\mu_b \sim m_b$

Status of the SM calculations for $\bar{B} \to X_s \gamma$ (Courtesy: M. Misiak)

Matching $(\mu_0 \sim M_W, m_t)$:

$$C_i(\mu_0) = C_i^{(0)}(\mu_0) + \frac{\alpha_s(\mu_0)}{4\pi} C_i^{(1)}(\mu_0) + \left(\frac{\alpha_s(\mu_0)}{4\pi}\right)^2 C_i^{(2)}(\mu_0)$$

$i = 1, ..., 6$: tree 1-loop 2-loop [Bobeth, Misiak, Urban, NPB 574 (2000) 291]

$i = 7, 8$: 1-loop 2-loop 3-loop [Steinhauser, Misiak, hep-ph/0401041]

The 3-loop matching has less than 2% effect on BR($\bar{B} \to X_s \gamma$)

Mixing:

$$\hat{\gamma} = \frac{\alpha_s}{4\pi} \begin{pmatrix} 1\mathrm{L} & 2\mathrm{L} \\ 0 & 1\mathrm{L} \end{pmatrix} + \left(\frac{\alpha_s}{4\pi}\right)^2 \begin{pmatrix} 2\mathrm{L} & 3\mathrm{L} \\ 0 & 2\mathrm{L} \end{pmatrix} + \left(\frac{\alpha_s}{4\pi}\right)^3 \begin{pmatrix} 3\mathrm{L} & 4\mathrm{L} \\ 0 & 3\mathrm{L} \end{pmatrix}$$

Haisch, Gorbahn, Gambino, Schröder, Czakon

Matrix elements $(\mu_b \sim m_b)$:

$$\langle O_i \rangle(\mu_b) = \langle O_i \rangle^{(0)}(\mu_b) + \frac{\alpha_s(\mu_b)}{4\pi} \langle O_i \rangle^{(1)}(\mu_b) + \left(\frac{\alpha_s(\mu_b)}{4\pi}\right)^2 \langle O_i \rangle^{(2)}(\mu_b)$$

$i = 1, ..., 6$: 1-loop 2-loop 3-loop [Bieri, Greub, Steinhauser, hep-ph/0302051]

$\mathcal{O}(\alpha_s^2 n_f)$, Steinhauser, Misiak

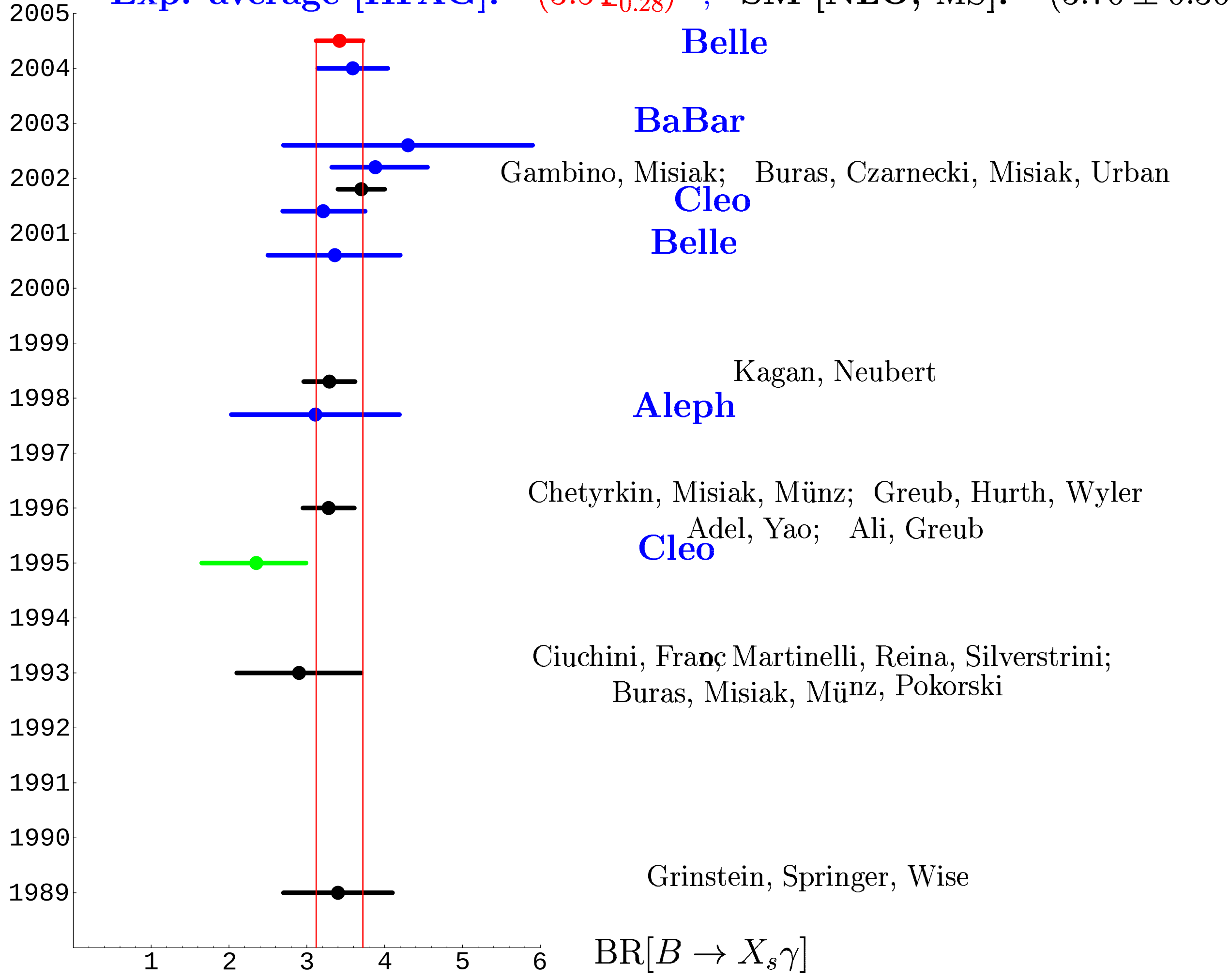
$i = 7, 8$: tree 1-loop 2-loop [Greub, Hurth, Asatrian]

Evolution in time

$\mathrm{BR}[\bar{B} \to X_s \gamma]$ **(units: 10^{-4})**

Measurements & the SM calculations

Exp. average [HFAG]: $(3.54^{+0.30}_{-0.28})$; **SM [NLO; $\overline{\mathrm{MS}}$]:** (3.70 ± 0.30)



$B \to (K^*, \rho)\,\gamma$ decay rates in NLO

- For Large $E_V \sim m_B/2$, symmetries in effective theory $\Longrightarrow$ relations among FFs:

$$f_k(q^2) = C_{\perp k}\xi_\perp(q^2) + C_{\parallel k}\xi_\parallel(q^2)$$

- Symmetries in effective theory broken by perturbative QCD

Factorization Ansatz:

[Beneke, Buchalla, Neubert, Sachrajda; Beneke & Feldmann]

$$f_k(q^2) = C_{\perp k}\xi_\perp(q^2) + C_{\parallel k}\xi_\parallel(q^2) + \Phi_B \otimes T_k \otimes \Phi_V$$

Perturbative Corrections:

$$C_i = C_i^{(0)} + \frac{\alpha_s}{\pi} C_i^{(1)} + \ldots$$

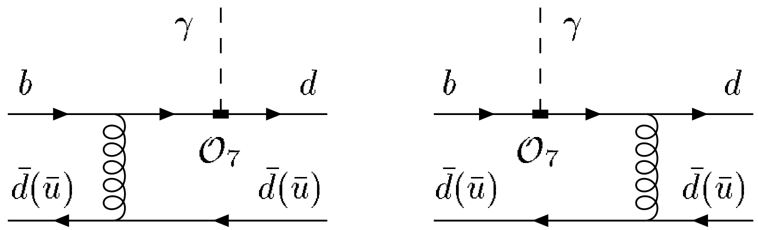
- T_k: Hard Spectator Corrections

$$\Delta\mathcal{M}^{(\mathrm{HSA})} \propto \int_0^1 du \int_0^\infty dl_+ \, M^{(B)} M^{(V)} \, T_k$$

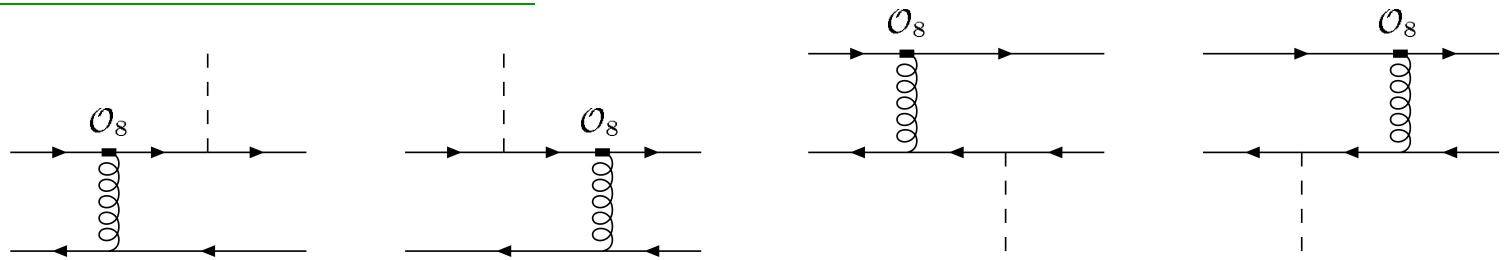
- $M^{(B)}$ and $M^{(V)}$ B-Meson & V-Meson Projection Operators

Hard spectator contributions in $B \to (K^*, \rho)\,\gamma$

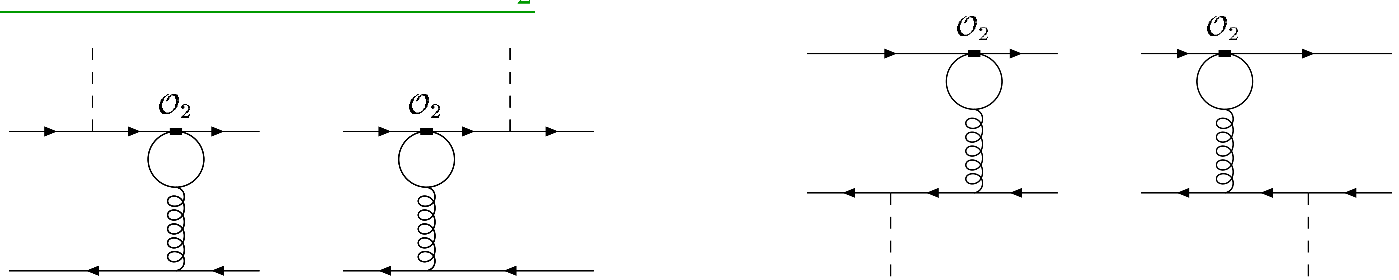
Spectator corrections due to $\mathcal{O}_7$



Spectator corrections due to $\mathcal{O}_8$



Spectator corrections due to $\mathcal{O}_2$



Comparison with data

$$\mathcal{B}_{\rm th}(B \to K^*\gamma) = \tau_B \frac{G_F^2 \alpha |V_{tb}V_{ts}^*|^2}{32\pi^4} m_{b,{\rm pole}}^2 M^3 \left[\xi_\perp^{(K^*)}\right]^2 \left(1 - \frac{m_{K^*}^2}{M^2}\right)^3 \left|C_7^{(0){\rm eff}} + A^{(1)}(\mu)\right|^2$$

$$K = \frac{\left|C_7^{(0){\rm eff}} + A^{(1)}(\mu)\right|^2}{\left|C_7^{(0){\rm eff}}\right|^2} \quad \text{with} \quad 1.5 \leq K \leq 1.7$$

[Beneke, Feldmann, Seidel; Bosch, Buchalla; Parkhomenko, A.A.]

$$\mathcal{B}_{\rm th}(B^0 \to K^{*0}\gamma) \simeq (6.9 \pm 1.1) \times 10^{-5} \left(\frac{m_{b,{\rm pole}}}{4.65\ {\rm GeV}}\right)^2 \left(\frac{\xi_\perp^{(K^*)}}{0.35}\right)^2$$

$$\mathcal{B}_{\rm th}(B^\pm \to K^{*\pm}\gamma) \simeq (7.4 \pm 1.2) \times 10^{-5} \left(\frac{m_{b,{\rm pole}}}{4.65\ {\rm GeV}}\right)^2 \left(\frac{\xi_\perp^{(K^*)}}{0.35}\right)^2$$

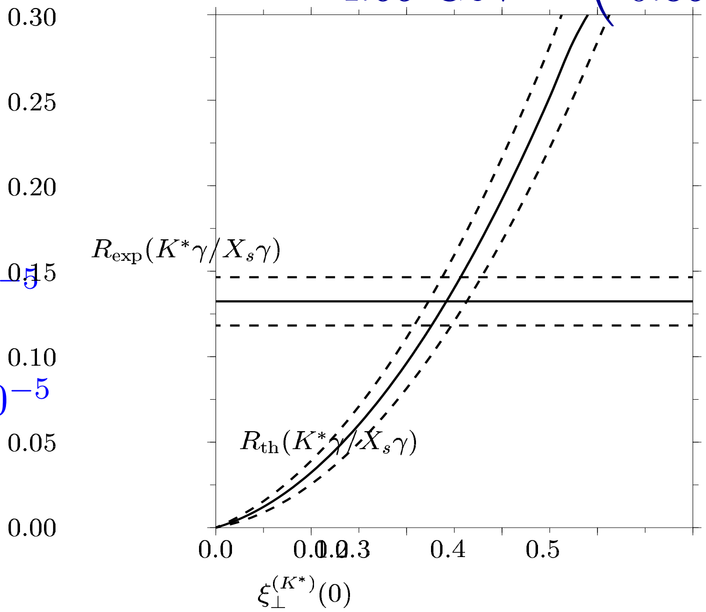
- $T_1^{K^*}(0) = (1 + O(\alpha_s))\xi_\perp^{(K^*)}(0)$
  [Beneke, Feldmann]

Current Experimental Average

$$\mathcal{B}(B^0 \to K^{*0}\gamma) = (4.14 \pm 0.26) \times 10^{-5}$$

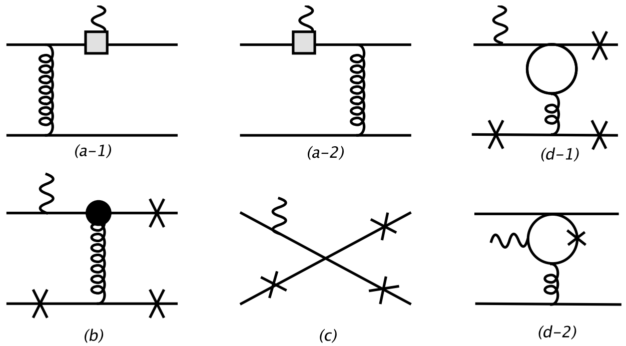
$$\mathcal{B}(B^\pm \to K^{*\pm}\gamma) = (3.98 \pm 0.35) \times 10^{-5}$$

$$\Longrightarrow \quad T_1^{K^*}(0) = 0.27 \pm 0.02$$



$B \to K^* \gamma$ *in PQCD*

[Keum, Matsumori, Sanda]



$$Br(B^0 \to K^{*0}\gamma) = (3.5^{+1.1}_{-0.8}) \times 10^{-5}$$
$$Br(B^\pm \to K^{*\pm}\gamma) = (3.4^{+1.2}_{-0.9}) \times 10^{-5}$$

$\Longrightarrow$ Form factor: $T_1^{K^*}(0) = 0.25 \pm 0.04$
in agreement with QCDF-based estimates of the same and data

- Isospin Symmetry Breaking :

$$\Delta_{0-} = \frac{\frac{\tau_{B^+}}{\tau_{B^0}} Br(B^0 \to \bar{K}^{0*}\gamma) - Br(B^- \to K^{*-}\gamma)}{\frac{\tau_{B^+}}{\tau_{B^0}} Br(B^0 \to \bar{K}^{0*}\gamma) + Br(B^- \to K^{*-}\gamma)} = (5.7^{+1.1}_{-1.3})\%$$

[Cf: $\mathbf{\Delta_{0-} = (8 \pm 4)\%}$ [Kagan, Neubert (QCDF)]]

- $\boldsymbol{\Delta_{0-}(K^*\gamma)^{exp} = (3.9 \pm 4.8)\%}$

$B \to \rho\gamma$ decay

Penguin amplitude $\mathcal{M}_{\rm P}(B \to \rho\gamma)$

$$-\frac{G_F}{\sqrt{2}} V_{tb} V_{td}^* C_7 \frac{e m_b}{4\pi^2} \epsilon^{(\gamma)\mu} \epsilon^{(\rho)\nu} \left(\epsilon_{\mu\nu\alpha\beta} p^\alpha q^\beta - i \left[g^{\mu\nu}(q.p) - p^\mu q^\nu \right] \right) T_1^{(\rho)}(0)$$

Annihilation amplitude $\mathcal{M}_{\rm A}(B^\pm \to \rho^\pm\gamma)$

$$e \frac{G_F}{\sqrt{2}} V_{ub} V_{ud}^* a_1 m_\rho \epsilon^{(\gamma)\mu} \epsilon^{(\rho)\nu} \left(\epsilon_{\mu\nu\alpha\beta} p^\alpha q^\beta F_A^{(\rho);\rm p.v.}(0) - i \left[g^{\mu\nu}(q.p) - p^\mu q^\nu \right] F_A^{(\rho);\rm p.c.}(0) \right)$$

- $F_A^{(\rho);\rm p.v.}(0) \simeq F_A^{(\rho);\rm p.c.}(0) = F_A^{(\rho)}(0)$ [more recently Byer, Melikhov, Stech]

$$\epsilon_{\rm A}(\rho^\pm\gamma) = \frac{4\pi^2 m_\rho a_1}{m_b C_7^{eff}} \frac{F_A^{(\rho)}(0)}{T_1^{(\rho)}} = 0.30 \pm 0.07$$

- Holds in factorization approximation
- $O(\alpha_s)$ corrections to annihilation amplitude $\mathcal{M}_{\rm A}(B^\pm \to \rho^\pm\gamma)$: Leading-twist contribution vanishes in the chiral limit [Grinstein, Pirjol]; non-factorizing annihilation contribution likely small; testable in $B^\pm \to \ell^\pm \nu_\ell \gamma$

Annihilation amplitude $\mathcal{M}_{\rm A}(B^0 \to \rho^0\gamma)$

• Suppressed due to the electric charges ($Q_d/Q_u = -1/2$) and colour factors (BSW Parameters: $a_2/a_1 \simeq 0.25$)

$$\Longrightarrow \epsilon_{\rm A}(\rho^0\gamma) \simeq 0.05$$

$B \to (\rho,\ \omega)\gamma$ decay rates

[Parkhomenko, A.A.; Bosch, Buchalla; Lunghi, Parkhomenko, AA; Beneke, Feldmann, Seidel]

$$R(\rho\gamma) \equiv \frac{\overline{\mathcal{B}}(B \to \rho\gamma)}{\overline{\mathcal{B}}(B \to K^*\gamma)} = S_\rho \left|\frac{V_{td}}{V_{ts}}\right|^2 \frac{(1 - m_\rho^2/M^2)^3}{(1 - m_{K^*}^2/M^2)^3} \zeta^2 \; [1 + \Delta R(\rho/K^*)]$$

$$R(\omega\gamma) \equiv \frac{\overline{\mathcal{B}}(B \to \omega\gamma)}{\overline{\mathcal{B}}(B \to K^*\gamma)} = 1/2 \left|\frac{V_{td}}{V_{ts}}\right|^2 \frac{(1 - m_\omega^2/M^2)^3}{(1 - m_{K^*}^2/M^2)^3} \zeta^2 \; [1 + \Delta R(\omega/K^*)]$$

- $S_\rho = 1 \;$ for $B^\pm \to \rho^\pm\gamma; \quad = 1/2 \;$ for $\; B^0 \to \rho^0\gamma$
- $\zeta = \frac{T_1^{(\rho)}(0)}{T_1^{(K^*)}(0)} \simeq 0.85 \pm 0.10 \;\; ; T_1^\omega(0) = T_1^{(\rho)}(0)$ [based on QCD − SRs, Lattice]
- $\Delta R(\rho^\pm/K^{*\pm}) = 0.12 \pm 0.10$
- $\Delta R(\rho^0/K^{*0}) \simeq \Delta R(\omega/K^{*0}) = 0.1 \pm 0.07$

Theoretical Branching Ratios

[Lunghi, Parkhomenko, AA]

- $R(\rho^\pm/K^{*\pm}) = (3.3 \pm 1.0) \times 10^{-2}$
- $R(\rho^0/K^{*0}) \simeq R(\omega/K^{*0}) = (1.6 \pm 0.5) \times 10^{-2}$
- $\mathrm{BR}(\mathrm{B}^\pm \to \rho^\pm\gamma) = (1.35 \pm 0.4) \times 10^{-6}$
- $\mathrm{BR}(\mathrm{B}^0 \to \rho^0\gamma) \simeq \mathrm{BR}(\mathrm{B}^0 \to \omega\gamma) = (0.65 \pm 0.2) \times 10^{-6}$

Comparison with data

Experimental Average

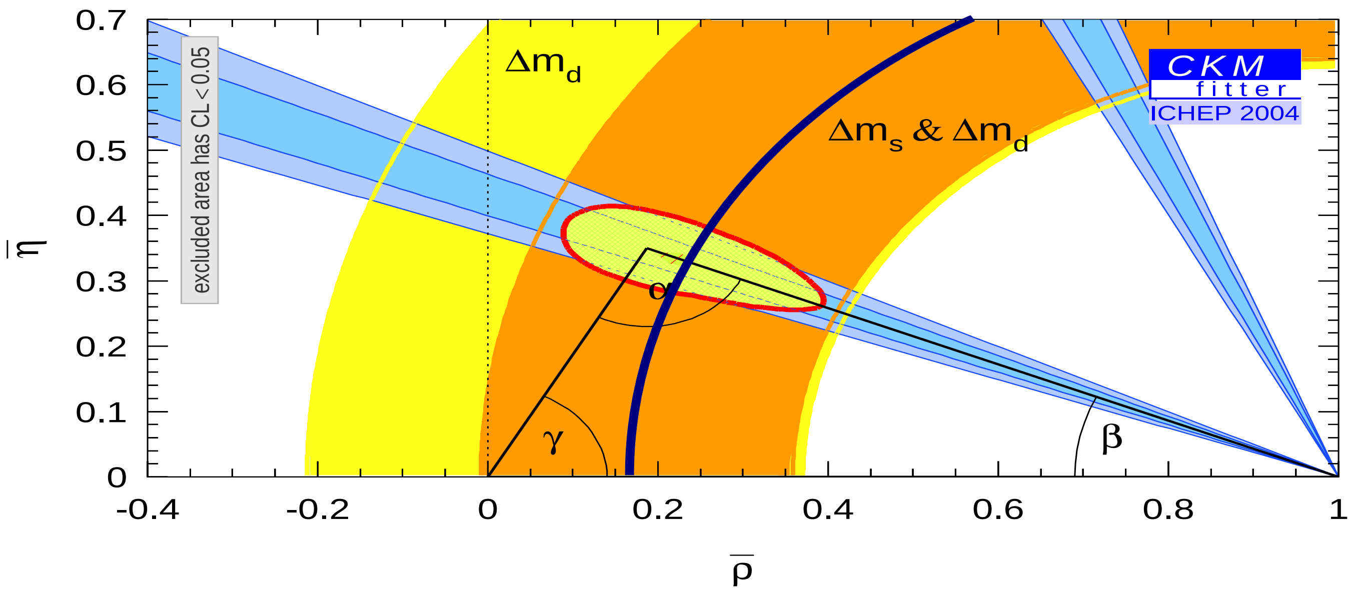
$$\bar{\mathcal{B}}[B \to (\rho,\omega)\,\gamma] \equiv \frac{1}{2}\left\{\mathcal{B}(B^+ \to \rho^+\gamma) + \frac{\tau_{B^+}}{\tau_{B^0}}\left[\mathcal{B}(B_d^0 \to \rho^0\gamma) + \mathcal{B}(B_d^0 \to \omega\gamma)\right]\right\}$$

Upper Limits (90% C.L.)

$\bar{\mathcal{B}}_{\rm exp}[B \to (\rho,\omega)\,\gamma] < 1.4 \times 10^{-6}$; $R[(\rho,\omega)/K^*] < 0.035$; $|V_{td}/V_{ts}| < 0.22$ [BELLE]

$\bar{\mathcal{B}}_{\rm exp}[B \to (\rho,\omega)\,\gamma] < 1.2 \times 10^{-6}$; $R[(\rho,\omega)/K^*] < 0.029$; $|V_{td}/V_{ts}| < 0.19$ [BABAR]

Constraints from $R[(\rho,\omega)/K^*] < 0.029$ on CKM Parameters [Berryhill (BABAR)]



$\bar{B} \to X_s l^+ l^-$

- The NNLO calculation of $\bar{B} \to X_s l^+ l^-$ corresponds to the NLO calculation of $\bar{B} \to X_s \gamma$, as far as the number of loops in the diagrams is concerned.

- Coefficients of the two additional operators

$$O_i = \frac{e^2}{16\pi^2}(\bar{s}_L \gamma_\mu b_L)(\bar{l}\gamma^\mu \gamma_5 l), \qquad i = 9, 10$$

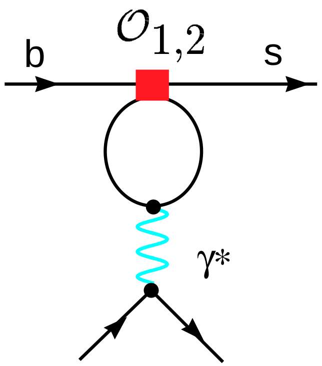
have the following perturbative expansion:

$$C_9(\mu) = \frac{4\pi}{\alpha_s(\mu)} C_9^{(-1)}(\mu) + C_9^{(0)}(\mu) + \frac{\alpha_s(\mu)}{4\pi} C_9^{(1)}(\mu) + \ldots$$

$$C_{10} = C_{10}^{(0)} + \frac{\alpha_s(M_W)}{4\pi} C_{10}^{(1)} + \ldots$$

- After an expansion in α_s, the term $C_9^{(-1)}(\mu)$ reproduces (the dominant part of) the electro-weak logarithm that originates from photonic penguins with charm quark loops:

$$\frac{4\pi}{\alpha_s(m_b)} C_9^{(-1)}(m_b) = \frac{4}{9} \ln \frac{M_W^2}{m_b^2} + \mathcal{O}(\alpha_s)$$



$C_9^{(-1)}(m_b) \simeq 0.033 \ll 1 \quad \Longrightarrow \quad \frac{4\pi}{\alpha_s(m_b)} C_9^{(-1)}(m_b) \simeq 2$

On the other hand: $\quad C_9^{(0)}(m_b) \simeq 2.2$

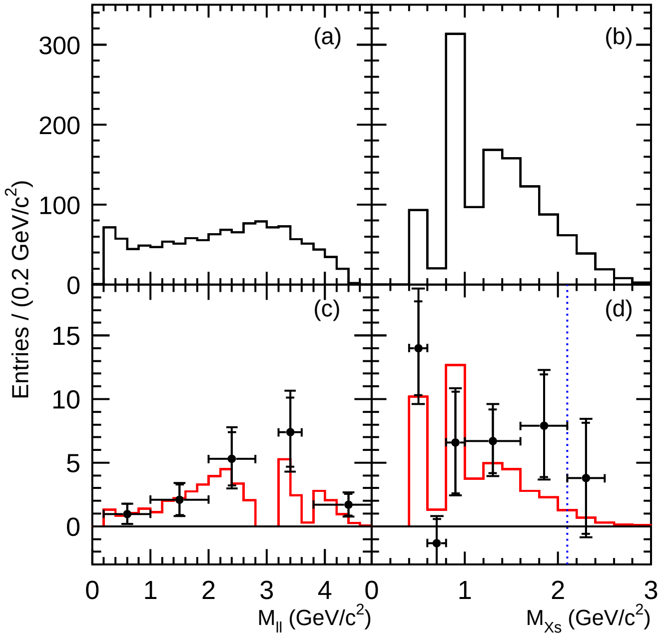
NNLO Calculations of BR($\bar{B} \to X_s \ell^+ \ell^-$)

- Two-loop matching, three-loop mixing and two-loop matrix elements have been completed
  - Matching: [Bobeth, Misiak, Urban]
  - Mixing: [Gambino, Gorbahn, Haisch]
  - Matrix elements:
    [Asatryan, Asatrian, Greub, Walker;
    Asatrian, Bieri, Greub, Hovhannissyan;
    Ghinculov, Hurth, Isidori, Yao;
    Bobeth, Gambino, Gorbahn, Haisch]
- Power corrections in $B \to X_s \ell^+ \ell^-$ decays
  - $1/m_b$ corrections [A. Falk et al.; AA, Handoko, Morozumi,Hiller; Buchalla, Isidori]
  - $1/m_c$ corrections [Buchalla, Isidori, Rey]
- NNLO Phenomenological analysis of $B \to X_s \ell^+ \ell^-$ decays [AA, Greub, Hiller, Lunghi]
  - $\mathrm{BR}(\bar{B} \to X_s \mu^+ \mu^-); \quad q^2 > 4m_\mu^2 = (4.2 \pm 1.0) \times 10^{-6}$
  - $\mathrm{BR}(\bar{B} \to X_s e^+ e^-) = (6.9 \pm 0.7) \times 10^{-6}$

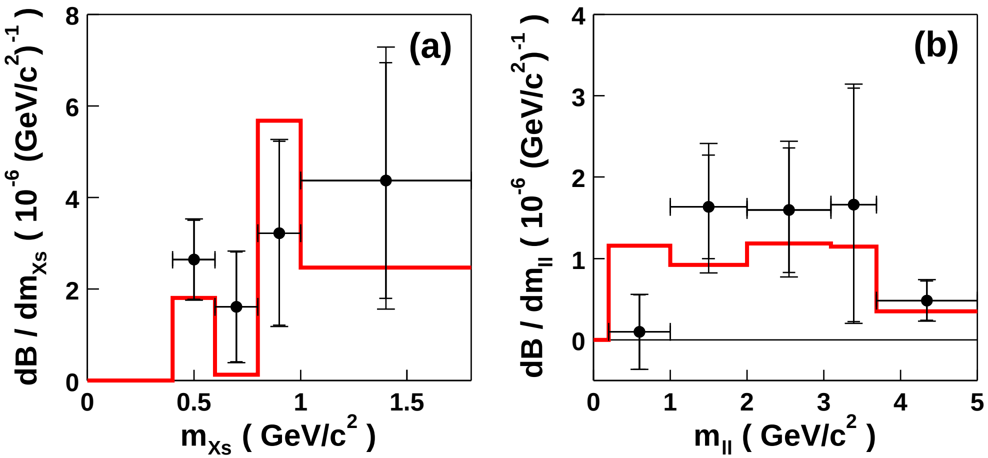
Decay distributions in $\bar{B} \to X_s \ell^+ \ell^-$

$M_{\ell\ell}$ and M_{X_s} Spectra

[BELLE]



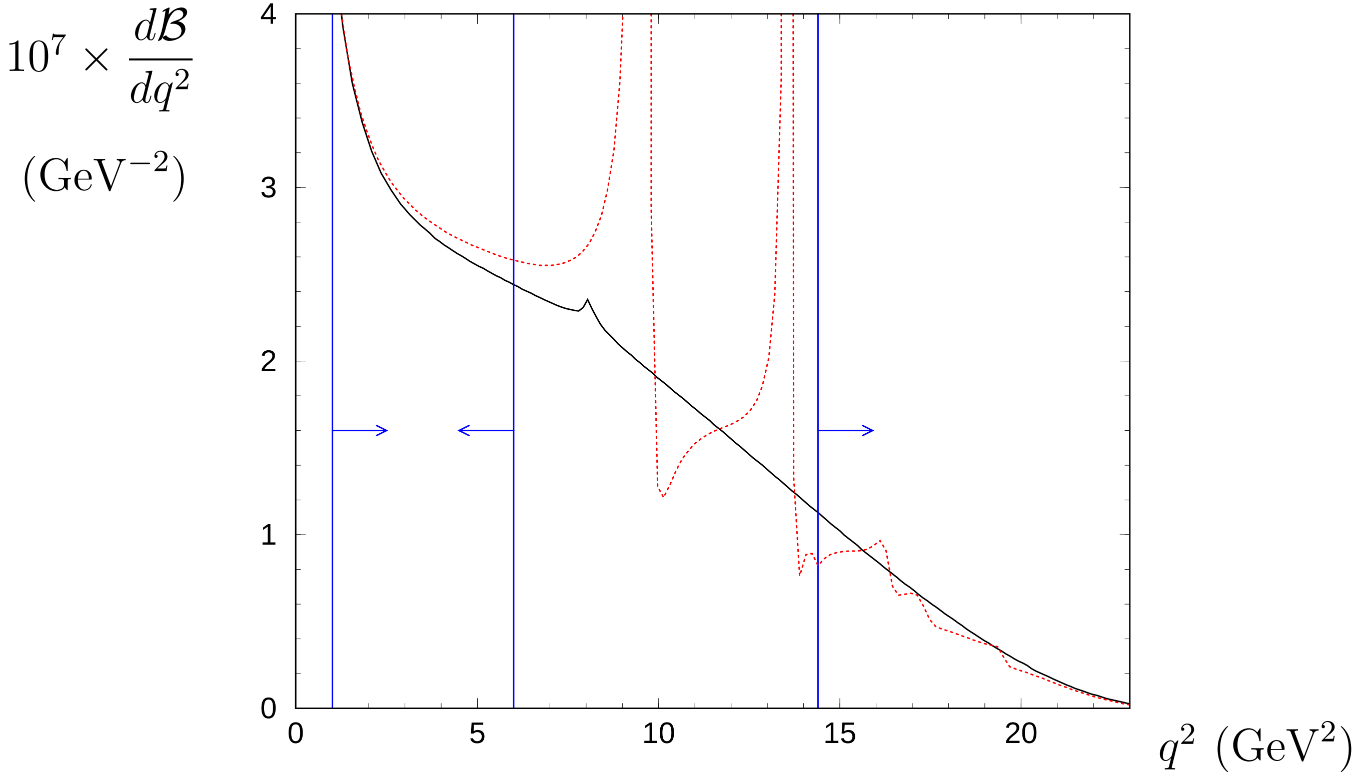
[BABAR]



- In agreement with the NNLO SM calculations

Dilepton invariant mass distribution in $\bar{B} \to X_s \ell^+ \ell^-$:

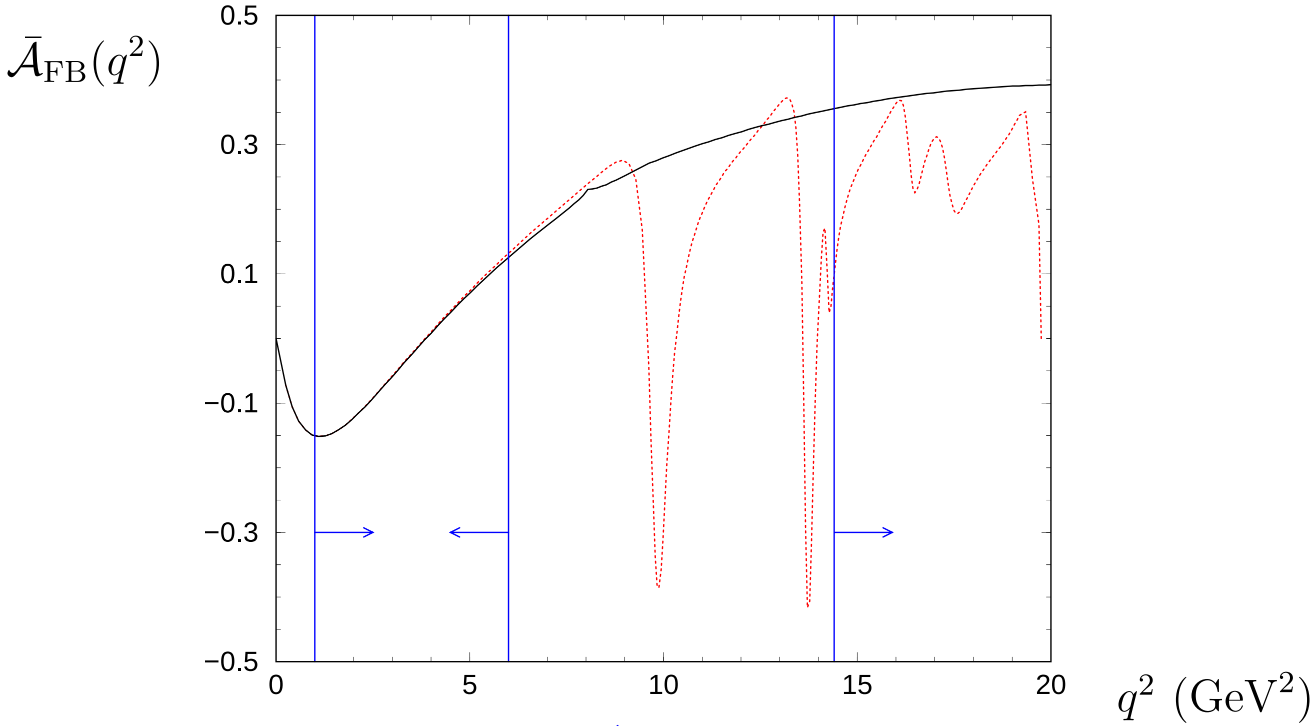
[Ghinculov, Hurth, Isidori, Yao 2004]



- $\mathbf{BR}(\bar{B} \to X_s \ell^+ \ell^-)$; $\quad q^2 \in [1,6]\ \mathrm{GeV}^2 = (1.63 \pm 0.20) \times 10^{-6}$
- $\mathbf{BR}(\bar{B} \to X_s \ell^+ \ell^-)$; $\quad q^2 > 14\ \mathrm{GeV}^2 = (4.04 \pm 0.78) \times 10^{-7}$
- $\mathbf{BR}(\bar{B} \to X_s \mu^+ \mu^-)$; $\quad q^2 > 4m_\mu^2 = (4.6 \pm 0.8) \times 10^{-6}$, in agreement with the earlier NNLO analysis [AA, Greub, Hiller, Lunghi 2001; Bobeth, Gambino, Gorbahn, Haisch, 2003]

Normalized FB-Asymmetry in $\bar{B} \to X_s \ell^+ \ell^-$:

[Ghinculov, Hurth, Isidori, Yao 2004]



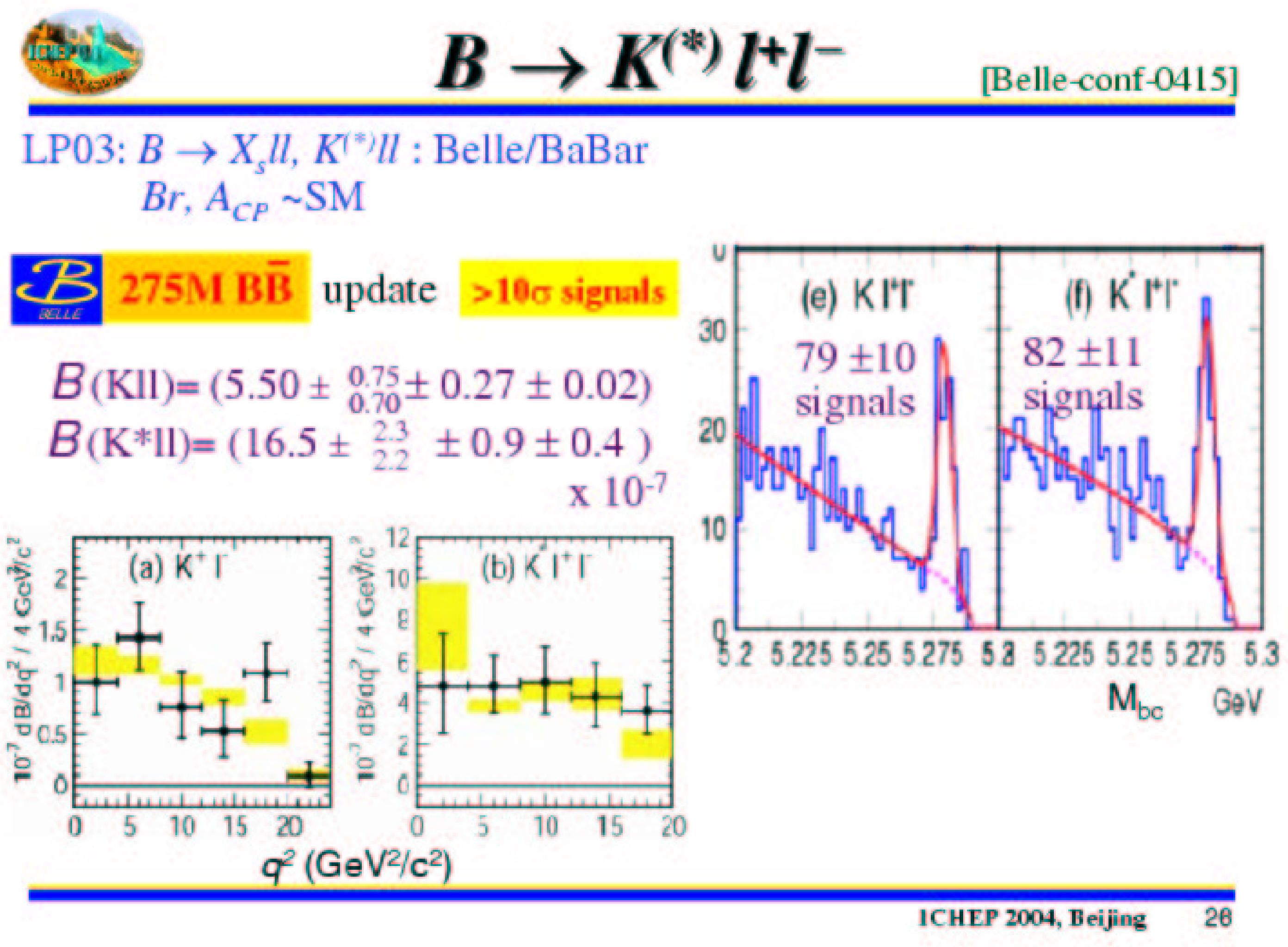
$$\bar{\mathcal{A}}_{\rm FB}(q^2) = \frac{1}{d\mathcal{B}(B \to X_s \ell^+ \ell^-)/dq^2} \int_{-1}^{1} d\cos\theta_\ell \, \frac{d^2\mathcal{B}(B \to X_s \ell^+ \ell^-)}{dq^2 \, d\cos\theta_\ell} \mathrm{sgn}(\cos\theta_\ell)$$

- Zero of the FB-Asymmetry is a precision test of the SM

$$q_0^2 = (3.90 \pm 0.25) \ \mathrm{GeV}^2 \quad \text{[Ghinculov, Hurth, Isidori, Yao 2004]}$$

$$q_0^2 = (3.76 \pm 0.22_{\rm theory} \pm 0.24_{m_b}) \ \mathrm{GeV}^2 \quad \text{[Bobeth, Gambino, Gorbahn, Haisch 2003]}$$

Sakai (FPCP 2004)



SM predictions at NNLO accuracy & Comparison with data

Expt. [HFAG]; SM: [AA, Greub, Lunghi, Hiller]

(in units of 10^{-6})

	Decay Mode	Theory (SM)	Expt. (BELLE & BABAR)
	$B \to K\ell^+\ell^-$	0.35 ± 0.12	$0.55\,^{+0.09}_{-0.08}$
	$B \to K^*e^+e^-$	1.58 ± 0.52	$1.25\,^{+0.37}_{-0.33}$
	$B \to K^*\mu^+\mu^-$	1.2 ± 0.4	$1.19\,^{+0.34}_{-0.29}$
$\longrightarrow$	$B \to X_s\mu^+\mu^-$	4.2 ± 0.7	4.8 ± 1.0
		4.6 ± 0.8 [1)]	$4.13 \pm 1.05^{+0.73}_{-0.69}$ [3)]
		4.6 ± 0.7 [2)]	
$\longrightarrow$	$B \to X_se^+e^-$	4.2 ± 0.7	5.0 ± 1.3
			$4.04 \pm 1.03^{+0.80}_{-0.76}$ [3)]
$\longrightarrow$	$B \to X_s\ell^+\ell^-$	4.18 ± 0.7	4.8 ± 1.0
			$4.11 \pm 0.83^{+0.74}_{-0.70}$ [3)]

[1)] Ghinculov et al. [2)] Bobeth et al. [3)] BELLE [ICHEP '04]

- Inclusive measurements and the SM rates include a cut $M_{\ell^+\ell^-} > 0.2$ GeV

Forward-Backward Asymmetry in $B \to K^*\ell^+\ell^-$

$$\frac{dA_{FB}}{d\hat{s}} = -\int_0^{\hat{u}(\hat{s})} d\hat{u}\frac{d\Gamma}{d\hat{u}d\hat{s}} + \int_{-\hat{u}(\hat{s})}^0 d\hat{u}\frac{d\Gamma}{d\hat{u}d\hat{s}}$$

$$\sim C_{10}[\mathrm{Re}(C_9^{eff})VA_1 + \frac{\hat{m}_b}{\hat{s}}C_7^{eff}(VT_2(1-\hat{m}_V) + A_1T_1(1+\hat{m}_V))]$$

- $\boldsymbol{T_1, T_2, V, A_1}$ form factors
  - Probes different combinations of WC's than dilepton mass spectrum; has a characteristic zero in the SM $(\hat{s}_0)$ below $m^2_{J/\psi}$

Position of the $A_{FB}(\hat{s})$ zero $(\hat{s}_0)$ in $B \to K^*\ell^+\ell^-$

$$\mathrm{Re}(\mathrm{C_9^{eff}}(\hat{\mathrm{s}}_0)) = -\frac{\hat{\mathrm{m}}_\mathrm{b}}{\hat{\mathrm{s}}_0}\mathrm{C_7^{eff}}(\frac{\mathrm{T_2}(\hat{\mathrm{s}}_0)}{\mathrm{A_1}(\hat{\mathrm{s}}_0)}(1-\hat{\mathrm{m}}_\mathrm{V}) + \frac{\mathrm{T_1}(\hat{\mathrm{s}}_0)}{\mathrm{V}(\hat{\mathrm{s}}_0)}(1+\hat{\mathrm{m}}_\mathrm{V}))$$

- Model-dependent studies $\Longrightarrow$ small FF-related uncertainties in $\hat{s}_0$ [Burdman '98]
- HQET provides a symmetry argument why the uncertainty in $\hat{s}_0$ is small. In leading order in $1/m_B$, $1/E$ ($E = \frac{m_B^2+m_{K^*}^2-q^2}{2m_B}$) and $O(\alpha_s)$:

$$\frac{T_2}{A_1} = \frac{1+\hat{m}_V}{1+\hat{m}_V^2-\hat{s}}(1-\frac{\hat{s}}{1-\hat{m}_V^2}); \quad \frac{T_1}{V} = \frac{1}{1+\hat{m}_V}$$

- No hadronic uncertainty in $\hat{s}_0$ [AA, Ball, Handoko, Hiller '99]: $C_9^{eff}(\hat{s}_0) = -\frac{2m_bM_B}{s_0}C_7^{eff}$

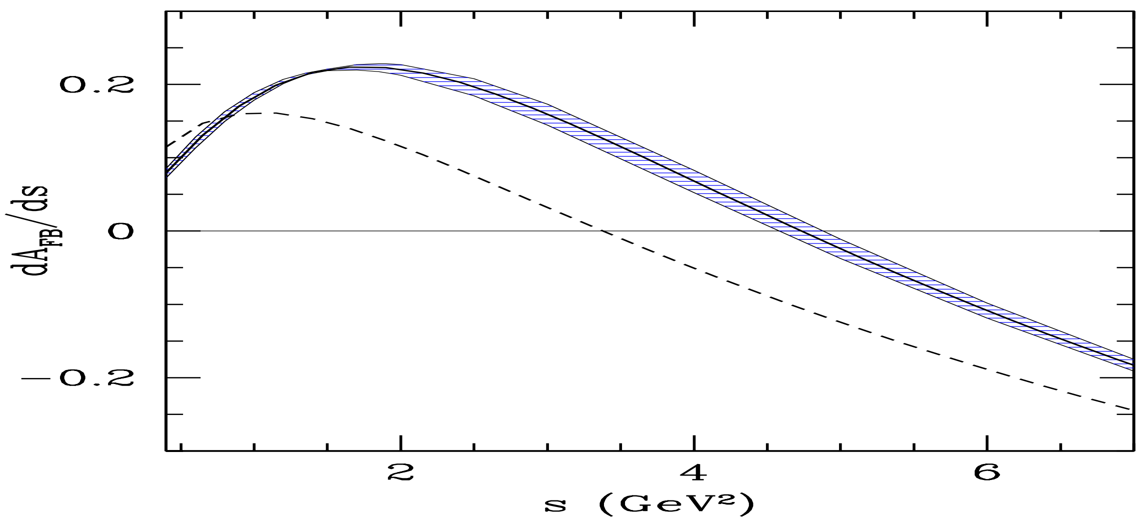
$O(\alpha_s)$ corrections to FB-Asymmetry in $B \to K^* \ell^+ \ell^-$

- $O(\alpha_s)$ corrections to the LEET-symmetry relations lead to substantial perturbative shift in $\hat{s}_0$ [Beneke, Feldmann, Seidel '01]

$$C_9^{eff}(\hat{s}_0) = -\frac{2m_b M_B}{s_0} C_7^{eff}(1 + \frac{\alpha_s C_F}{4\pi}[\ln \frac{m_b^2}{\mu^2} - L] + \frac{\alpha_s C_F}{4\pi} \frac{\Delta F_\perp}{\xi_\perp(s_0)})$$

[AA, A.S. Safir (hep-ph/02054)]

H



Forward-backward asymmetry $dA_{FB}(B \to K^* l^+ l^-)/ds$ at next-to-leading order (solid center line) and leading order (dashed)

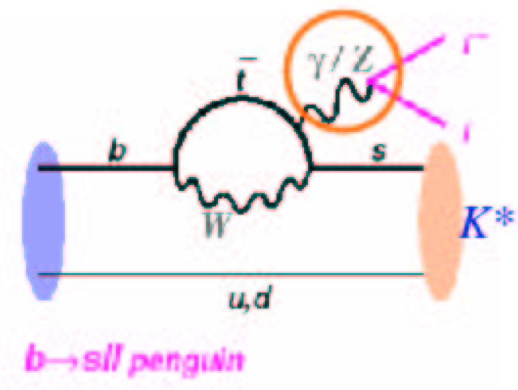
Sakai (FPCP 2004)

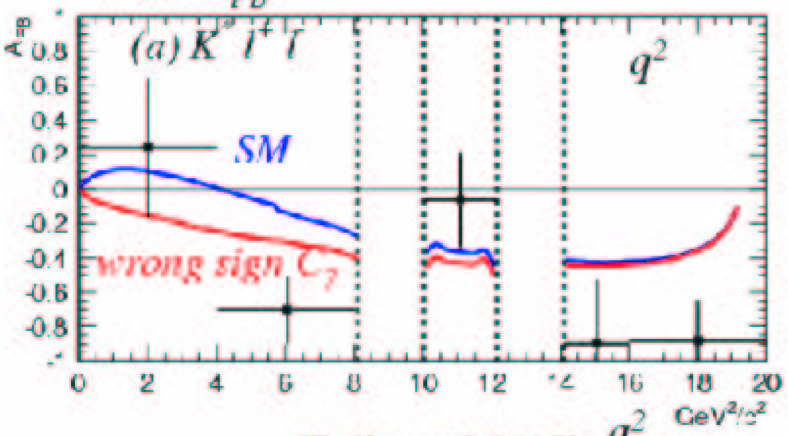
$B \to K^* l^+ l^-$:FB Asymmetry

275M $B\bar{B}$

$A_{FB}(K^*ll)$: very sensitive to NP
that may not be seen in $\mathcal{B}(b \to s\gamma)$

$$A_{FB} = \frac{\Gamma(\theta_{Bl^+} < \pi/2) - \Gamma(\theta_{Bl^+} > \pi/2)}{\Gamma(\theta_{Bl^+} < \pi/2) + \Gamma(\theta_{Bl^+} > \pi/2)}$$





First Look !

[Belle-conf-0415]

ICHEP 2004, Beijing 27

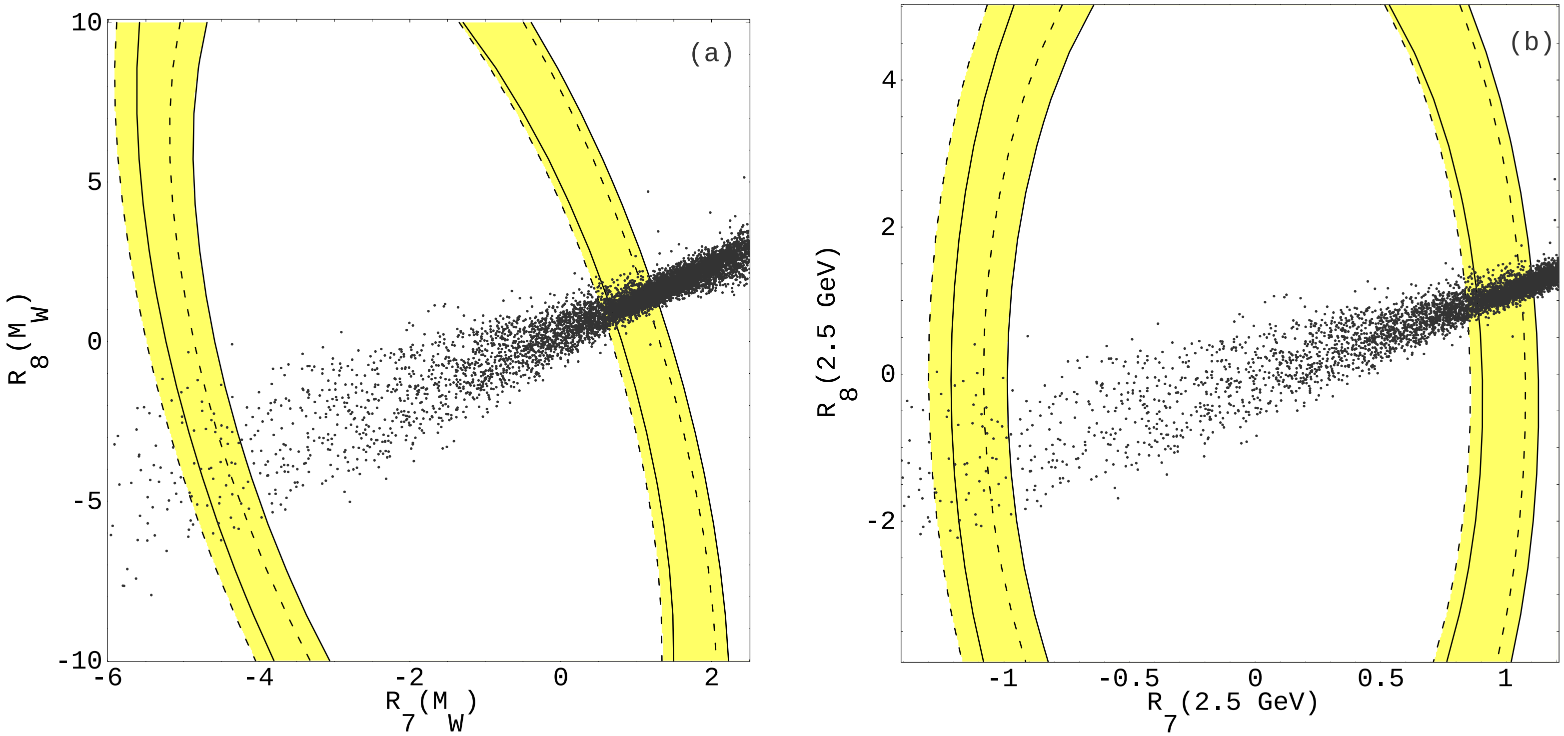
A Model-independent Analysis of $B \to X_s\gamma$ & $B \to X_s\ell^+\ell^-$

- Assume $\mathcal{H}_{eff}^{\rm SM}$ a sufficient operator basis also for Beyond-the-SM physics
- Shifts due to Beyond-the-SM [BSM] physics only in $C_7(\mu_W)$,$C_8(\mu_W)$, $C_9(\mu_W)$, and $C_{10}(\mu_W)$
- BSM Coefficients: $R_7 - 1$, $R_8 - 1$, $C_9^{\rm NP}$, & $C_{10}^{\rm NP}$
- Define: $R_{7,8}(\mu_W) \equiv \frac{C_{7,8}^{\rm tot}(\mu_W)}{C_{7,8}^{\rm SM}(\mu_W)}$
  with $C_{7,8}^{\rm tot}(\mu_W) = C_{7,8}^{\rm SM}(\mu_W) + C_{7,8}^{\rm NP}(\mu_W)$
- Set the scale $\mu_W = M_W$, and use RGE to evolve
  $$R_{7,8}(\mu_W) \to R_{7,8}(\mu_b) = \frac{A_{7,8}^{\rm tot}(\mu_b)}{A_{7,8}^{\rm SM}(\mu_b)}$$
- Impose constraints from $R_7(\mu_b)$ and $R_8(\mu_b)$ from $B \to X_s\gamma$ Data
- Use Data on $B \to (X_s, K^*, K)\ell^+\ell^-$ BRs to constrain $C_9^{\rm NP}$ and $C_{10}^{\rm NP}$
- Two-fold ambiguity due to the sign of C_7^{eff} can be resolved by data on $\boldsymbol{B \to (X_s, K, K^*)\ell^+\ell^-}$

Simulation of $B \to X_s\gamma$ in SUSY-MFV Models

- 90% C.L. bounds in the $[R_7(\mu), R_8(\mu)]$ plane from the $\mathcal{B}(B \to X_s\gamma)$

$\mu = m_W$ (left-hand plot); $\mu = 2.5$ GeV (right-hand plot)

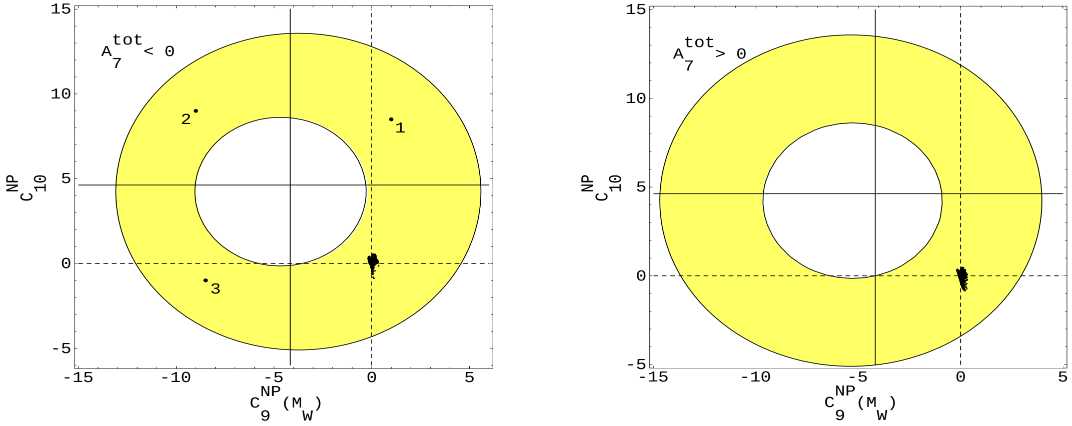


$$A_7^{\rm tot} - {\rm negative}: -0.37 \leq A_7^{{\rm tot},<0}(2.5\ \text{GeV}) \leq -0.17$$
$$A_7^{\rm tot} - {\rm positive}: 0.21 \leq A_7^{{\rm tot},>0}(2.5\ \text{GeV}) \leq 0.43$$

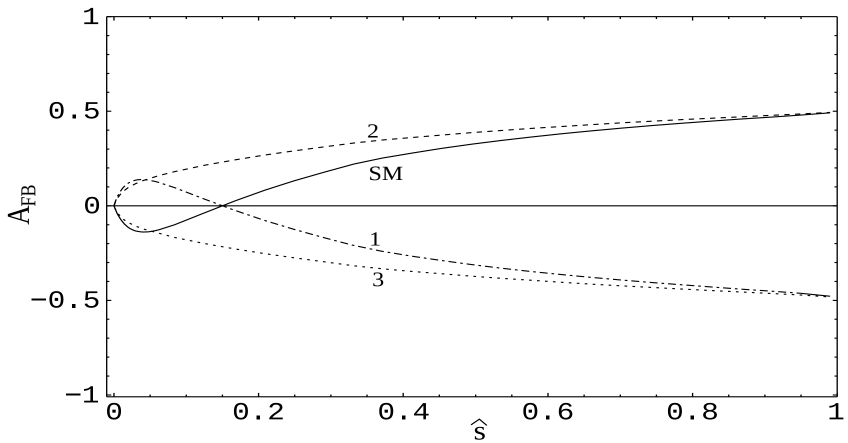
Combined analysis of $B \to X_s\gamma$ & $B \to (X_s, K, K^*)\ell^+\ell^-$

[A.A., Lunghi, Greub, Hiller; DESY 01-217; hep-ph/0112300]

- Constraints from radiative and semileptonic rare decays (Points: SUSY-MFV Model)



- FB asymmetry for $\bar{B} \to X_s\ell^+\ell^-$, corresponding to the points indicated above

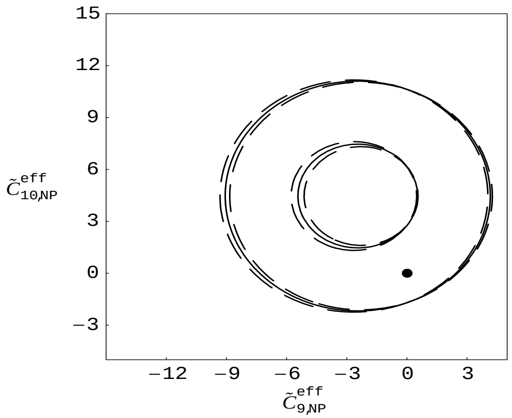


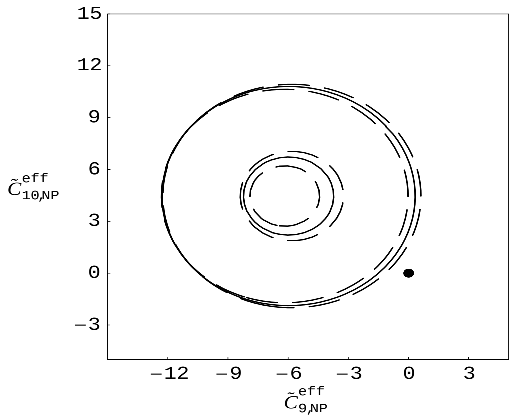
First hints on the sign of the $B \to X_s\gamma$ amplitude

[Gambino, Haisch, Misiak; hep-ph/0410155]

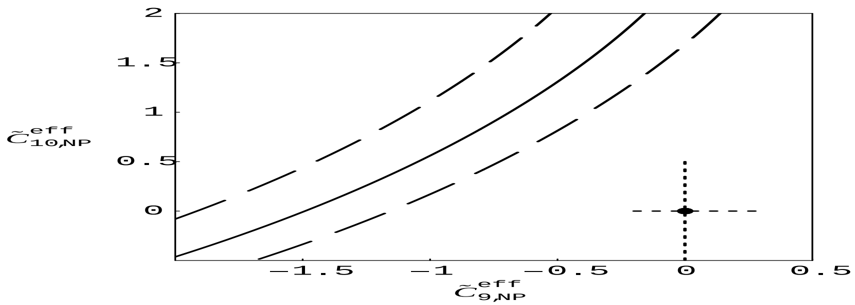
90% C.L. constraints from $\boldsymbol{B \to X_s\gamma}$ and $\boldsymbol{B \to X_s\ell^+\ell^-}$

C_7 SM-like (left frame) C_7 opposite sign (right frame)





Surroundings of the origin in the right frame above; dashed lines: MFV-MSSM



LHC-B MC Studies

Rare Semileptonic Decays at LHCb

Source: P. Koppenburg; LHCb 2002-017

Annual Yields

$B \to \mu^+\mu^- X_s = 24200 \pm 800$	$S/B = 8.1 \pm 2.1$
$B \to \mu^+\mu^- X_d = 550 \pm 30$	$S/B = 1.3^{+0.7}_{-0.9}$
$B^\pm \to \mu^+\mu^- K^\pm = 7750 \pm 600$	$S/B = 7.3 \pm 2.0$
$B^0 \to \mu^+\mu^- K^{*0} = 8600 \pm 300$	$S/B > 15$
$B^\pm \to \mu^+\mu^- \pi^\pm = 310 \pm 20$	$S/B = 1.0^{+0.6}_{-0.7}$
$B^0 \to \mu^+\mu^- \rho^0 = 220 \pm 20$	$S/B > 2.2$

Errors on Physical Constants

$\sigma(\frac{V_{td}}{V_{ts}})/(\frac{V_{td}}{V_{ts}})$	$(11.5^{+2.8}_{-3.2})\%$ **at** $(\lvert V_{ts}\rvert/\lvert V_{td}\rvert)^2 = 30$
$\sigma(A_{\rm CP})(B \to \mu^+\mu^- X_s)$	$(2.3 \pm 0.2)\%$
$\sigma(A_{\rm FB})(B \to \mu^+\mu^- K)$	1.2% **at** $0 \leq \hat{s} \leq 0.32$
$\sigma(\frac{\mathcal{R}(C_{9V}^{\rm eff}(s_0))}{C_7^{\rm eff}})/\frac{\mathcal{R}(C_{9V}^{\rm eff}(s_0))}{C_7^{\rm eff}}$	$(6.0 \pm 0.3)\%$ **at** $\hat{s}_0 = 0.1307$

LHC-B MC Studies

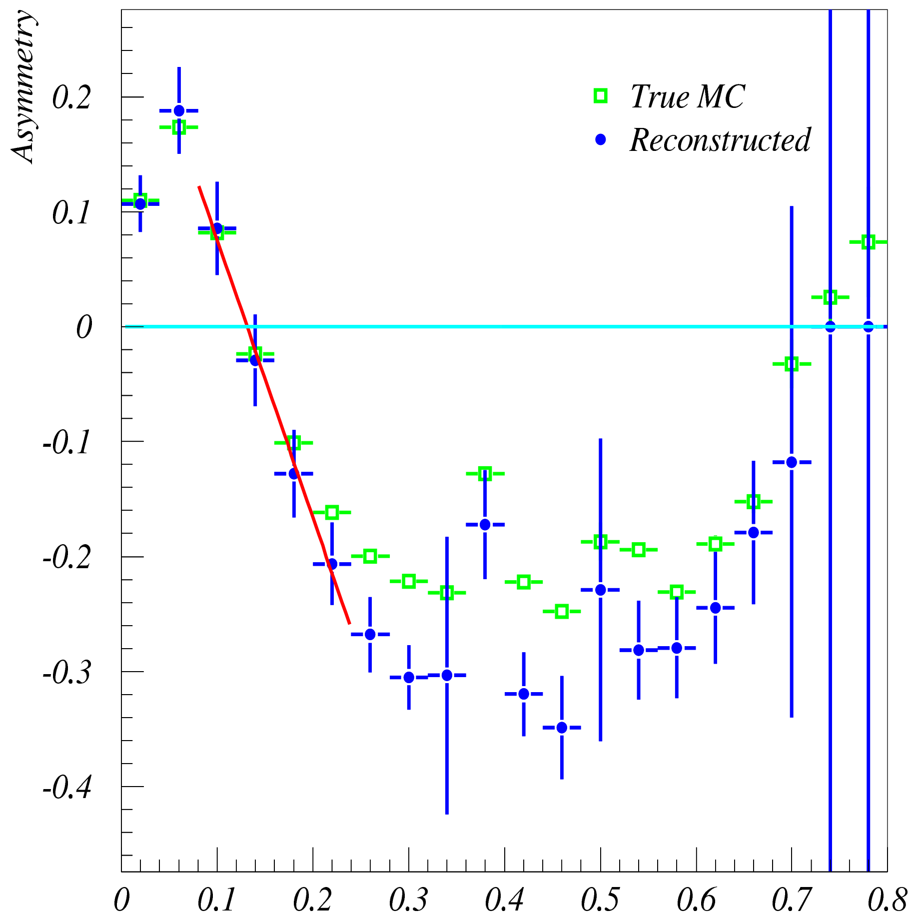


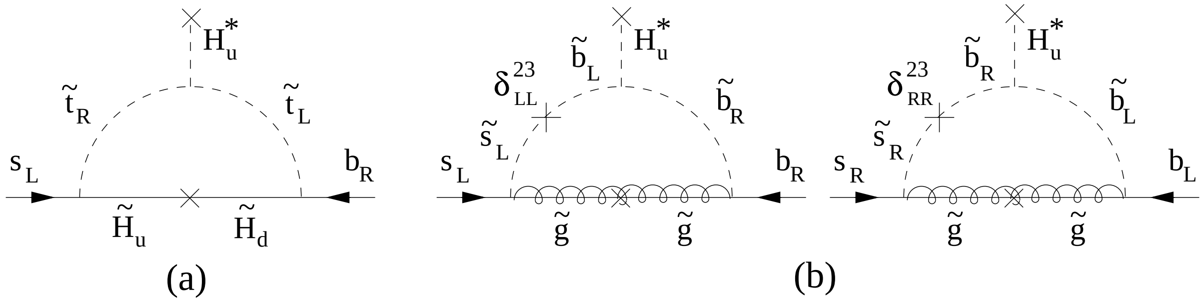
Figure 4: FB Asymmetry versus $\hat{s}$ for $B \xrightarrow{\hat{s}} \mu^+\mu^- K^*$ (from Koppenburg)

$B_s \to \mu^+\mu^-$ in Supersymmetric Models

- The decay $B_s \to \mu^+\mu^-$ probes essentially the Higgs sector of Supersymmetry, a type-II two-Higgs doublet model; One Higgs field (H_u) couples to the up-type quarks, the other (H_d) couples to the down-type quarks

$$\mathcal{L} = \overline{Q} Y_U U_R H_u + \overline{Q}_L Y_D D_R H_d$$

- Supersymmetry does not have discrete symmetries to protect the alignment of the Higgs boson interaction eigenbasis with the fermion mass eigenbasis; Higgs-induced FCNC interactions are generated through loops



- As H_u gets a VEV (v_u), it contributes an off-diagonal piece to the down-type fermion mass matrix, mixing s_L and b_L by an angle θ
$$\sin\theta = y_b \epsilon v_u / m_b; \quad \text{as } m_b = y_b \langle H_d \rangle v_d, \;\; \sin\theta = \epsilon \tan\beta$$

- $\mathcal{A}(b\bar{s} \to \mu^+\mu^-) \simeq \sin\theta \mathcal{A}(b\bar{b} \to \mu^+\mu^-) \;\propto \tan\beta/\cos^2\beta \Longrightarrow \tan^3\beta$ for large-$\tan\beta$

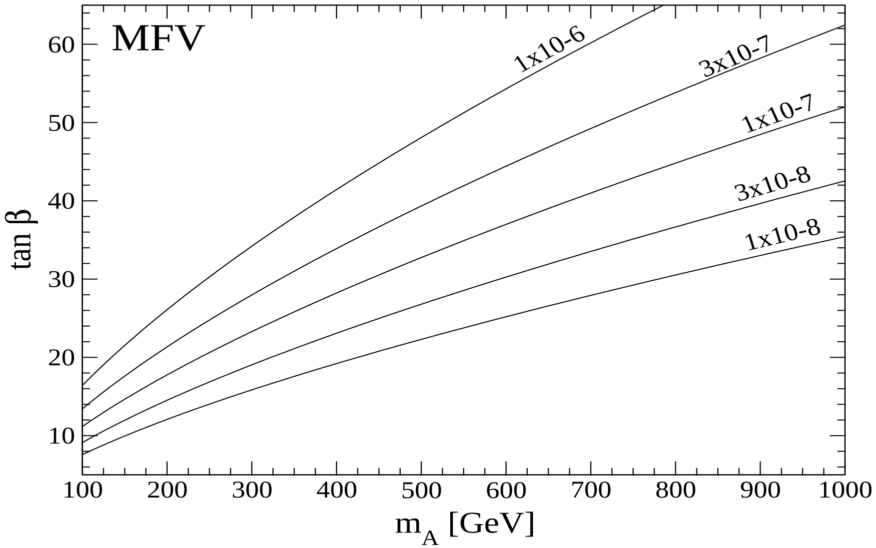
$B_s \to \mu^+\mu^-$ in Minimal Flavor Violation SUSY Models

- Higgsino contribution to $\mathcal{B}(B_s \to \mu^+\mu^-)$ [Babu, Kolda;...]

$$\mathcal{B}(B_s \to \mu\mu) \simeq \frac{G_F^2}{8\pi}\,\eta_{\rm QCD}^2 m_{B_s}^3 f_{B_s}^2 \tau_{B_s} m_b^2\, m_\mu^2 \left(\frac{\tan^2\beta}{\cos^4\beta}\right)\left(\frac{\kappa_{\widetilde{H}}^2}{m_A^4}\right).$$

- $\eta_{\rm QCD} \simeq 1.5$ is the QCD correction due to the RG between the SUSY and B_s scales

$$\kappa_{\widetilde{H}} = -\frac{G_F\, m_t^2\, V_{ts}V_{tb}}{4\sqrt{2}\pi^2 \sin^2\beta}\,\mu A_t\, f(\mu^2, m_{\tilde{t}_L}^2, m_{\tilde{t}_R}^2)$$



Constraints from $BR(B_s \to \mu^+\mu^-)$

$B_s \to \mu\mu$: Physics Reach

D0 $B_s \to \mu^+\mu^-$ result: 240pb^{-1}

$$BF(B_s \to \mu^+\mu^-) < 3.8\times10^{-7}\ 90\%\ \mathrm{CL}$$

CDF $B_{(s,d)} \to \mu^+\mu^-$ results: 171pb^{-1}

$$BF(B_s \to \mu^+\mu^-) < 5.8\times10^{-7}\ 90\%\ \mathrm{CL}$$

$$BF(B_d \to \mu^+\mu^-) < 1.5\times10^{-7}\ 90\%\ \mathrm{CL}$$

Combined: Bayesian approach with a flat prior. Systematic error on fs correlated. Combination by M. Herndon

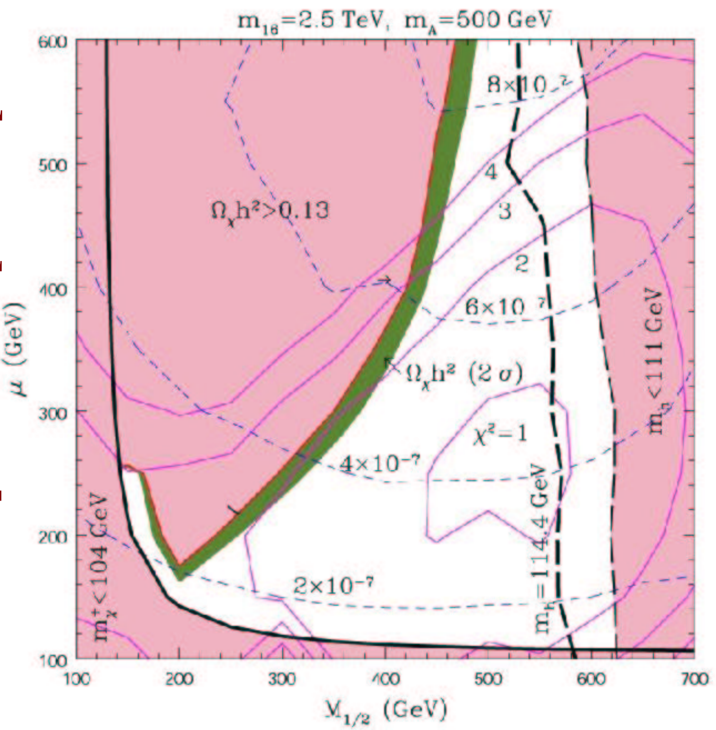
$$BF(B_s \to \mu^+\mu^-) < 2.7\times10^{-7}\ 90\%\ \mathrm{CL}$$

SM predictions

$BF(B_{s(d)} \to \mu^+\mu^-)$ 3.5x10^{-9} (1.0x10^{-10})

- No sensitivity for SM decay rate

- BSM predictions Limiting many models
- Example SUSY S0(10)
  - Allows for massive neutrino
  - Accounts for relic density of cold dark matter



BF $B_s \to \mu^+\mu^-$: Dashed blue

Excludes scenarios where M_A is light and tanβ ~ 50: M_A> 450GeV/c^2

R. Dermisek hep-ph/0304101,2003

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Summary

- Radiative rare B-decays are in agreement with the SM; determine $|V_{ts}|$ and $|V_{td}|$, though not precisely. At LHC, these CKM matrix elements as well as the CPV phases will be measured with greatly improved accuracy
- $B \to X_s\gamma$ and $B \to X_s\ell^+\ell^-$ provide sensitive tests of the SM and SUSY. First measurements of the dilepton mass spectra in $B \to X_s\ell^+\ell^-$ and $B \to (K, K^*)\ell^+\ell^-$ are now available from the B-factories. BELLE has taken a first shot at the FB asymmetry in $B \to K^*\ell^+\ell^-$. Data hints that the signs and magnitudes of the Wilson coeffs. are SM-like.
- Dilepton mass spectra and FB asymmetries will be measured with greatly improved accuracy at the B-factories and LHC, constraining the Wilson coefficients $C_7 - C_{10}$ and providing information on the Flavour structure of SUSY
- Improved upper limit on $BR(B_s \to \mu^+\mu^-)$ from CDF/D0 probes interesting SUSY parameter space. All three Experiments at the LHC (ATLAS, CMS, LHC-B) can reach the SM sensitivity and will test the Higgs sector of SUSY in an independent way
- With some of the supersymmetric particles measured directly at LHC, disentangling the SUSY-flavour structure will become a sharply focussed enterprise. The synergy of the B-factory/LCH data may open a whole new flavour world.