

The golden modes of rare K decays

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- Introduction
- General properties of $K \rightarrow (\ell\ell, \nu\nu) + n\pi$ decays
- Status and perspectives of the four golden modes:

$$K^+ \rightarrow \pi^+ \nu\nu$$

$$K_L \rightarrow \pi^0 e^+e^-$$

$$K_L \rightarrow \pi^0 \nu\nu$$

$$K_L \rightarrow \pi^0 \mu^+\mu^-$$

- Some comments on New Physics contributions

• Introduction

Several arguments/observations suggest that the SM is an **effective theory** or the low-energy limit of a more fundamental theory with new degrees of freedom appearing above some energy threshold $\Lambda \geq v \sim 250 \text{ GeV}$



$$\mathcal{L} = \mathcal{L}_{\text{gauge}}(A_i, \psi_i) + \mathcal{L}_{\text{Higgs}}(\phi_i, A_i, \psi_i; Y, v) + \sum \frac{c_n}{\Lambda^{d-4}} \mathcal{O}_n^{(d \geq 5)}$$

Key questions:

- How large can Λ be?
- Which is the nature (\Leftrightarrow symmetries) of the new degrees of freedom?

general parameterization of the possible new heavy d.o.f. valid as long as we perform low-energy experiments

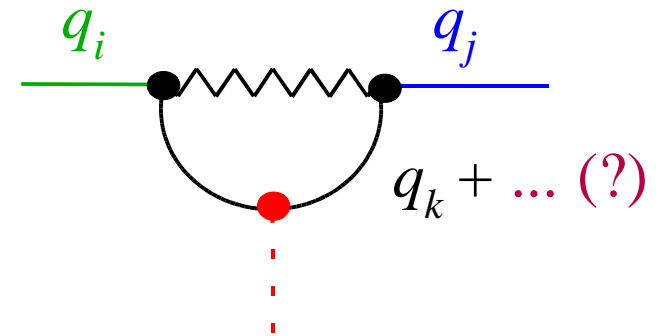
Flavour physics - and particularly precision studies of rare decays - provides a key ingredient to answer these questions

Precision studies of rare decays can (slightly) help to improve our knowledge about the SM Yukawa interaction but their main interest is in probing the flavour structure of new physics:



Rare processes mediated by Flavor Changing Neutral Currents are the ideal candidates

$$q_i \rightarrow q_j + \gamma, l^+l^-, \nu\nu$$



- no SM tree-level contribution
- strong suppression within the SM by CKM hierarchy
- calculable with high precision within the SM if dominated by short-distance dynamics [*key point*]



precise determination of flavor
mixing within the SM



enhanced sensitivity to
[*the flavour structure of*]
physics beyond the SM

Towards a model independent approach to the flavour problem:

Anatomy of a typical $O_i^{(6)}$ relevant to FCNC rare decays:

$$Q_\gamma^{bs} = W_\gamma^{bs} \underbrace{D_R^b \sigma_{\mu\nu} F^{\mu\nu} H Q_L^s}_{\text{flavour-blind electroweak structure}} \sim m_b b_R \sigma_{\mu\nu} F^{\mu\nu} s_L$$

flavour coupling

e.g.: $W_\gamma^{bs} \sim y_b y_t^2 V_{tb}^* V_{ts}$
for the SM short-distance contr.

The most restrictive choice is the so-called **MFV** hypothesis

[= same CKM / Yukawa suppression as in the SM]

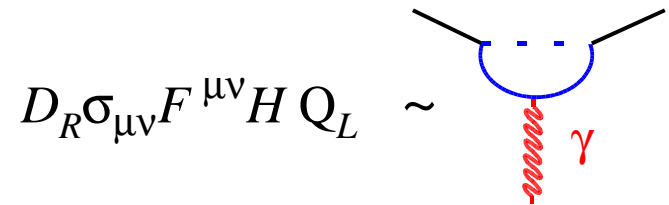
it cannot be worse than this without serious fine-tuning problems

[Chivukula & Georgi, '86; Buras *et al.* '00; D'Ambrosio, Giudice, G.I., Strumia '02]

flavour-blind electroweak structure

Limited number of independent terms once we impose $SU(3)_c \times SU(2)_L \times U(1)_Y$ gauge invariance

closely related to specific loop topologies, e.g.:



Towards a model independent approach to the flavour problem:

FLAVOUR COUPLING:

ELECTROWEAK STRUCTURE

	$b \rightarrow s (\sim\lambda^2)$	$b \rightarrow d (\sim\lambda^3)$	$s \rightarrow d (\sim\lambda^5)$
$\Delta F=2$ box	$(Q_L^b \Gamma Q_L^s)^2$...	
$\Delta F=1$ 4-quark box	⋮		
gluon penguin			
γ penguin			
Z^0 penguin			
H^0 penguin			

The FCNC matrix:

each box correspond to an indep. combination of dim.-6 $SU(3) \times SU(2) \times U(1)$ -invariant operators

Towards a model independent approach to the flavour problem:

- th. error $\lesssim 10\%$
- = exp. error $\lesssim 10\%$
- = exp. error $\sim 30\%$

FLAVOUR COUPLING:

ELECTROWEAK STRUCTURE

	$b \rightarrow s (\sim \lambda^2)$	$b \rightarrow d (\sim \lambda^3)$	$s \rightarrow d (\sim \lambda^5)$
$\Delta F=2$ box	ΔM_{B_s} $A_{CP}(B_s \rightarrow \psi \phi)$	ΔM_{B_d} $A_{CP}(B_d \rightarrow \psi K)$	$\Delta M_K, \epsilon_K$
$\Delta F=1$ 4-quark box	$B_d \rightarrow \phi K, B_d \rightarrow K\pi, \dots$	$B_d \rightarrow \pi\pi, B_d \rightarrow \rho\pi, \dots$	$\epsilon'/\epsilon, K \rightarrow 3\pi, \dots$
gluon penguin	$B_d \rightarrow X_s \gamma, B_d \rightarrow \phi K,$ $B_d \rightarrow K\pi, \dots$	$B_d \rightarrow X_d \gamma, B_d \rightarrow \pi\pi, \dots$	$\epsilon'/\epsilon, K_L \rightarrow \pi^0 l^+ l^-, \dots$
γ penguin	$B_d \rightarrow X_s l^+ l^-, B_d \rightarrow X_s \gamma,$ $B_d \rightarrow \phi K, B_d \rightarrow K\pi, \dots$	$B_d \rightarrow X_d l^+ l^-, B_d \rightarrow X_d \gamma,$ $B_d \rightarrow \pi\pi, \dots$	$\epsilon'/\epsilon, K_L \rightarrow \pi^0 l^+ l^-, \dots$
Z^0 penguin	$B_d \rightarrow X_s l^+ l^-, B_s \rightarrow \mu\mu,$ $B_d \rightarrow \phi K, B_d \rightarrow K\pi, \dots$	$B_d \rightarrow X_d l^+ l^-, B_d \rightarrow \mu\mu,$ $B_d \rightarrow \pi\pi, \dots$	$\epsilon'/\epsilon, K_L \rightarrow \pi^0 l^+ l^-,$ $K \rightarrow \pi\nu\nu, K \rightarrow \mu\mu, \dots$
H^0 penguin	$B_s \rightarrow \mu\mu$	$B_d \rightarrow \mu\mu$	$K_{L,S} \rightarrow \mu\mu$

- General properties of $K \rightarrow (ll, \nu\nu) + n\pi$ decays

I. Clean electroweak short-distance amplitude

[similar -within the SM- for all the channels]

II. Long-distance amplitude of e.m. origin

[$K \rightarrow ll + n\pi$ modes only]

I. The clean electroweak short-distance amplitude

Electroweak penguins and box diagrams determine the *initial conditions* of the effective FCNC Hamiltonian:

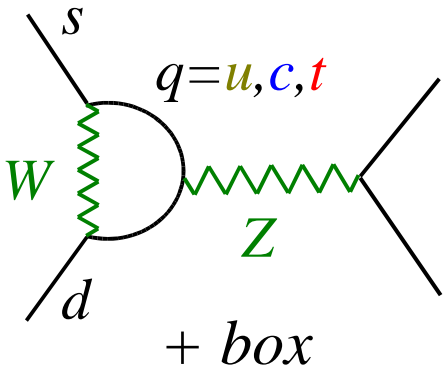
$$\mathcal{H}_{\text{eff}} = \sum_i C_i(M_W) Q_i$$

$$Q_v = (sd)_{V-A} (\nu\nu)_{V-A}$$

$$Q_{9V} = (sd)_{V-A} (ll)_V$$

$$Q_{10A} = (sd)_{V-A} (ll)_A$$

Thanks to the "hard" GIM mechanism, Z-peng. and box diagrams give rise to a scale-independent amplitude which is dominated by the top-quark exchange:



$$\Rightarrow C_i(M_W) \sim m_q^2 \underbrace{V_{qs}^* V_{qd}}_{\lambda_q} \sim \begin{cases} \Lambda_{\text{QCD}}^2 \lambda & (u) \\ m_c^2 \lambda + i m_c^2 \lambda^5 & (c) \\ m_t^2 \lambda^5 + i m_t^2 \lambda^5 & (t) \end{cases}$$

- QCD corr. small and known beyond LO
- large CPV-phase

$$[\lambda = \sin \theta_c]$$

I. The clean electroweak short-distance amplitude

Electroweak penguins and box diagrams determine the *initial conditions* of the effective FCNC Hamiltonian:

$$\mathcal{H}_{\text{eff}} = \sum_i C_i(M_W) Q_i$$

$$Q_\nu = (sd)_{V-A} (\nu\nu)_{V-A}$$

$$Q_{9V} = (sd)_{V-A} (ll)_V$$

$$Q_{10A} = (sd)_{V-A} (ll)_A$$



- Hadronic matrix element: $\langle \pi | (sd)_{V-A} | K \rangle$
known (from K_{l3}) with excellent accuracy
- Lepton pair in a CP eigenstate: the contrib. of \mathcal{H}_{eff} to $K_L \rightarrow \pi^0 + ll (\nu\nu)$ is CPV

I. The clean electroweak short-distance amplitude

Electroweak penguins and box diagrams determine the *initial conditions* of effective FCNC Hamiltonian:

$$\mathcal{H}_{\text{eff}} = \sum_i C_i(M_W) Q_i$$

QCD corrections
below the e.w. scale

[RGE]

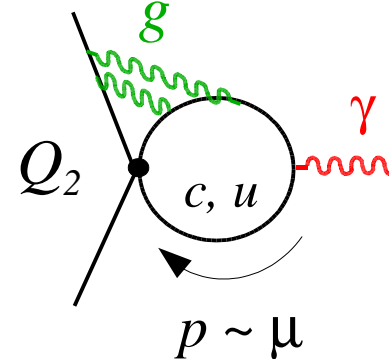
mixing with
4-q operators:

$$\mathcal{H}_{\text{eff}} = \sum_i C_i(\mu \sim 1 \text{ GeV}) Q_i$$

$$Q_V = (sd)_{V-A} (\nu\nu)_{V-A}$$

$$Q_{9V} = (sd)_{V-A} (ll)_V$$

$$Q_{10A} = (sd)_{V-A} (ll)_A$$



large effect in
CPC γ -penguin
amplitudes

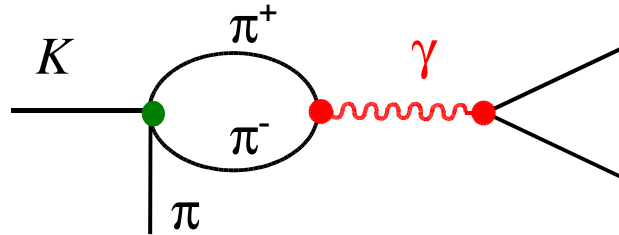
- Negligible corrections for $\text{Im}(C_V)$ & $\text{Im}(C_{10A})$ $K_L \rightarrow \pi^0 \nu \nu$
- Small & calculable [*charm loops*] for $\text{Im}(C_{9V})$ $K_L \rightarrow \pi^0 e^+ e^-$
- Small & calc. [*charm loops*] for $\text{Re}(C_V)$ & $\text{Re}(C_{10A})$ $K^+ \rightarrow \pi^+ \nu \nu, K_L \rightarrow \mu^+ \mu^-$
- Huge and not stable [*long distance*] for $\text{Re}(C_{9V})$ $K^+ \rightarrow \pi^+ e^+ e^-$

II. The e.m. long-distance amplitude in $K \rightarrow (\pi) ll$ modes

Qualitative picture:

["XL": 1γ] $K^\pm (K_S) \rightarrow \pi^\pm (\pi^0) l^+ l^-$

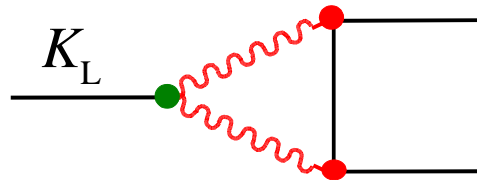
Hopeless to disentangle
short-distance effects



$$A_{\text{short}}/A_{\text{long}} \sim 10^{-2}$$

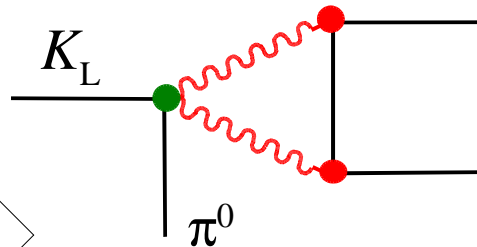
["L": $2\gamma, J=0$] $K_L \rightarrow \mu^+ \mu^-$

Possible to obtain significant
constraints on realistic
(but non-MFV) NP models



$$A_{\text{short}}/A_{\text{long}} \sim 1$$

["S": $2\gamma, J=2$] $K_L \rightarrow \pi^0 e^+ e^-$



$$A_{\text{short}}/A_{\text{long}} < 1$$

Possible to perform precision
tests of short-distance dynamics

NEW!

Quantitative analysis possible
using low-energy EFT approaches

- Status and perspectives of the four golden modes

Neutrino modes:

- No leading long distance contributions [only Z-penguin & W-box \Rightarrow hard GIM suppression effective also for the leading l.d. terms]
- Dominant uncertainty from the perturbative charm contribution [NNLO corr.] + subleading long distance terms [power-suppressed higher-dim. operators]

$K^+ \rightarrow \pi^+ \nu \bar{\nu}$

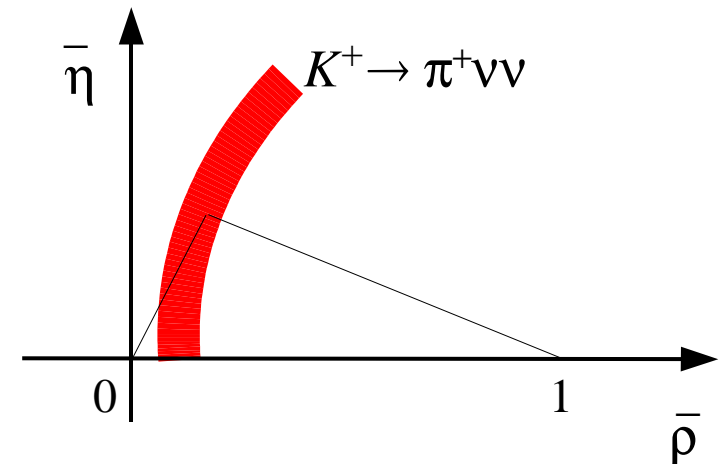
Buchalla & Buras '97-'99
Lu & Wise '94
Falk *et al.* '00

large fraction of the present error still due to parametric CKM uncertainties

$$\text{BR}(K^+)^{[\text{SM}]} = C |V_{cb}|^4 [(\bar{\rho} - \rho_c)^2 + (\sigma \bar{\eta})^2] = (8.0 \pm 1.0) \times 10^{-11}$$

$\rho_c = 1.40 \pm 0.06 \Rightarrow \delta \text{BR}_{\text{th}} \approx 8\%$

On-going theoretical activity to reduce $\delta \text{BR}_{\text{th}}$ below the 5% level: [Munich](#), [Frascati](#)



- Status and perspectives of the four golden modes

Neutrino modes:

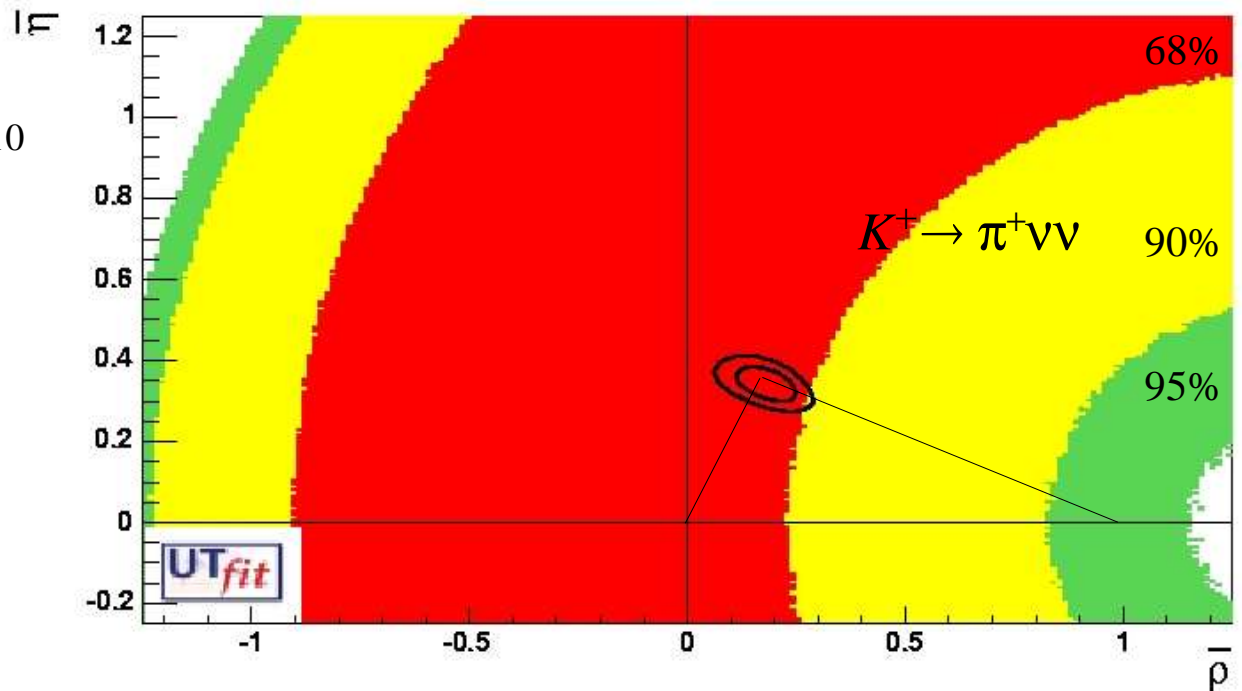
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$$\text{BR}(K^+)^{\text{exp}} = (1.47^{+1.9}_{-0.9}) \times 10^{-10}$$

E787+E949 [BNL] '04



• Status and perspectives of the four golden modes

Neutrino modes:

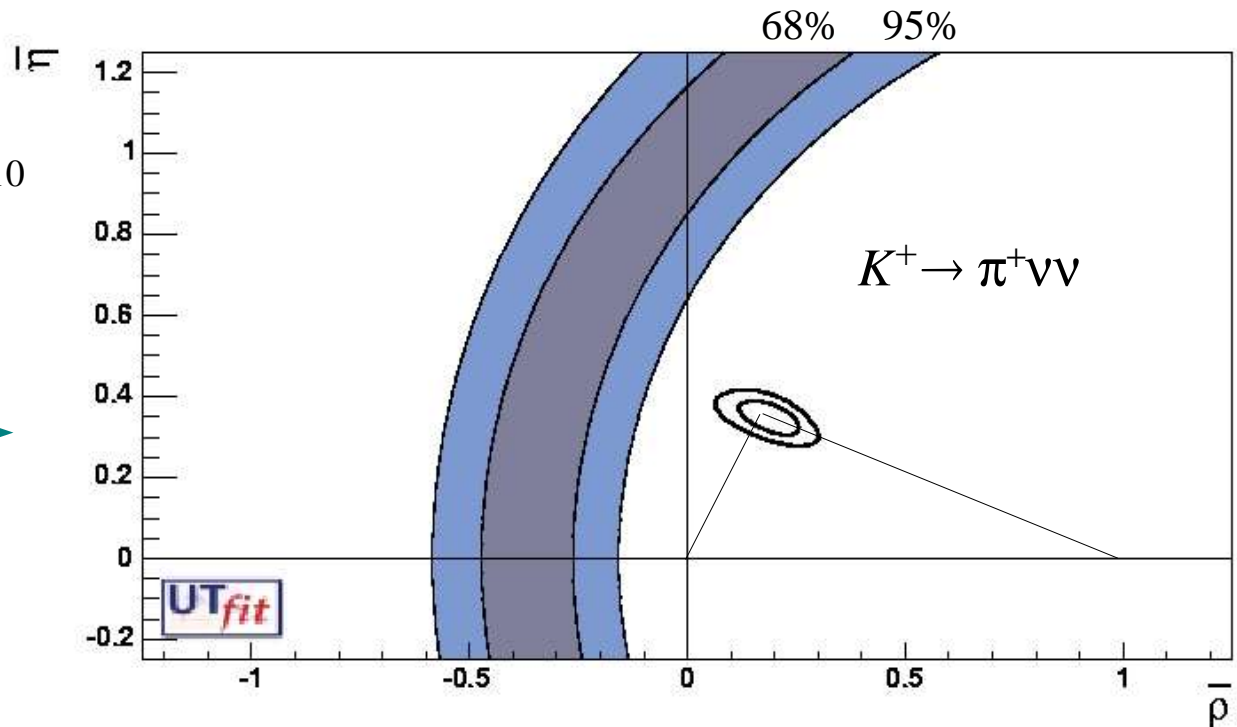
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$K^+ \rightarrow \pi^+ \nu \bar{\nu}$ $\text{BR}(K^+)^{[\text{SM}]} = C |V_{cb}|^4 [(\bar{\rho} - \rho_c)^2 + (\sigma \bar{\eta})^2] = (8.0 \pm 1.0) \times 10^{-11}$

if we could decrease the error at the 10% level...

$\text{BR}(K^+)^{\text{exp}} = (1.47_{-0.9}^{+1.9}) \times 10^{-10}$
 \downarrow
 $\pm 10\%$ \rightarrow

The situation could become quite interesting...



- Status and perspectives of the four golden modes

Neutrino modes:

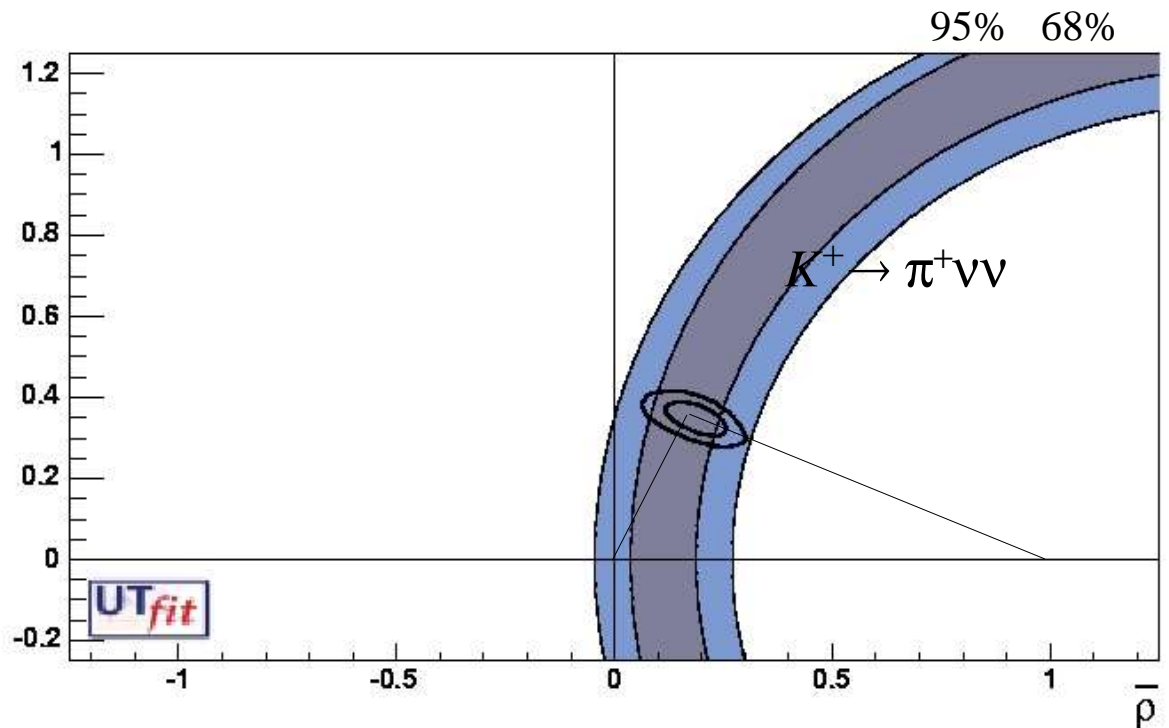
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$$K^+ \rightarrow \pi^+ \nu \bar{\nu}$$

$$\text{BR}(K^+)^{[\text{SM}]} = C |V_{cb}|^4 [(\bar{\rho} - \rho_c)^2 + (\sigma \bar{\eta})^2] = (8.0 \pm 1.0) \times 10^{-11}$$

N.B.: plotting the $\text{BR}(K^+)^{\text{exp}}$ contour in the ρ - η plane is only a fast way to compare it with the SM prediction:

the **main interest** of such measurement is not a more precise determination of V_{td} but the extraction of a key **information about NP**



- Status and perspectives of the four golden modes

$$K_L \rightarrow \pi^0 \nu\nu$$

CPV transition if the lepton pair is in $J^{CP} = 1^{--}, 1^{++}$ [leading dim.-6 operators]
 \Rightarrow charm & long-distance effects totally negligible

Littenberg, '89
 Buchalla & Buras '97
 Buchalla & G.I. '98

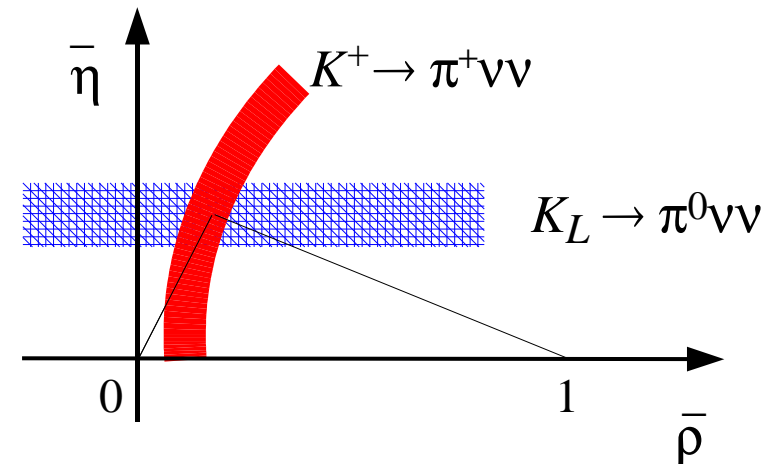
$$\text{BR}(K_L)^{[\text{SM}]} = 1.48 \times 10^{-11} \left[\frac{m_t(m_t)}{166 \text{ GeV}} \right]^{2.3} \left[\frac{\text{Im}(V_{ts}^* V_{td})}{10^{-4}} \right]^2 = (3.0 \pm 0.6) \times 10^{-11}$$

th. error ~ 2% !

control the amount
of CPV within the SM

Best exp. bound [KTeV '99]
still very far from the SM level:

$$B(K_L \rightarrow \pi^0 \nu\nu) < 5.9 \times 10^{-7} \quad [\text{using } \pi^0 \rightarrow \gamma e^+ e^-]$$

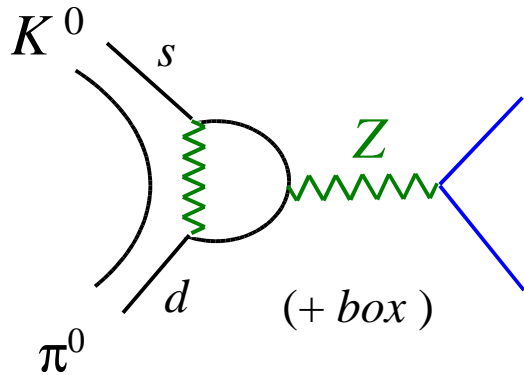


$$K_L \rightarrow \pi^0 l^+ l^-$$

The 3 components of the $K_L \rightarrow \pi^0 l^+ l^-$ amplitude:

A. direct CPV amplitude

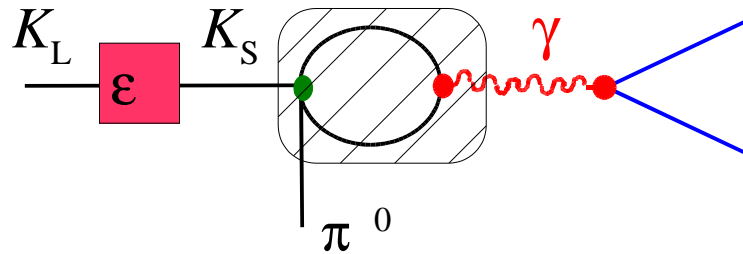
- short-distance dominated
- very similar to $K_L \rightarrow \pi^0 \nu \nu$



\longleftrightarrow
interference

B. indirect CPV

- determined by $K_S \rightarrow \pi^0 l^+ l^-$
- + theory to fix the sign

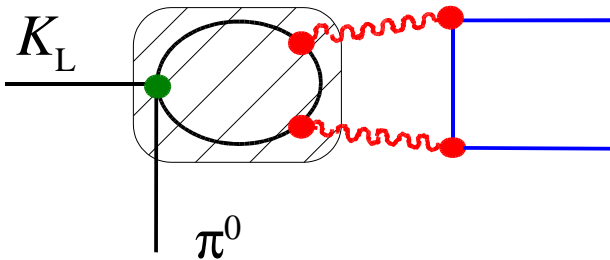


need exp. input

C. CPC amplitude

- no interference & different Dalitz plot
- predicted by theory with good accuracy

in terms of rate & spectrum of $K_L \rightarrow \pi^0 \gamma \gamma$



need exp. input

$$K_L \rightarrow \pi^0 l^+ l^-$$

Thanks to some recent results by NA48-NA48/1:

$$B(K_S \rightarrow \pi^0 e^+ e^-)_{m_{ee} > 165 \text{ MeV}} = (3.0_{-1.2}^{+1.5} \pm 0.2) \times 10^{-9}$$

$$B(K_S \rightarrow \pi^0 \mu^+ \mu^-) = (2.9_{-1.2}^{+1.4} \pm 0.2) \times 10^{-9}$$

$$B(K_L \rightarrow \pi^0 \gamma\gamma)_{m_{\gamma\gamma} < 110 \text{ MeV}} < 0.9 \times 10^{-8}$$

+

Some related th. works:

Buchalla, D'Ambrosio, G.I. '03
G.I., Smith, Unterdorfer '04
Friot, Grenat, de Rafael '04

We finally have a clear picture of the various terms:

$$B(K_L \rightarrow \pi^0 l^+ l^-)^{[SM]} = [C_{\text{mix}} + C_{\text{int}} y_t + C_{\text{dir}} y_t^2 + C_{\text{CPC}}] \times 10^{-12} \quad y_t = \frac{\text{Im}(V_{ts}^* V_{td})}{10^{-4}}$$

$$(e^+ e^-) \approx 23 + (10 + 4) + 0 \Rightarrow (3.7 \pm 1.0) \times 10^{-11}$$

$$(\mu^+ \mu^-) \approx 5.4 + (2.5 + 1.8) + 5.2 \Rightarrow (1.5 \pm 0.3) \times 10^{-11}$$

$$B(K_L \rightarrow \pi^0 e^+ e^-)^{[\text{SM}]} = (3.7 \pm 1.0) \times 10^{-11} \quad [\approx 50\% \text{ due to short dist.}]$$

$$B(K_L \rightarrow \pi^0 \mu^+ \mu^-)^{[\text{SM}]} = (1.5 \pm 0.3) \times 10^{-11} \quad [\approx 30\% \text{ due to short dist.}]$$

Errors on SM predictions dominated by the large (exp.) uncertainty on $B(K_S \rightarrow \pi^0 l^+ l^-)$, but irreducible theoretical error below 10%



$$B(K_L \rightarrow \pi^0 e^+ e^-)^{\text{exp}} < 2.8 \times 10^{-10} \quad [90\% \text{ CL}] \quad \text{KTeV '03}$$

$$B(K_L \rightarrow \pi^0 \mu^+ \mu^-)^{\text{exp}} < 3.8 \times 10^{-10} \quad [90\% \text{ CL}] \quad \text{KTeV '00} \quad \text{not too far...}$$



Very interesting candidates for future dedicated experiments

- More observables to be studied [Dalitz plot, time-dependent distrib.]
- Different sensitivity to NP with respect to $K_L \rightarrow \pi^0 \nu \nu$

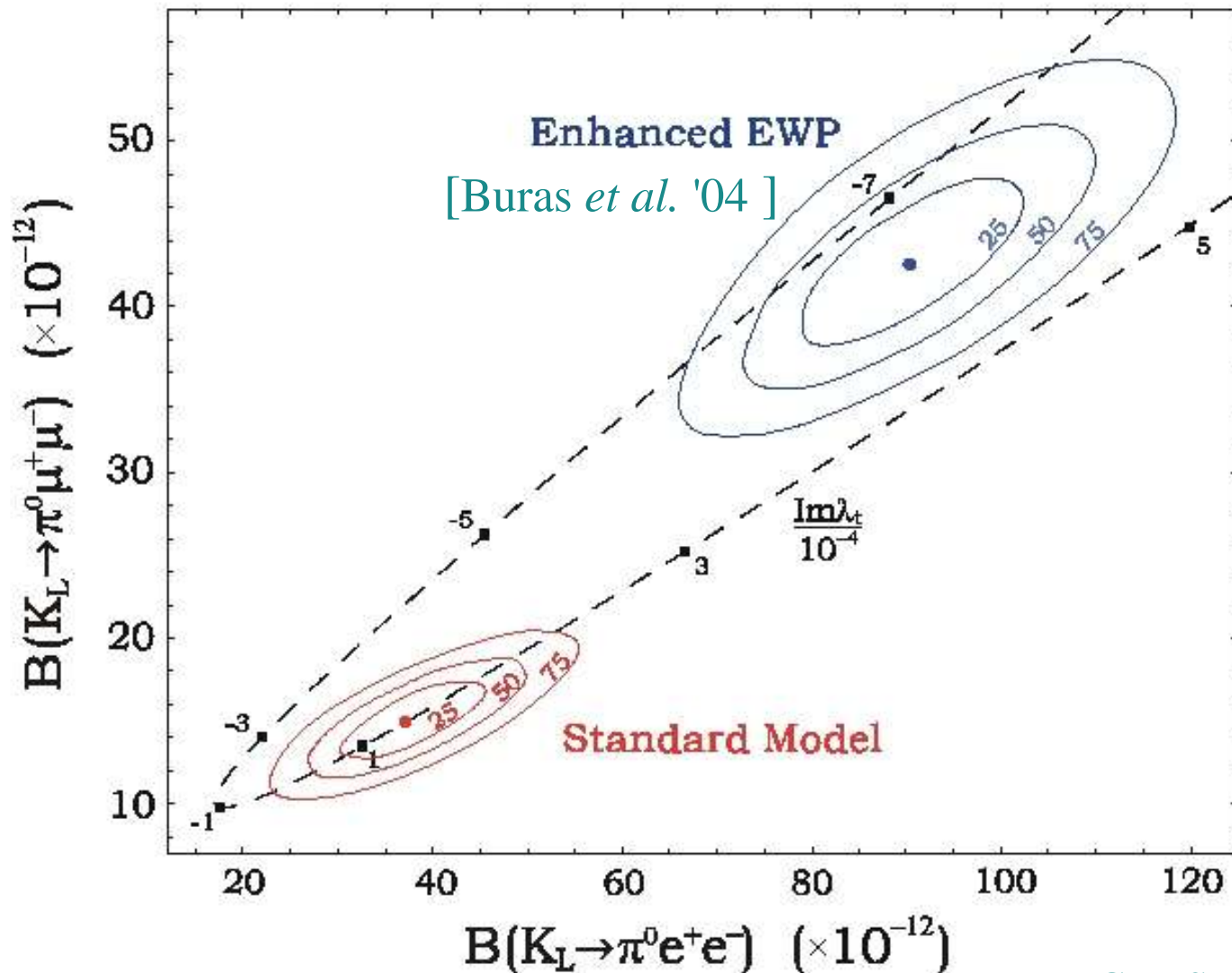
the 3 decay modes $K_L \rightarrow \pi^0 + e^+ e^-, \mu^+ \mu^-, \nu \nu$
 are sensitive to different short-distance structures
 \Rightarrow **3 independent info** on CPV beyond the SM

$$\begin{aligned} Q_\nu &= (\text{sd})_{V-A} (\nu \nu)_{V-A} \\ Q_{9V} &= (\text{sd})_{V-A} (ll)_V \\ Q_{10A} &= (\text{sd})_{V-A} (ll)_A \end{aligned}$$

Discriminating power of the combined measurements

$$B(K_L \rightarrow \pi^0 e^+ e^-) + B(K_L \rightarrow \pi^0 \mu^+ \mu^-)$$

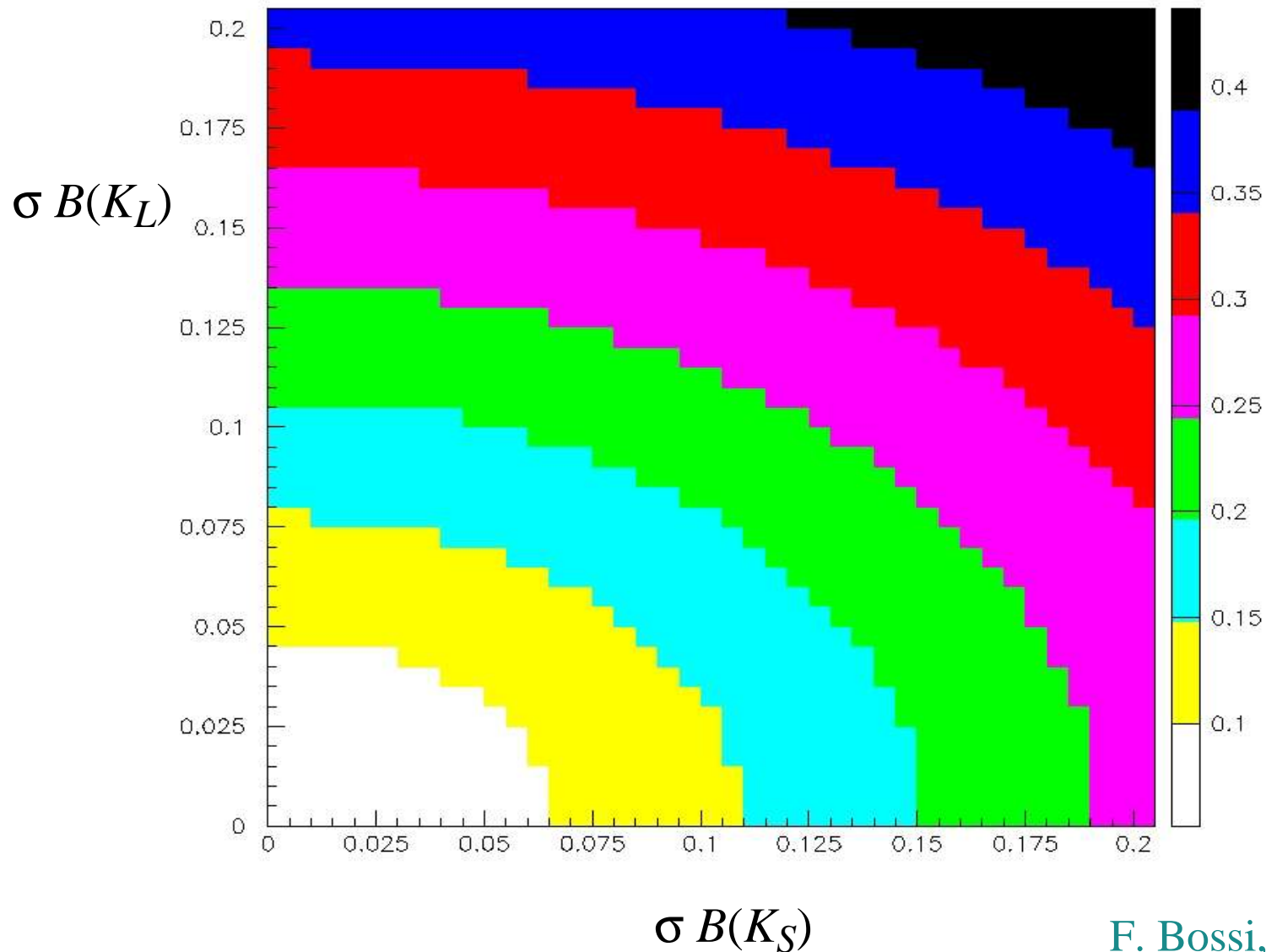
with respect to non-SM scenarios:



Relative error on $\text{Im}(V_{ts}^* V_{td})$

vs.

relative exp. errors on $B(K_S \rightarrow \pi^0 e^+ e^-)$ & $B(K_L \rightarrow \pi^0 e^+ e^-)$



F. Bossi, V. Patera [KLOE]
& G.I., work in prog.

- Rare FCNC decays beyond the SM

Natural solution of the flavour (+hierarchy) problem:

$\Lambda \sim 1 \text{ TeV}$ + flavor-mixing protected by additional symmetries

As long as we are interested only in low-energy rare processes, the most important feature of the NP model is the nature of this symmetry

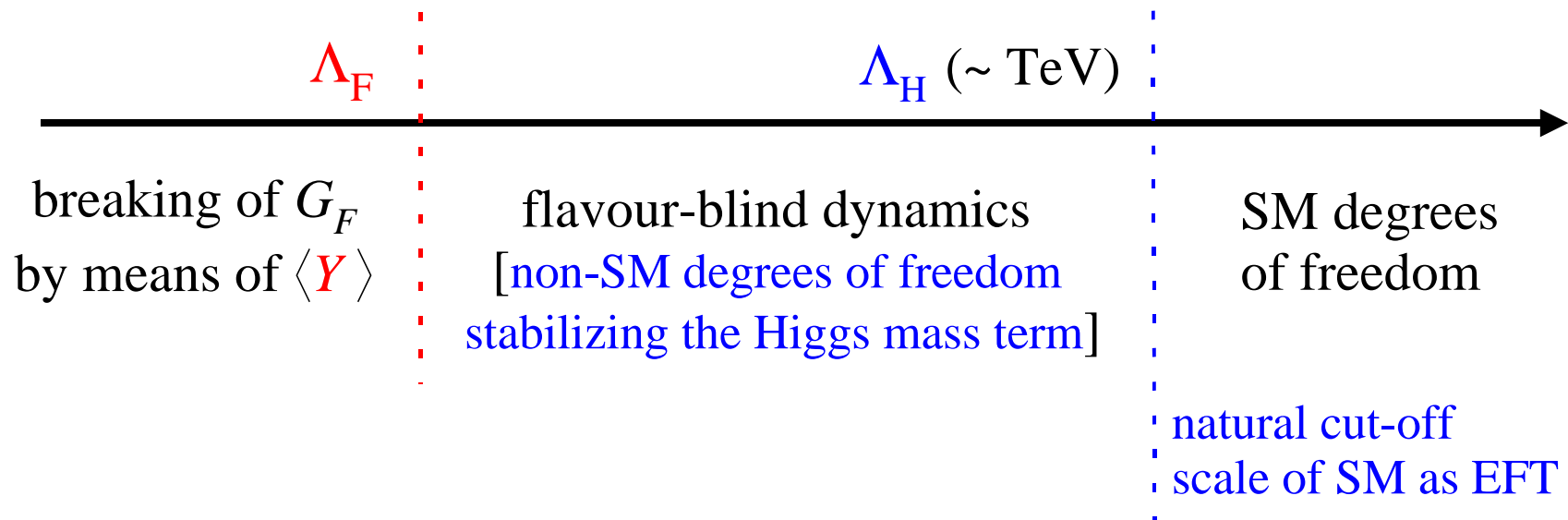
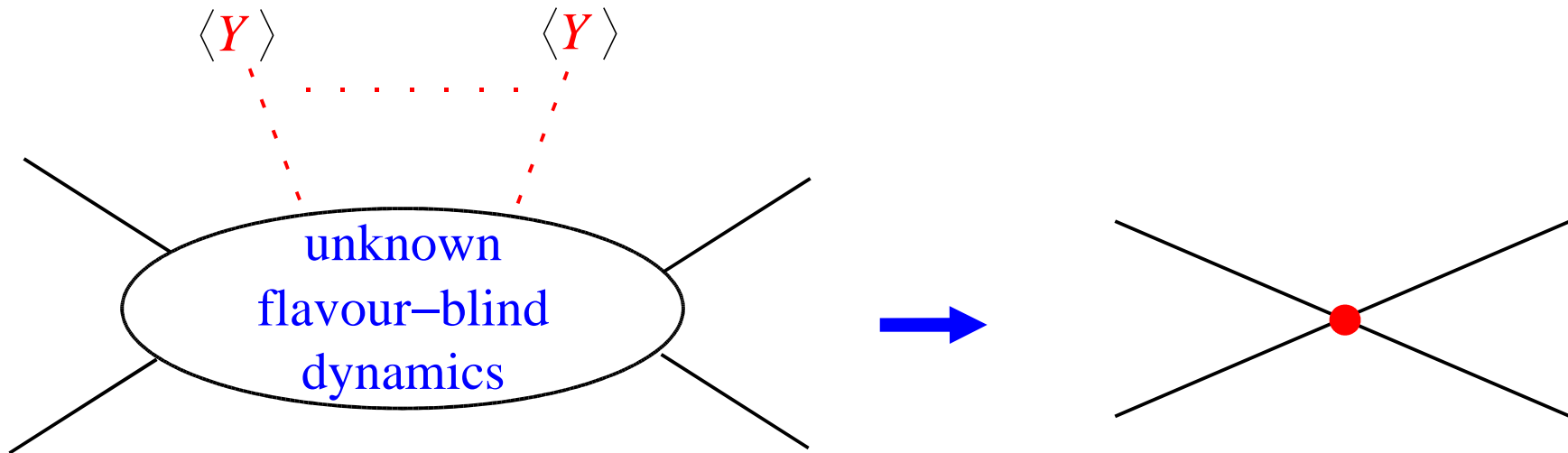


most restrictive possibility

Minimal Flavour Violation (MFV) hypothesis:

The breaking of the flavour symmetry occurs at very high scales and is mediated at low energies only by terms prop. to SM Yukawa couplings

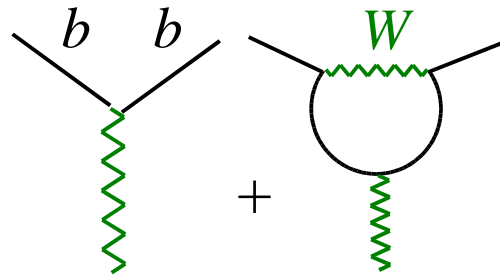
- natural implementation in many consistent scenarios [SUSY, technicolour, extra dimensions,...]
- possible to build a predictive low-energy EFT
model-independent approach



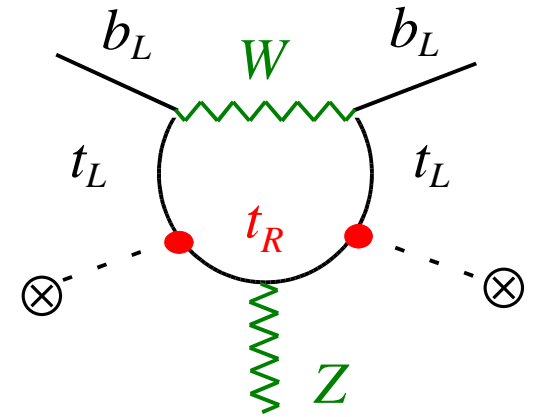
The MFV hypothesis can be considered as the most pessimistic scenario:
 \Rightarrow deviations from the SM in FCNCs bounded by flavour-conserving e.w. precision observables

E.g.: Z-penguins & R_b

$$R_b = \frac{\Gamma(Z \rightarrow bb)}{\Gamma(Z \rightarrow \text{had})}$$



breaking of
universality
→
driven by



$$R_b = R_d \left[1 - \boxed{G_f m_t^2 / 2\pi^2 \sqrt{2}} + \dots \right] \approx 0.2200 - 0.0024 + \dots$$

$R_b^{\text{exp}} = 0.2163 \pm 0.0007$

↑
tree + flavour univ. terms

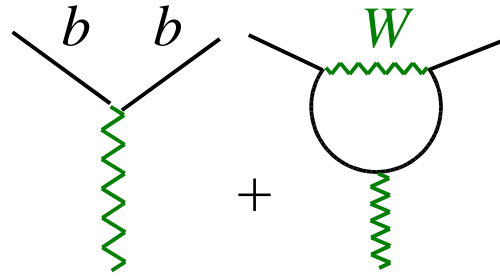
↓
same e.w.-Yukawa
structure of the leading
 $K \rightarrow \pi \nu \nu$ amplitude

The $O(10^{-3})$ accuracy on R_b of LEP let us to probe the genuine e.w.-Yukawa loop amplitude only at the 20-30% level

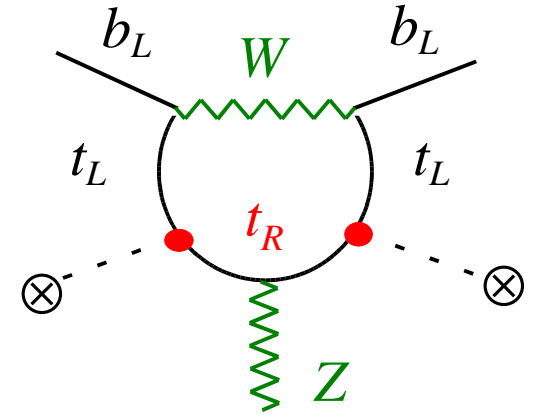
A 10% measurement of $B(K \rightarrow \pi \nu \nu)$ [or $B \rightarrow \pi \nu \nu$] would probe the same e.w.-Yukawa structure (assuming MFV) at the 6-8% level

E.g.: Z-penguins & R_b

$$R_b = \frac{\Gamma(Z \rightarrow bb)}{\Gamma(Z \rightarrow \text{had})}$$



breaking of
universality
→
driven by



$$R_b = R_d [1 - \boxed{G_f m_t^2 / 2\pi^2 \sqrt{2}} + \dots] \approx 0.2200 - 0.0024 + \dots$$

$$R_b^{\text{exp}} = 0.2163 \pm 0.0007$$

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tree + flavour univ. terms

↓
same e.w.-Yukawa
structure of the leading
 $K \rightarrow \pi \nu \nu$ amplitude

- Even within the most pessimistic NP scenario O(30-50%) deviations from SM possible in BR(rare short-distance dominated FCNC decays)
- O(10%) measurements of BR(rare) probe NP parameter space of NP not cover yet by LEP

Beyond Minimal Flavour Violation

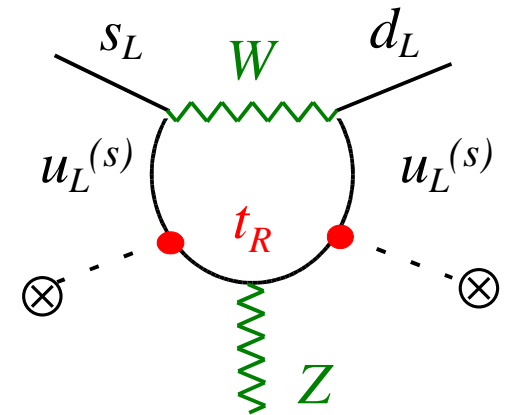
[new sources of flavour symmetry breaking at the TeV scale]

- A priori the most natural possibility naturally appearing in several specific scenarios [e.g. SUSY: [huge literature](#)]
- challenged -at present- by the good agreement with SM in $\Delta F=2$ sector

General features:

- Some decoupling between $\Delta F=2$ & $\Delta F=1$ [i.e.: $\delta_{\text{NP}}(\Delta F=1) \sim 100\%$ vs. $\delta_{\text{NP}}(\Delta F=2) \sim 10\%$] possible thanks to the interplay between SU(2) · U(1) & flavour symm. breaking

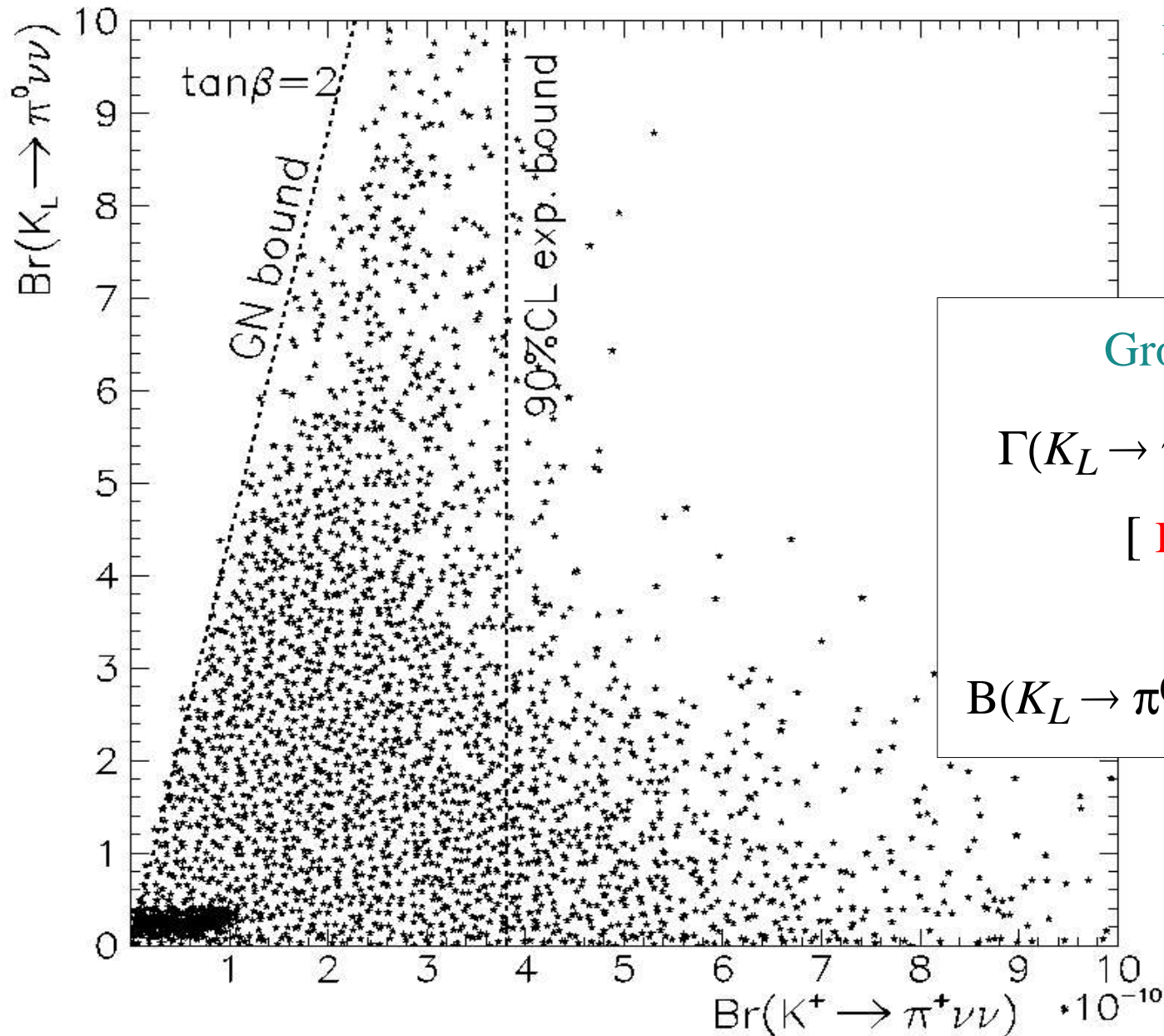
Colangelo & G.I. '98,
Nir & Worah '97;
Buras, Romanino & Silvestrini, '97



- Rare kaon decays are particularly sensitive to new sources of flavour symm. breaking because of the severe CKM suppression [$V_{ts}^* V_{td} \sim \lambda^5$]

E.g.: $B(K \rightarrow \pi \nu \nu)$ within generic MSSM

[including all the phenomenological constraints from ϵ_K , ΔM_K , $b \rightarrow s \gamma$, ...]



Buras *et al.* '04

Grossman-Nir bound:

$$\Gamma(K_L \rightarrow \pi^0 \nu \nu) < \Gamma(K^+ \rightarrow \pi^+ \nu \nu)$$

$$[\text{Im}(A) < |A|]$$



$$B(K_L \rightarrow \pi^0 \nu \nu) < 4.4 B(K^+ \rightarrow \pi^+ \nu \nu)$$

bounds on $\frac{1}{\Lambda^2} (Q_L Y_U Y_U \gamma_\mu Q_L) L_L \gamma_\mu L_L$ within MFV [$\sigma(V_{ij})=1\%$]

