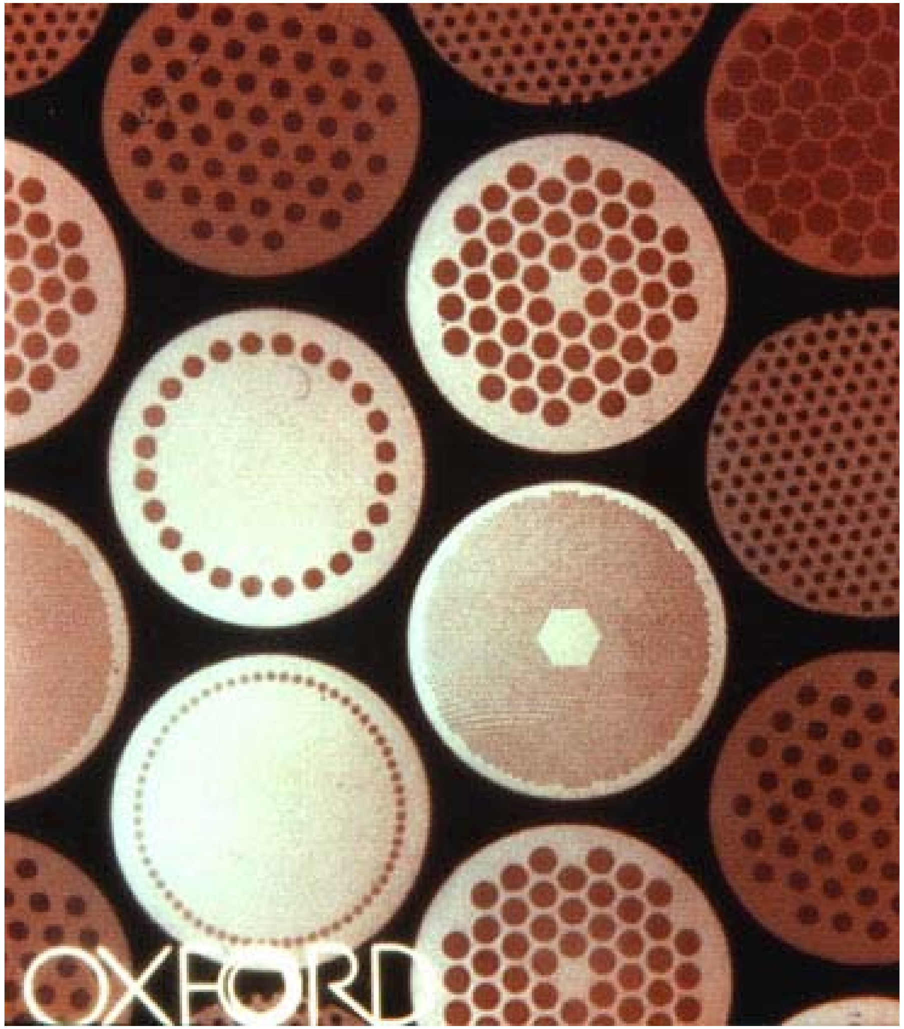
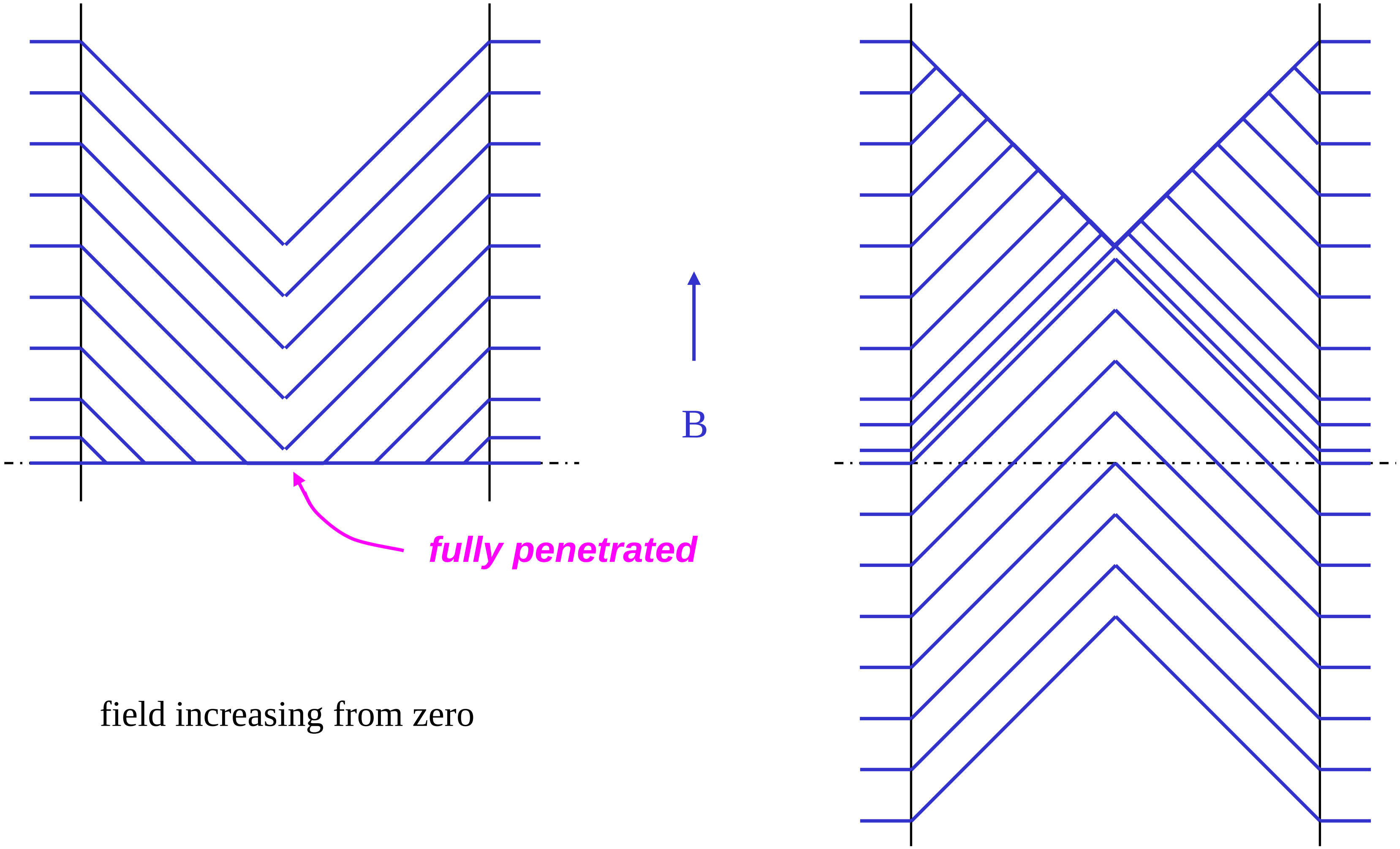
Lecture 2: Magnetization, AC Losses and Filamentary Wires

- magnetization from screening currents, irreversibility and hysteresis loops
- field errors caused by screening currents
- flux jumping
- general formulation of ac loss in terms of magnetization
- ac losses caused by screening currents
- the need for fine filaments; composite wires
- coupling between filaments via currents crossing the matrix



Recap the flux penetration process

plot field profile across the slab



field increasing from zero

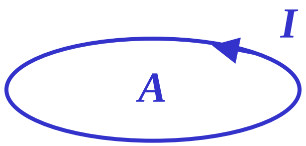
field decreasing through zero

Magnetization of the Superconductor

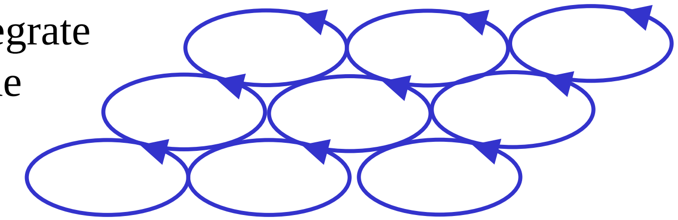
When viewed from outside the sample, the persistent currents produce a magnetic moment.

As for any magnetic material, we can define a **magnetization** of the superconductor (magnetic moment per unit volume)

$$M = \sum_V \frac{I.A}{V}$$

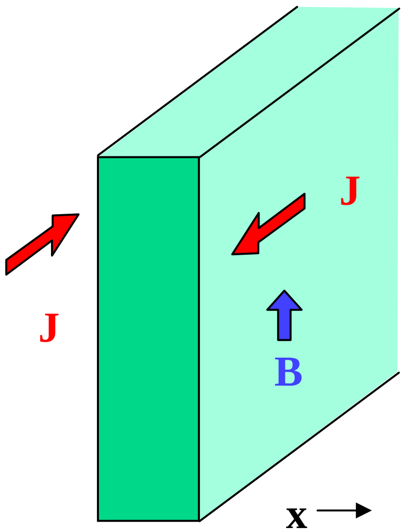


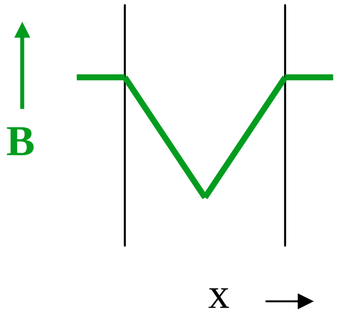
where distributed currents flow, we must integrate over the bulk of the material



NB: M is units of H

For a fully penetrated superconducting slab





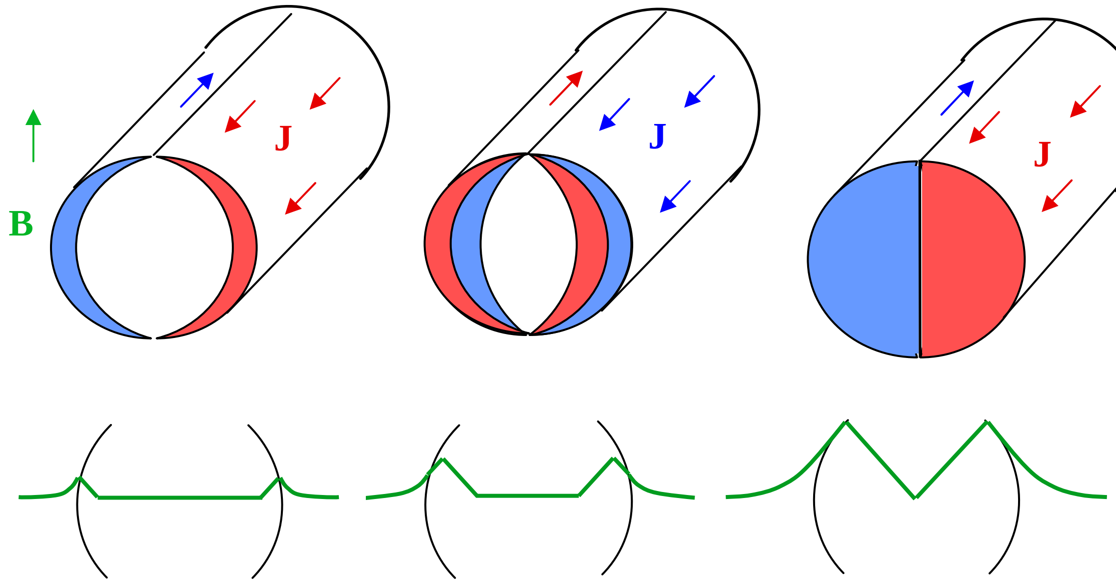
by symmetry integrate half width

$$M = \frac{1}{a}\int_0^a J_c.x.dx = \frac{J_c.a}{2}$$

Note: J_c varies with field, so does M

Magnetization of a Superconducting wire

for **cylindrical** filaments the inner current boundary of screening current penetration is roughly elliptical (controversial)



when fully penetrated, the magnetization is $M = \frac{4}{3\pi} J_c \, a$ (compare $M = \frac{1}{2} J_c \, a$ for the slab)

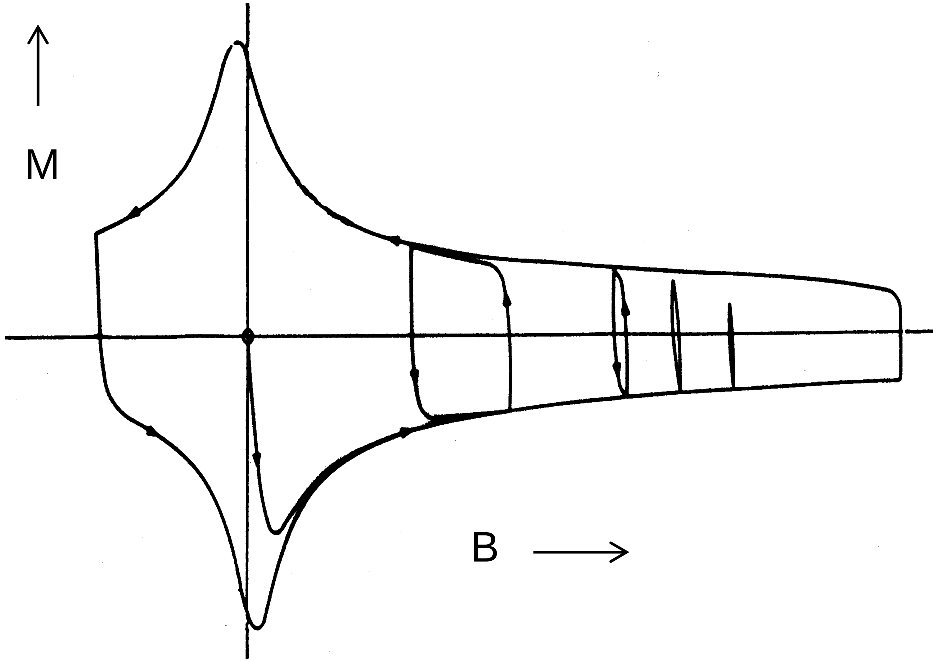
more commonly $$M = \frac{2}{3\pi} J_c \, d_f$$

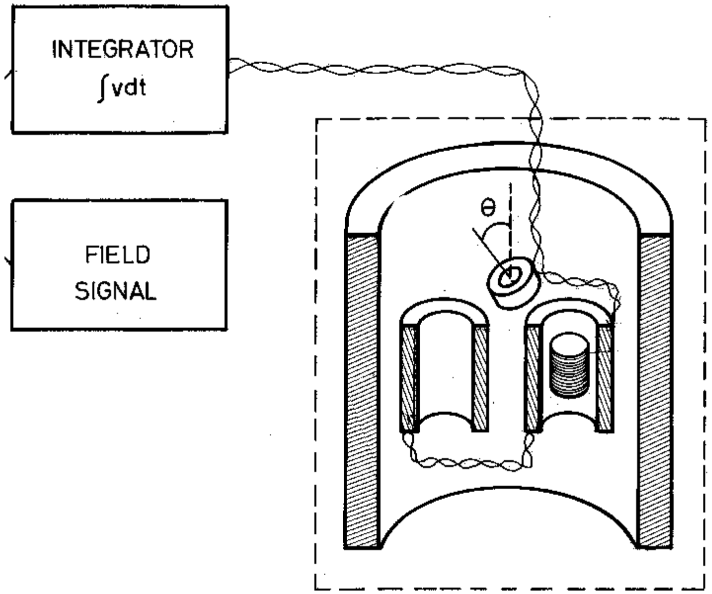
where $\boldsymbol{a}$ and $\boldsymbol{d_f}$ = filament radius and diameter

Recap: M is defined per unit volume of NbTi filament

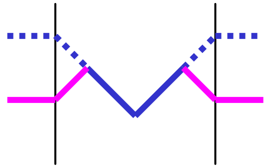
Measurement of magnetization

In field, the superconductor behaves just like a magnetic material. We can plot the magnetization curve using a magnetometer. It shows hysteresis - just like iron only in this case the magnetization is both diamagnetic and paramagnetic.





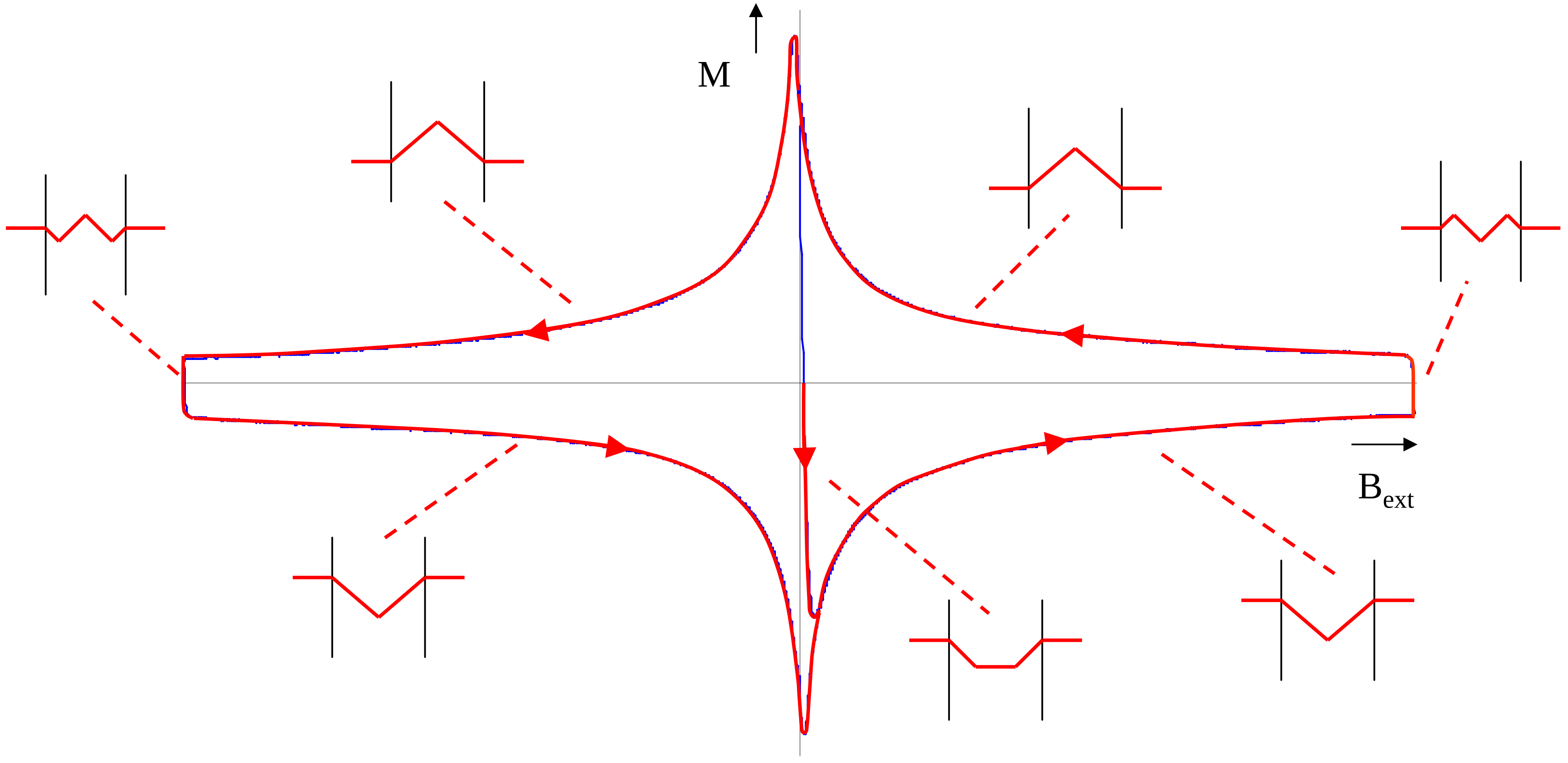
Note the minor loops, where field and therefore screening currents are reversing



The magnetometer, comprising 2 balanced search coils, is placed within the bore of a superconducting solenoid. These coils are connected in series opposition and the angle of small balancing coil is adjusted such that, with nothing in the coils, there is no signal at the integrator. With a superconducting sample in one coil, the integrator measures magnetization when the solenoid field is swept up and down

Magnetization of NbTi

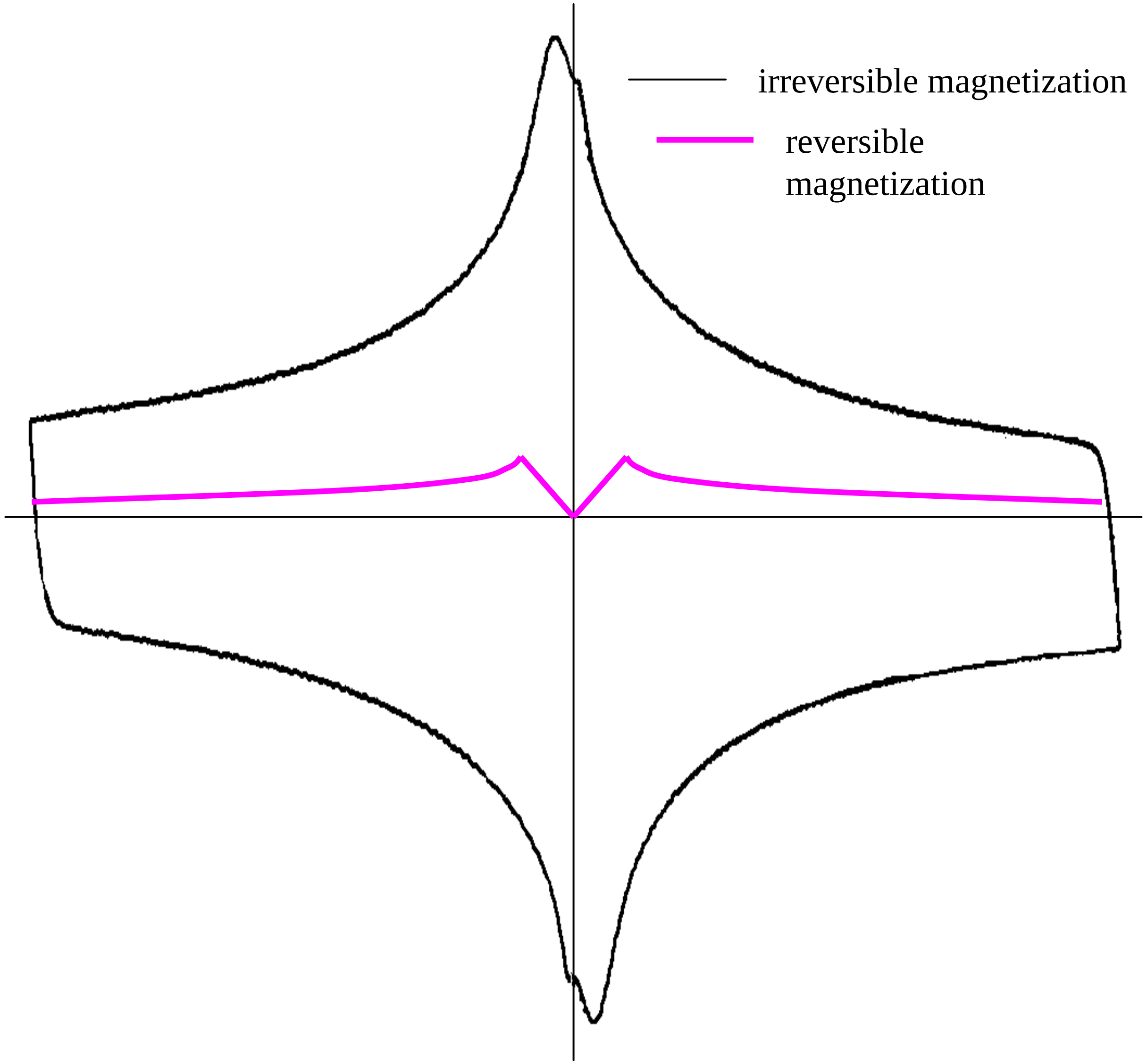
The induced currents produce a magnetic moment and hence a magnetization
= magnetic moment per unit volume



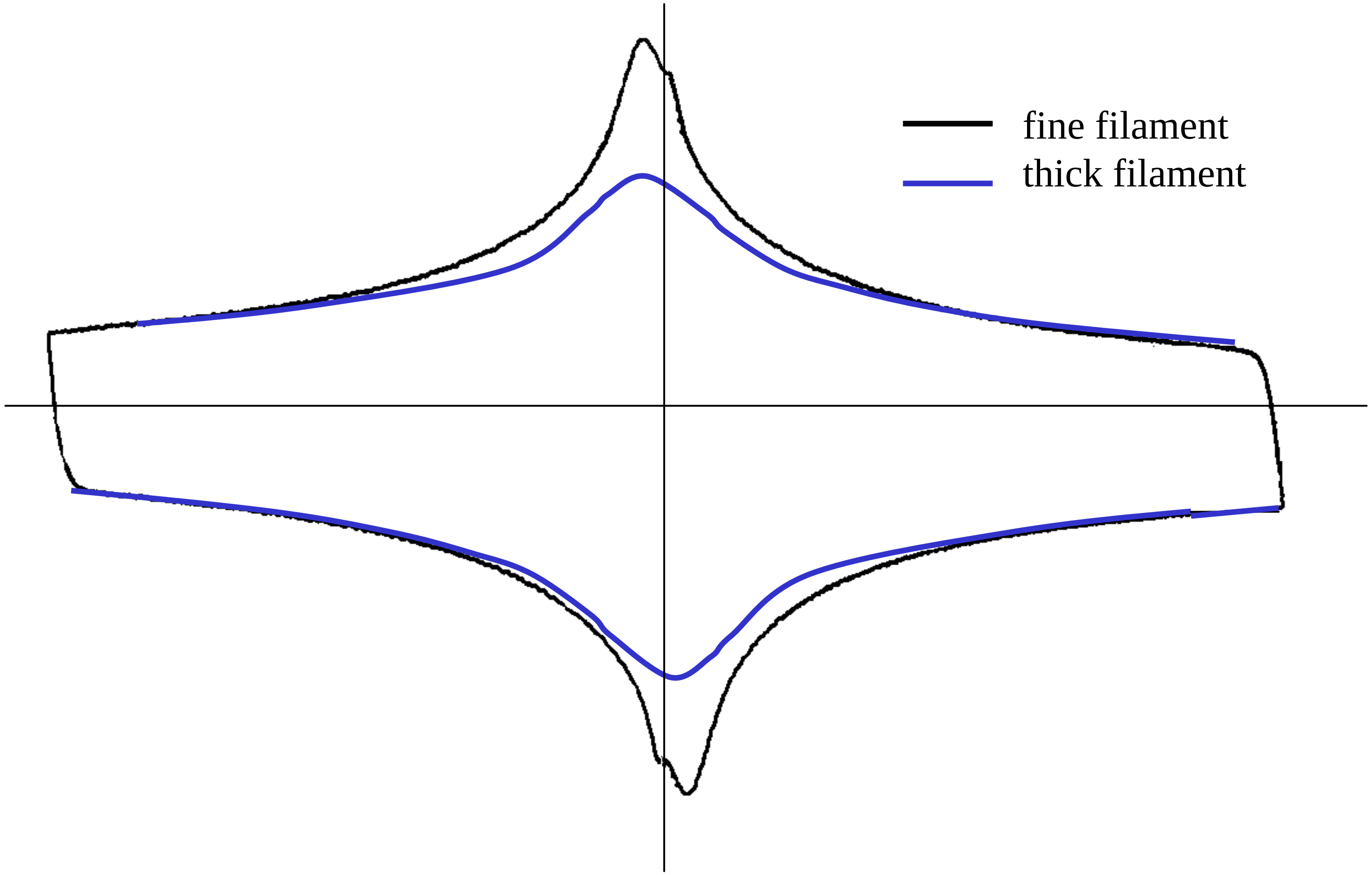
Reversible magnetization

•We have been discussing the irreversible magnetization, produced by bulk currents and thus by flux pinning.

•In addition, there is another component - the reversible (non hysteretic) magnetization. It is shown by all type 2 superconductors even if they have no flux pinning. In technical (strong pinning) materials however it is negligible in comparison with the irreversible component



Very fine filaments



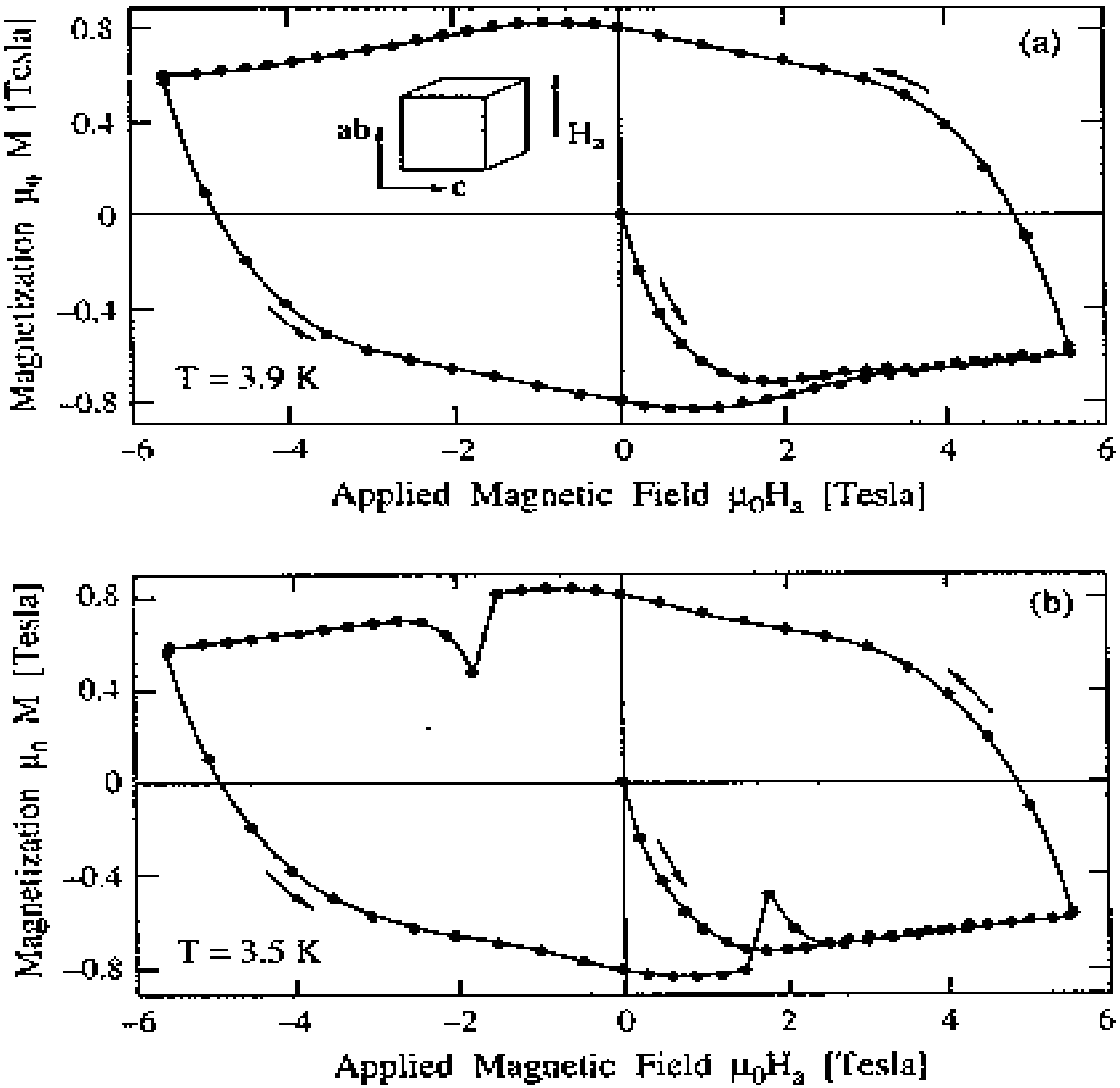
At zero external field, magnetization currents ensure that there is still a field inside the filament.
Magnetization is smaller with fine filaments, so the average field within the filament is smaller.
For this reason, the *Jc* at zero field measured on fine filament is greater than thick filaments.

HTS magnetization

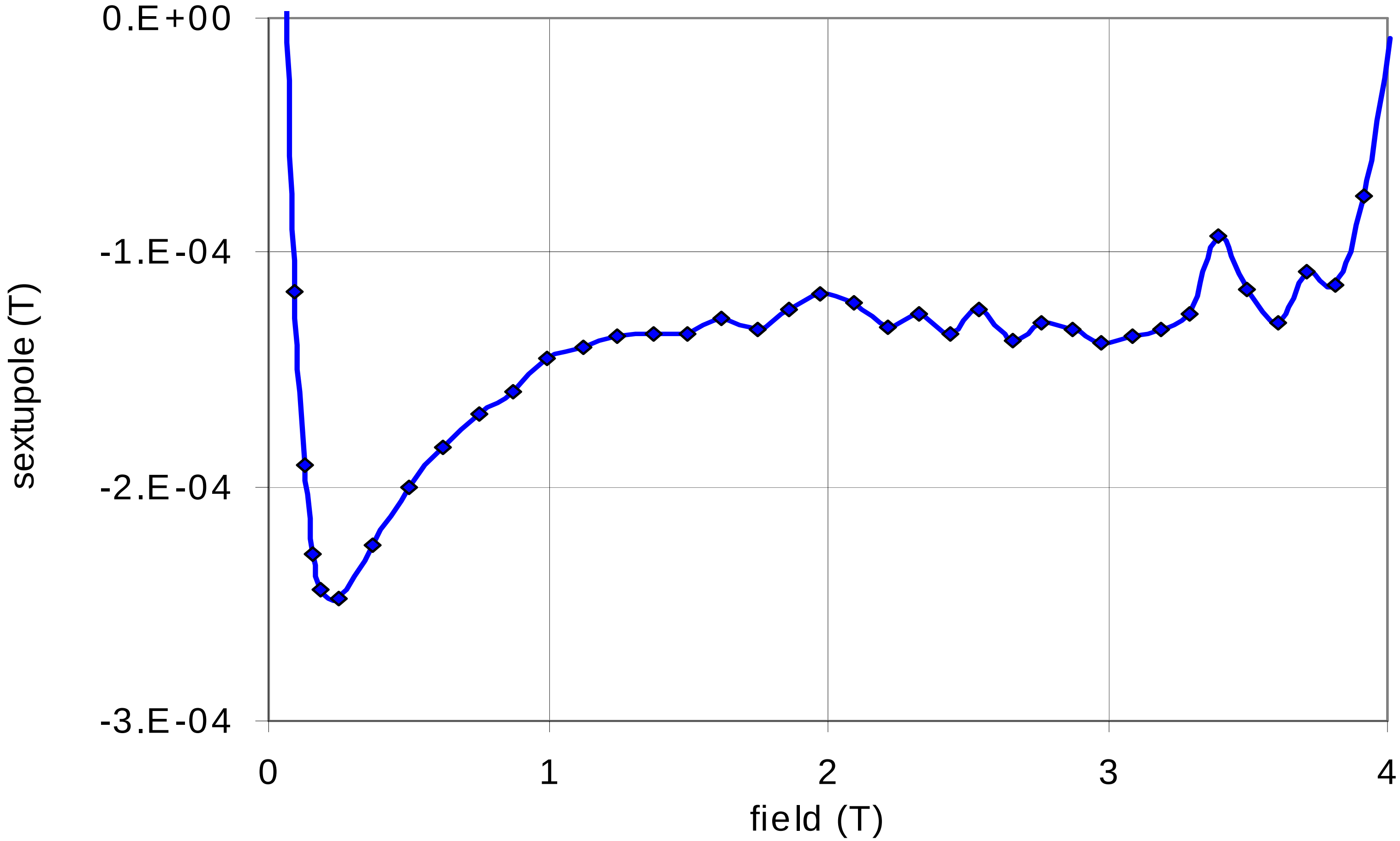
Magnetization is shown by all superconducting materials.

Here we see the magnetization of a high temperature superconductor.

The glitches in the lower curve are flux jumps - see lecture 8

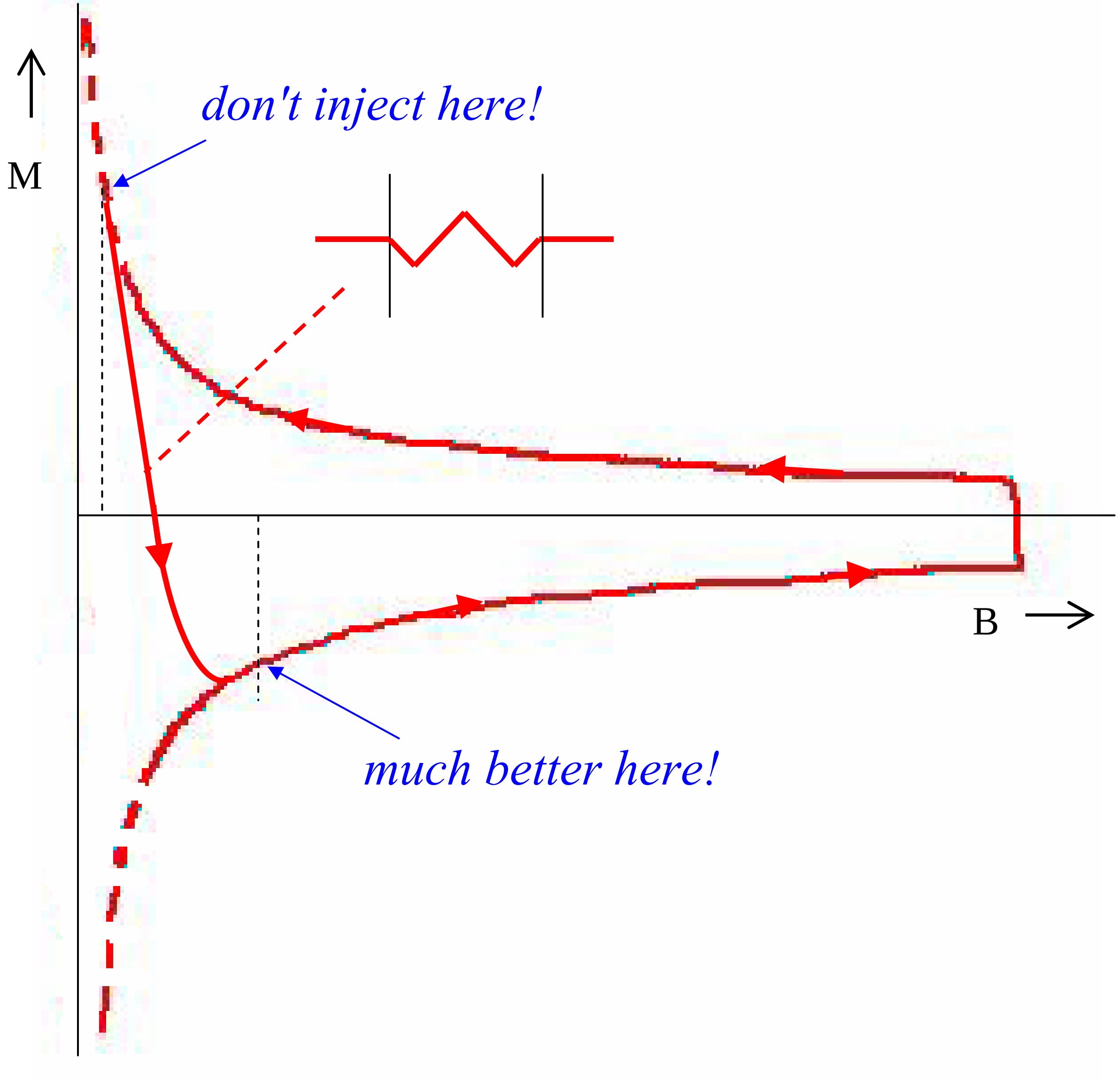


Magnetization in superconductor ⇒ field error in magnet



sextupole field in the GSI FAIR prototype dipole D001 without the iron yoke, dc field
in Tesla at radius 25mm measured at BNL by Animesh Jain

Synchrotron injection



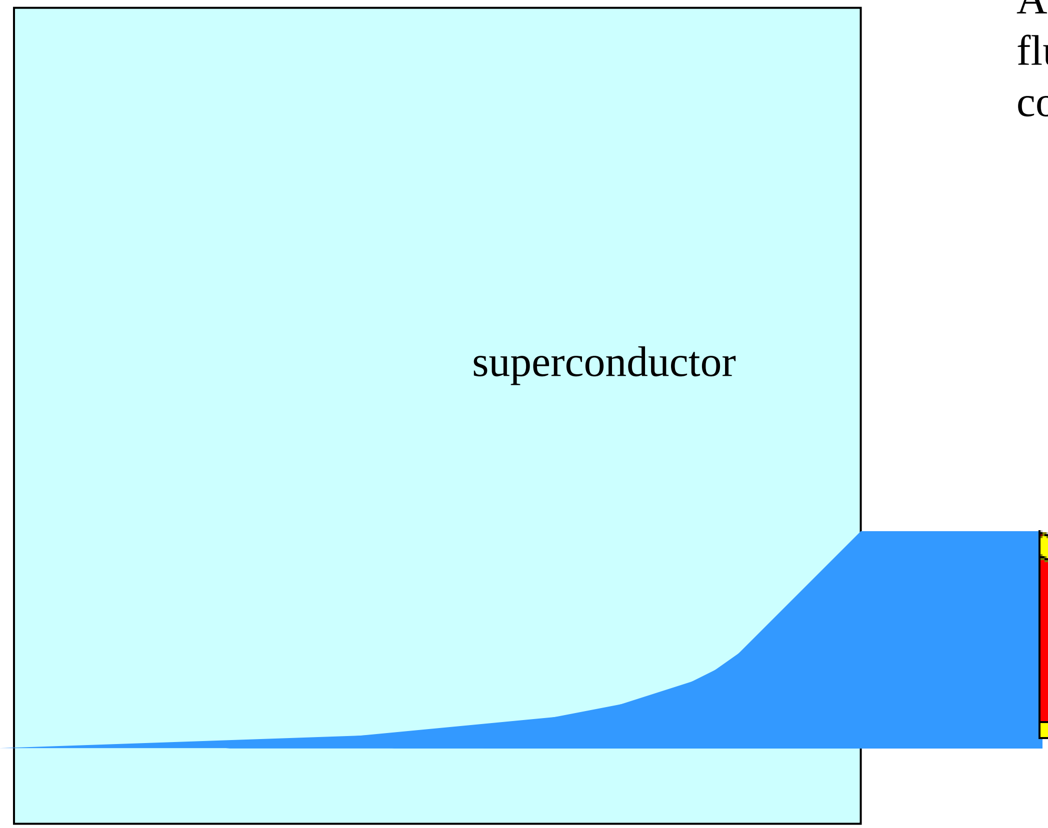
- synchrotron injects at low field, ramps to high field and then back down again
- magnetization error is worst at injection because M is largest and B is smallest, so $\mu_o M/B$ is largest .
- note how quickly the magnetization changes when we start the ramp up
- so better to ramp up a little way, then stop to inject

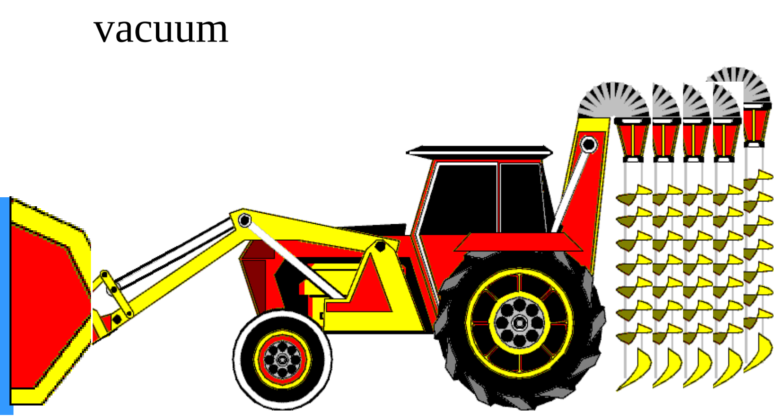
Flux penetration from another viewpoint

Think of the screening currents, in terms of a gradient in fluxoid density within the superconductor. Pressure from the increasing external field pushes the fluxoids against the pinning force, and causes them to penetrate, with a characteristic gradient in fluxoid density

At a certain level of field, the gradient of fluxoid density becomes unstable and collapses

– a flux jump





Flux jumping: why it happens

Unstable behaviour is shown by all type 2 and HT superconductors when subjected to a magnetic field

It arises because:-

magnetic field induces screening currents, flowing at critical density J_c

*** reduction in screening currents allows flux to move into the superconductor**

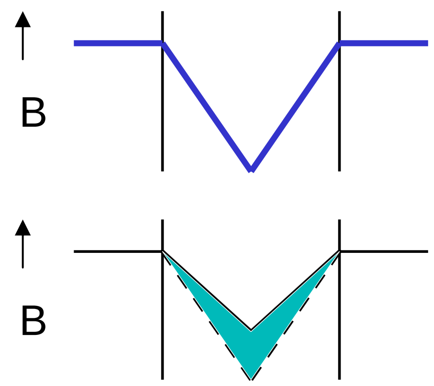
flux motion dissipates energy

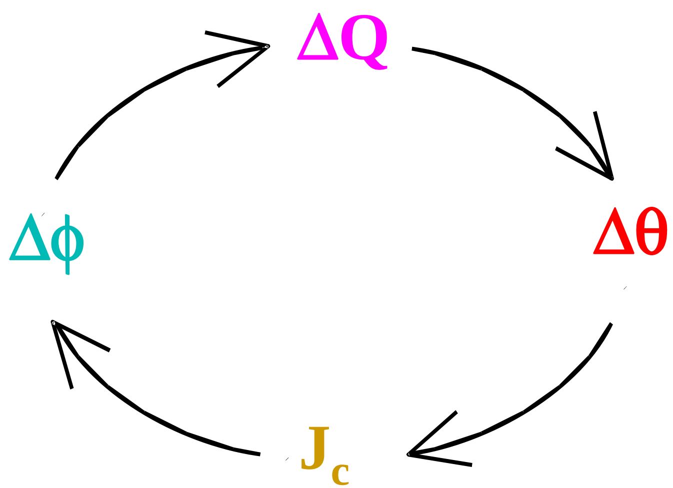
thermal diffusivity in superconductors is low, so energy dissipation causes local temperature rise

critical current density falls with increasing temperature

go to *

Cure flux jumping by making superconductor in the form of fine filaments – weakens $\Delta J_c \Rightarrow \Delta\phi \Rightarrow \Delta Q$





Flux jumping: the numbers for NbTi

criterion for stability against flux jumping
a = half width of filament

$$a \leq \frac{1}{J_c}\left\{\frac{3\gamma C(\theta_c - \theta_o)}{\mu_o}\right\}^{\frac{1}{2}}$$

typical figures for NbTi at 4.2K and 1T

J_c critical current density = 7.5 x 10^9 Am^{-2}

γ density = 6.2 x 10^3 $kg.m^3$

C specific heat = 0.89 $J.kg^{-1}K^{-1}$

θ_c critical temperature = 9.0K

so *a* = 33µm, ie 66µm diameter filaments

Notes:

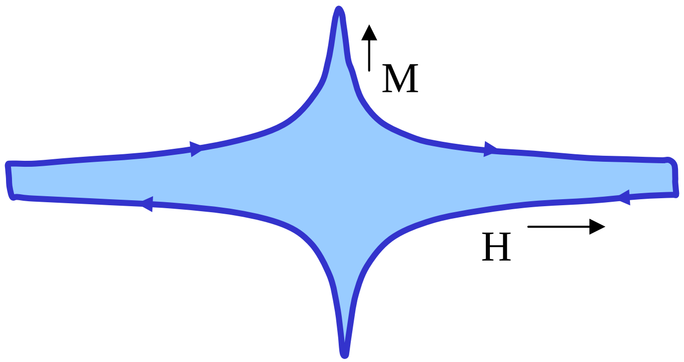
- least stable at low field because J_c is highest
- instability gets worse with decreasing temperature because J_c increases and C decreases
- criterion gives the size at which filament is just stable against infinitely small disturbances
  - still sensitive to moderate disturbances, eg mechanical movement
- better to go somewhat smaller than the limiting size
- in practice 50µm diameter seems to work OK

Flux jumping is a solved problem ✓

Magnetization and Losses: General

in general, the change in magnetic field energy $\delta E = H \delta B$

(see textbooks on electromagnetism)

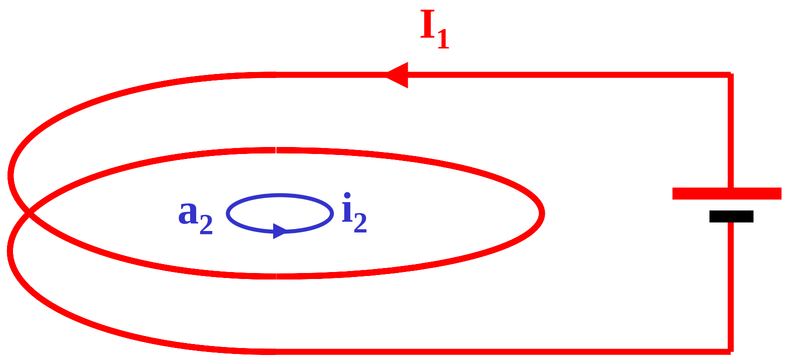


so work done on magnetic material

$$W = \int \mu_o H dM$$

around a closed loop, energy dissipated in material

$$E = \int \mu_o H dM = \int \mu_o M dH$$



work done by battery to raise current in solenoid

$$W = \int V_1 I_1 dt = \int I_1 L_{11} \frac{dI_1}{dt} dt - \int I_1 L_{21} \frac{di_2}{dt} dt$$

$$= \tfrac{1}{2} L_{11} I_1^2 - \int I_1 L_{21} di_2$$

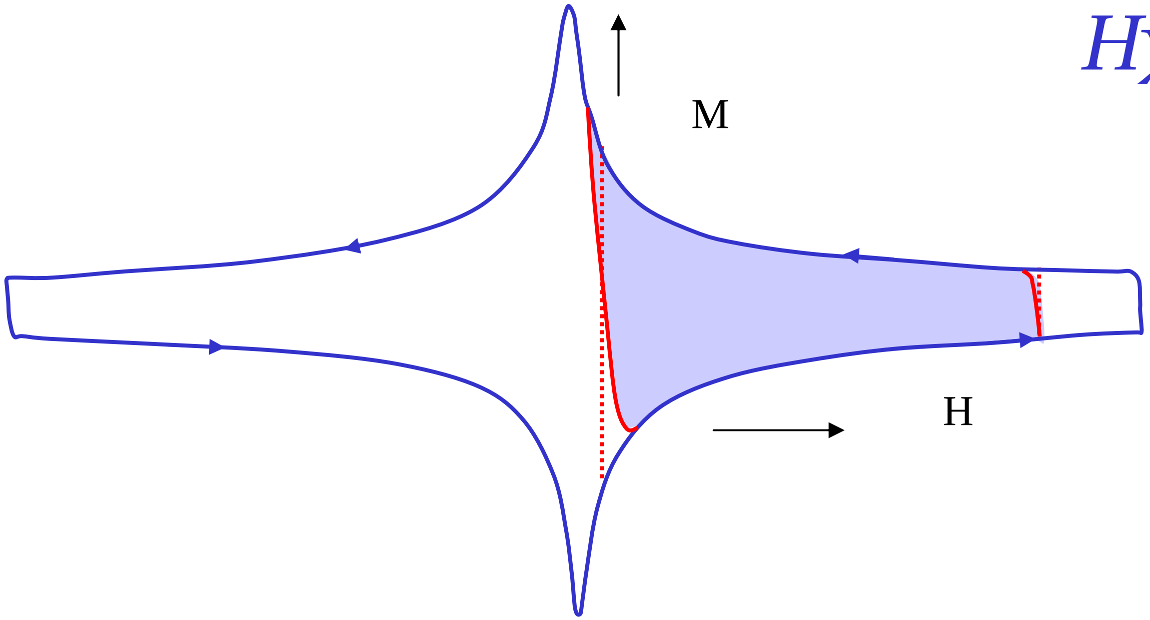
first term is change in stored energy of solenoid

$I_1 L_{21}$ is the flux change produced in loop 2

$$\int I_1 L_{21} di_2 = \int \mu_o H_1 a_2 di_2 = \int \mu_o H_1 dM$$

so battery work done on loop $= \int \mu_o H_1 dM$

Hysteresis Losses



With the approximation of vertical lines at the **'turn around points'** and saturation magnetization in between, the hysteresis loss per cycle is

$$E = \oint \mu_o M dH \cong \oint M dB$$

$$W = \int \mu_o H dM = \int \mu_o M dH$$

This is the work done on the sample
Strictly speaking, we can only say it is a heat dissipation if we integrate round a loop and come back to the same place
- otherwise the energy just might be stored

Around a loop the red 'crossover' sections are complicated, but we usually approximate them as straight vertical lines (dashed)

In the (usual) situation where ***dH>>M,*** we may write the loss between two fields $\boldsymbol{B_1}$ and $\boldsymbol{B_2}$ as

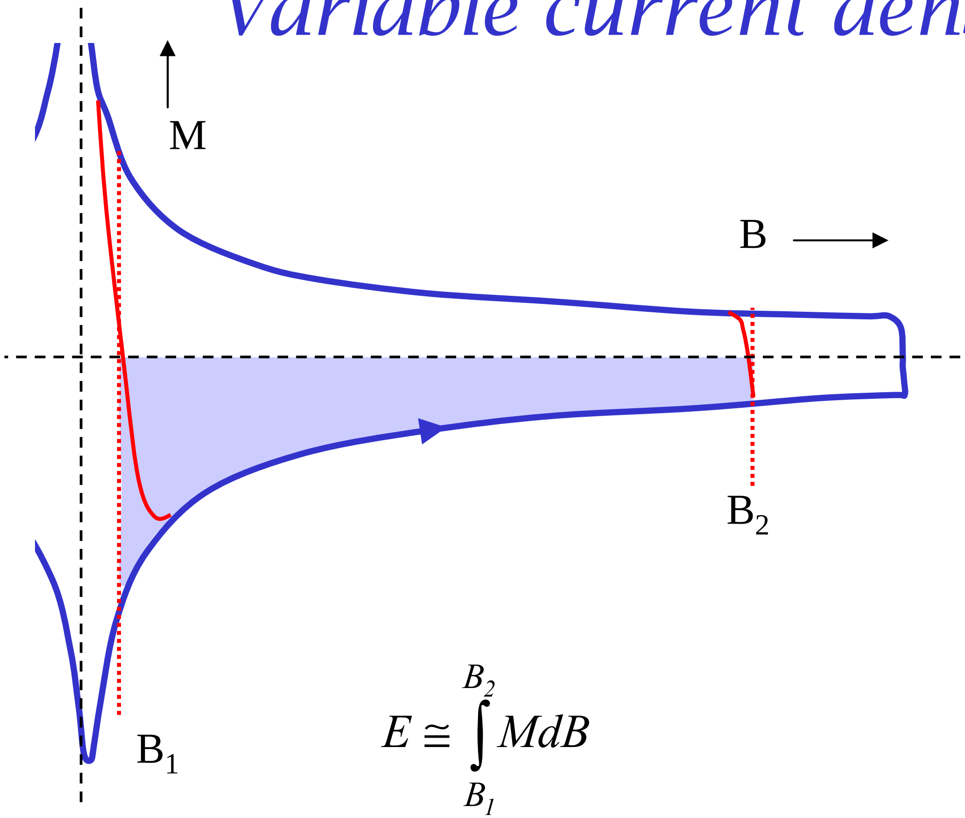
$$E \cong \int_{B_1}^{B_2} M dB$$

so the loss power is

$$P = M\dot{B} = \frac{2}{3\pi} J_c d_f \dot{B}$$

losses in Joules per m^3 and Watts per m^3 of superconductor

Variable current density in superconductor



$$E \cong \int_{B_1}^{B_2} M dB$$

recap $M = \frac{2}{3\pi} J_c d_f$

$$E = \int_{B_1}^{B_2} \frac{2}{3\pi} J_c d_f . dB$$

To evaluate need $J_c(B)$
Kim Anderson approximation

$$J_c(B) = \frac{J_o B_o}{(B + B_o)}$$

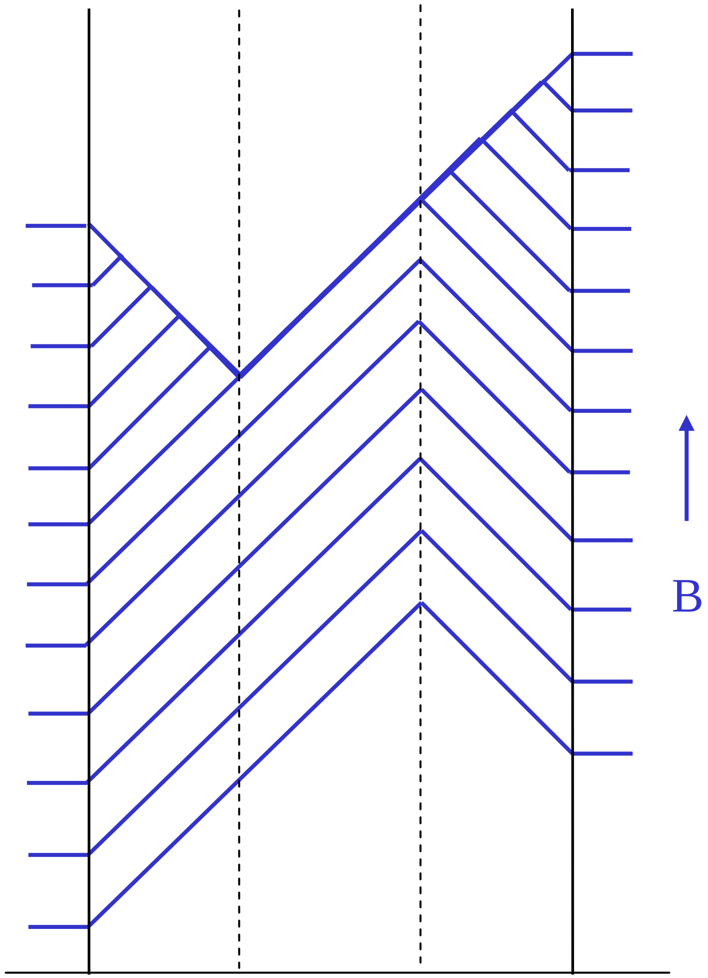
loss for ramp up from B_1 to B_2

$$E = \frac{2}{3\pi} \int_{B_1}^{B_2} \frac{J_o B_o}{(B + B_o)} d_f dB$$

$$E = \frac{2}{3\pi} d_f J_o B_o \ln\left\{\frac{B_2 + B_o}{B_1 + B_o}\right\}$$

loss in Joules per m^3
of superconductor

The effect of transport current



plot field profile across the slab

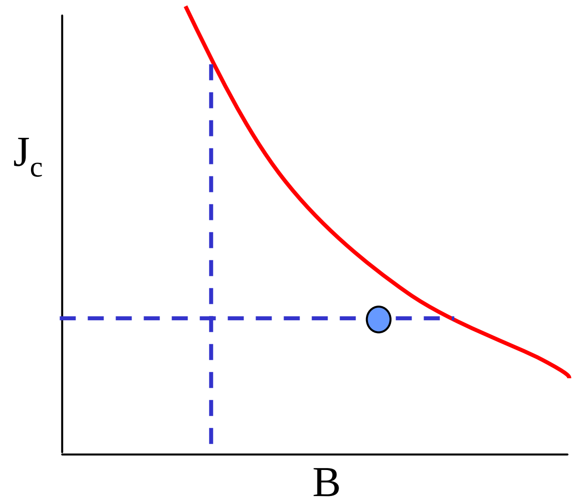
- in magnets there is a transport current, coming from the power supply, in addition to magnetization currents.
- because the transport current 'uses up' some of the available J_c the magnetization is reduced.
- but the loss is increased because the power supply does work and this adds to the work done by external field

total loss is increased by factor $(1+i^2)$ where $i = I_{max} / I_c$

$$E = \frac{2}{3\pi} d_f J_o B_o \ln\left\{\frac{B_2 + B_o}{B_1 + B_o}\right\}(1+i^2)$$

usually not such a big factor because

- *design for a margin in Jc*
- *most of magnet is in a field much lower than the peak*



The need for fine filaments

Magnetization

$$M = \frac{2}{3\pi} J_c d_f$$

- d as small as possible, typically 7µm
- critical current of a 7µm filament in 5T at 4.2K = 0.1A

Flux Jumping

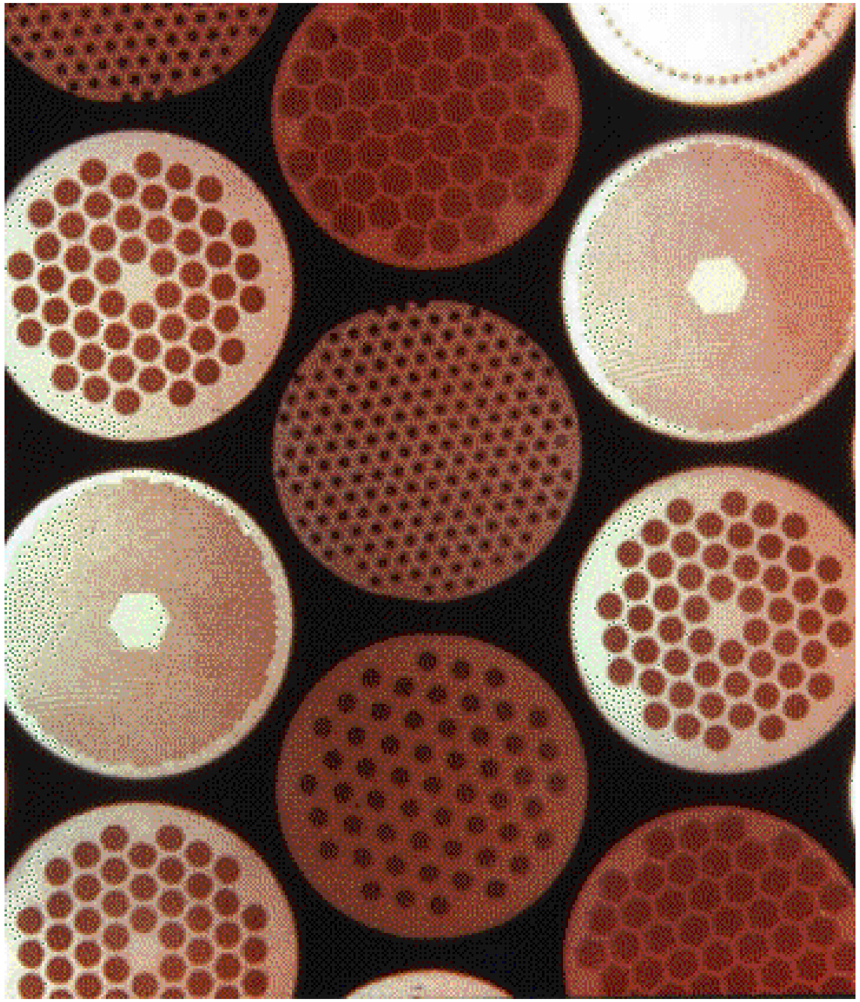
$$a \leq \frac{1}{J_c}\left\{\frac{3\gamma C(\theta_c - \theta_o)}{\mu_o}\right\}^{\frac{1}{2}}$$

- for NbTi d < 50µm
- critical current of a 50µm filament in 5T at 4.2K = 0.1A

AC Losses

$$E = \frac{4}{3\pi} d_f J_o B_o \ln\left\{\frac{B_2 + B_o}{B_1 + B_o}\right\}$$

- d_f as small as possible, typically 7µm, for FAIR we are thinking of 3µm
- critical current of a 7µm filament in 5T at 4.2K I_c = 0.1A; for a 3µm filament I_c = 0.02A

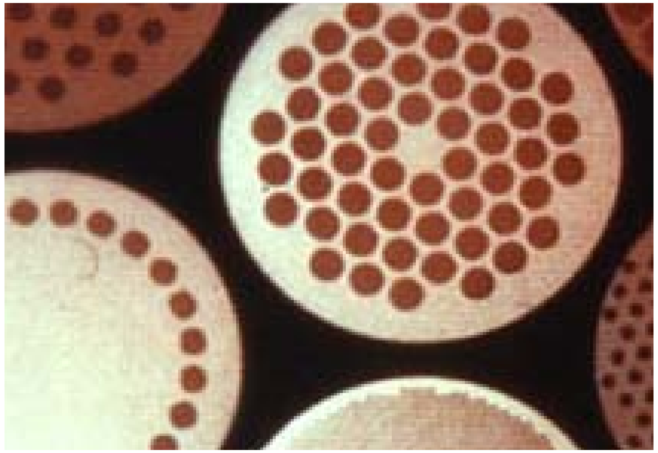


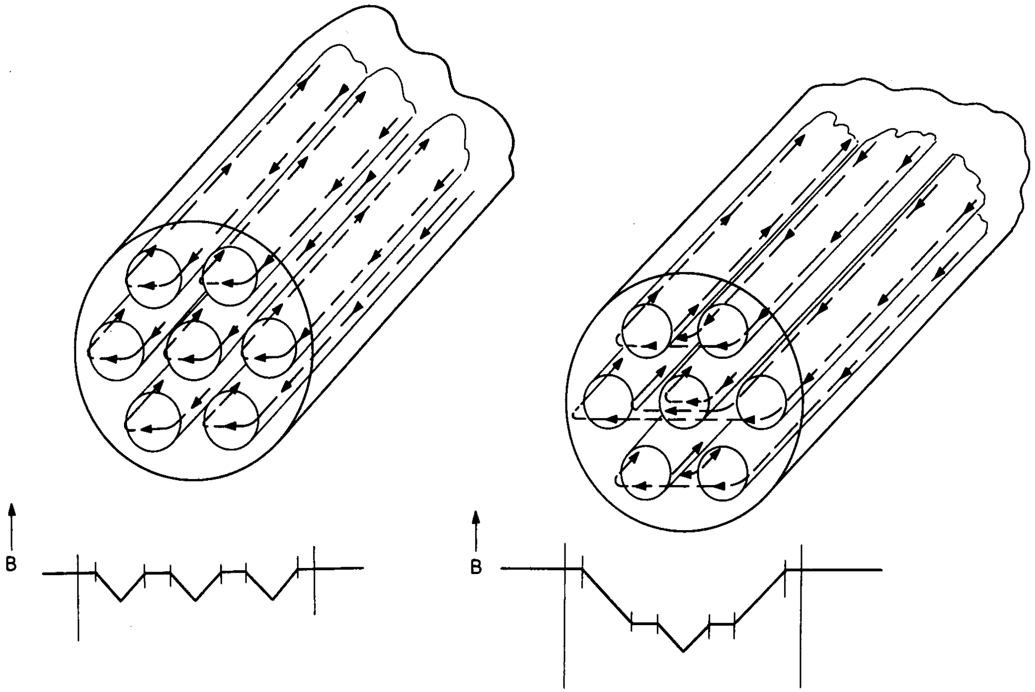
so we need multi-filamentary wires

Fine filaments

recap $$M = \frac{2}{3\pi} J_c \, d_f$$

We can reduce M by making the superconductor as fine filaments. For ease of handling, an array of many filaments is embedded in a copper matrix





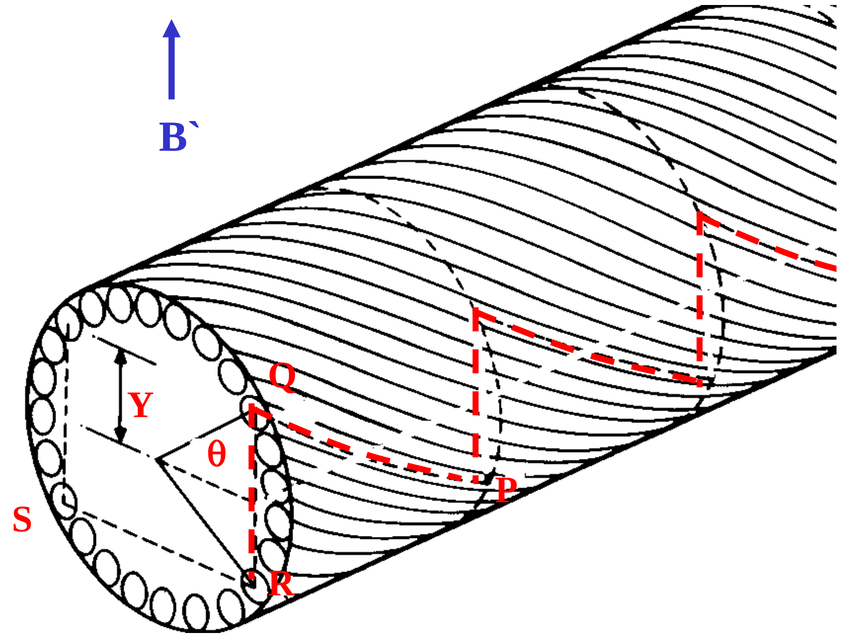
Unfortunately, in changing fields, the filament are coupled together; screening currents go up the LHS filaments and return down the RHS filaments, crossing the copper at each end.

In time these currents decay, but for wires ~ 100m long, the decay time is years!

So the advantages of subdivision are lost

Twisting 1

- coupling may be reduced by twisting the wire
- coupling currents now flow along the filaments and cross over the resistive matrix every ½ twist pitch
- now the matrix crossing currents flow vertically, parallel to the changing field
- at each end of the wire, the current crosses over horizontally and then returns along the other side of the wire
- we assume the filaments have not reached J_c and so there is no electric field along them
- thus the electric field due to flux change linked by the filament lies entirely on the vertical path Y
- thus we have a ***uniform*** electric field in the matrix

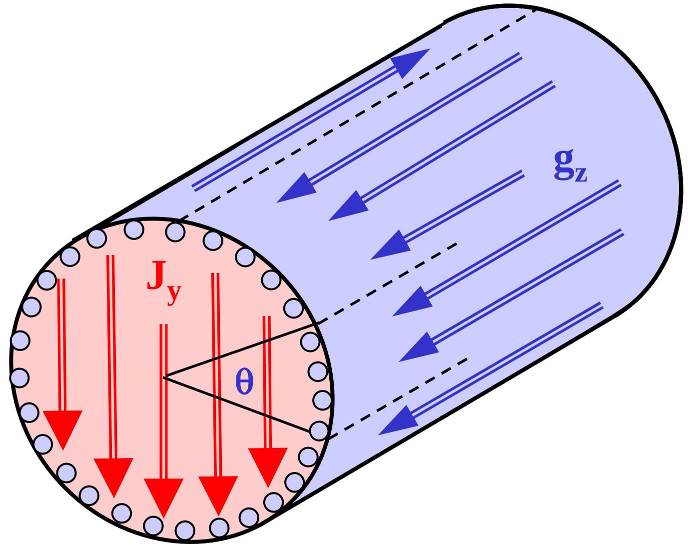


$$\int_Q^R E\,dl = \int_P^Q B_i'\,a\cos\theta\,dz = \frac{2B_i'\,p\,Y}{2\pi}$$

where p is the twist pitch

$$E_y = \frac{B_i'\,P}{2\pi}$$

Twisting 2



- a uniform electric field across the resistive matrix implies a uniform vertical current density J_y
- a consideration of how this current enters and leaves the outer ring of filaments shows that there must be a ***linear*** current density g_z (A/m) along the wire where

$$g_z(\theta) = g_{zo}\cos\theta = \frac{B_i'}{\rho_t}\left\{\frac{p}{2\pi}\right\}^2\cos\theta$$

where ρ_t is the transverse resistivity across the matrix and ***p*** is the twist pitch.

- note that, because of twist, ***g*** reverses on left hand side
- recap from theory of fields in magnets that a *Cos*θ current distribution around a cylinder produces a perfect dipole field inside

$$B_{dip} = \frac{\mu_o}{2} g_{zo}$$

- so Ohm's law has given us the exact field needed to screen the external changing field and the internal field B_i is less than the external field B_e

$$B_i = B_e - \frac{\mu_o}{2} g_z(0) = B_e - \frac{\mu_o}{2}\frac{B_i'}{\rho_t}\left(\frac{p}{2\pi}\right)^2$$

Twisting 3

we may define a time constant (compare with eddy currents)

$$\tau = \frac{\mu_o}{2\rho_t}\left(\frac{p}{2\pi}\right)^2$$

so that

$$B_i = B_e - B_i' \, \tau$$

and integrate the magnetic moment of the screening currents to calculate a coupling component of magnetization

$$M = \frac{4}{\pi a^2}\int_0^{\frac{\pi}{2}} g_z(\theta)\, a\cos\theta \, a\, d\theta$$

$$M = \frac{2}{\mu_o} B_i' \, \tau$$

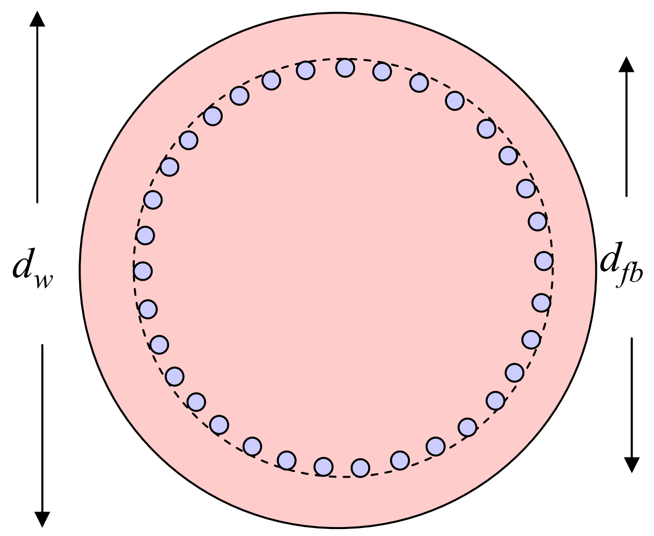
provided the external field has changed by more than M_{cp} we may take $B`_i \sim B`_e$

Note that the coupling magnetization is defined over the volume enclosed by the filament boundary.

To define over the wire volume must multiply by a filling factor

$$\lambda_{fb} = \frac{d_{fb}^2}{d_w^2}$$

$$M_c = \frac{2}{\mu_o}\lambda_{fb} B_i' \, \tau$$



Twisting 4

Summing the persistent current and coupling current components, we get the total magnetization of the wire.

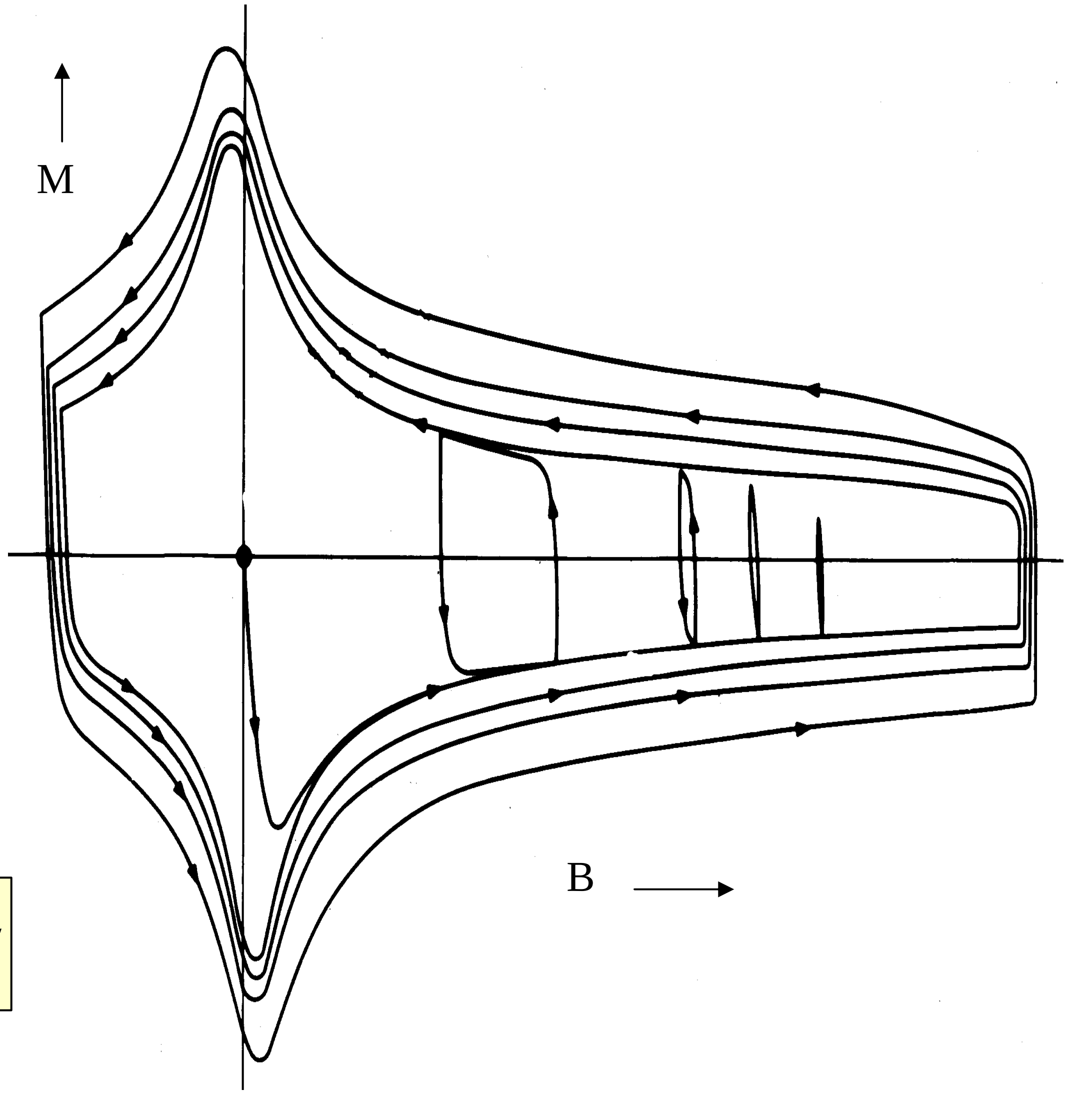
To define over the wire volume, we need a fill factor for the NbTi filaments

$$\lambda_f = \frac{volume\ NbTi}{volume\ wire}$$

so the total wire magnetization

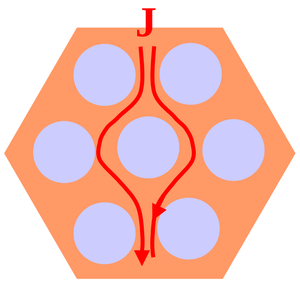
$$M_w = \lambda_f M_p + \lambda_{fb} M_c$$

$$M_w = \frac{2}{3\pi} \lambda_f J_c(B) d_f + \frac{2}{\mu_o} \lambda_{fb} B_i' \tau$$

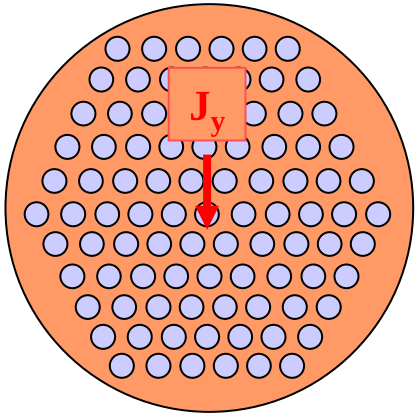


Transverse resistivity

Poor contact to filaments

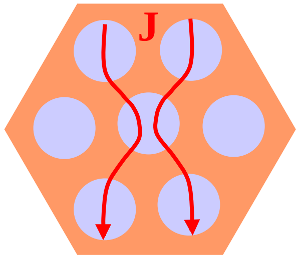


$$\rho_t = \rho_{Cu} \frac{1+\lambda}{1-\lambda}$$



where λ is the fraction of superconductor (after J Carr)

Good contact to filaments

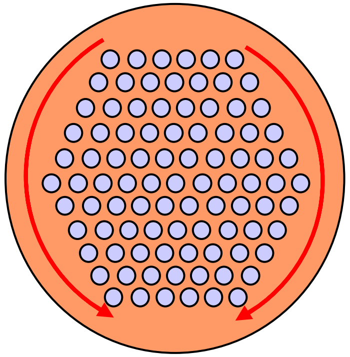


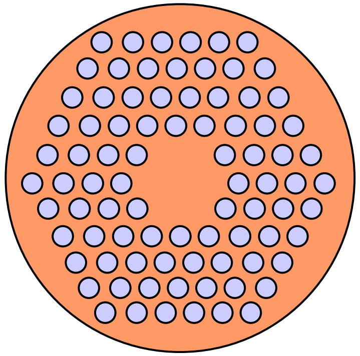
$$\rho_t = \rho_{Cu} \frac{1-\lambda}{1+\lambda}$$

Some complications

Thick copper jacket

include the copper jacket as a resistance in parallel



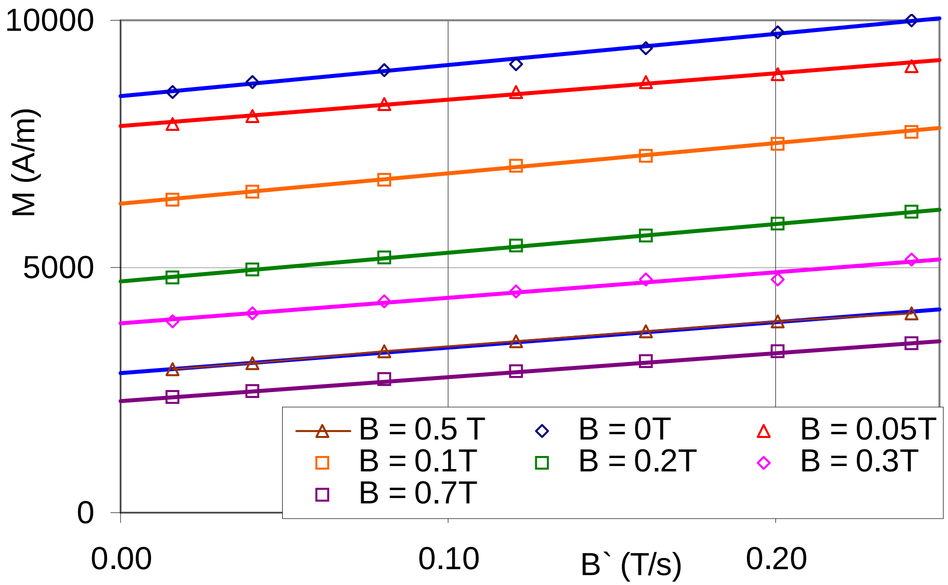


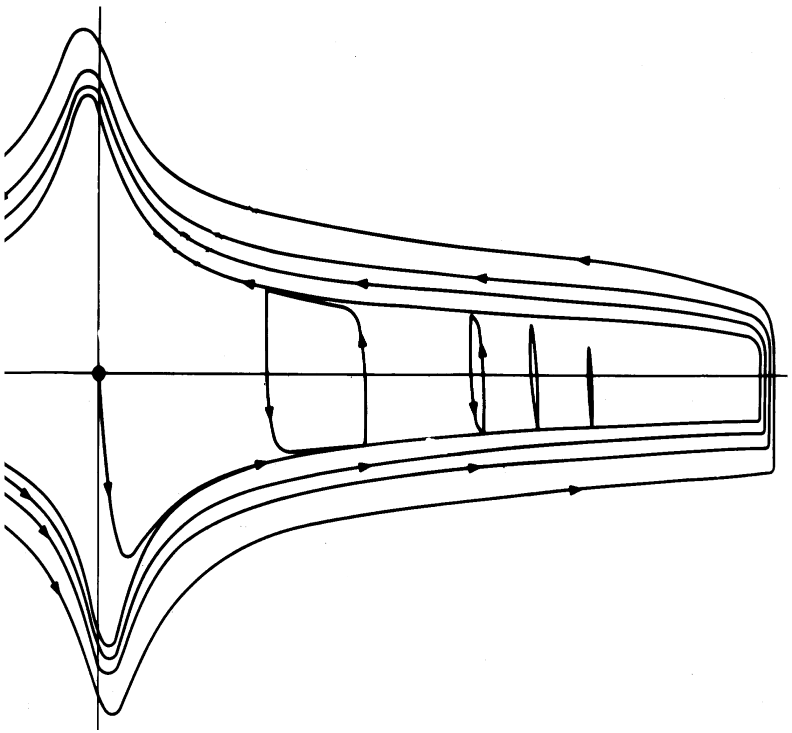
Copper core

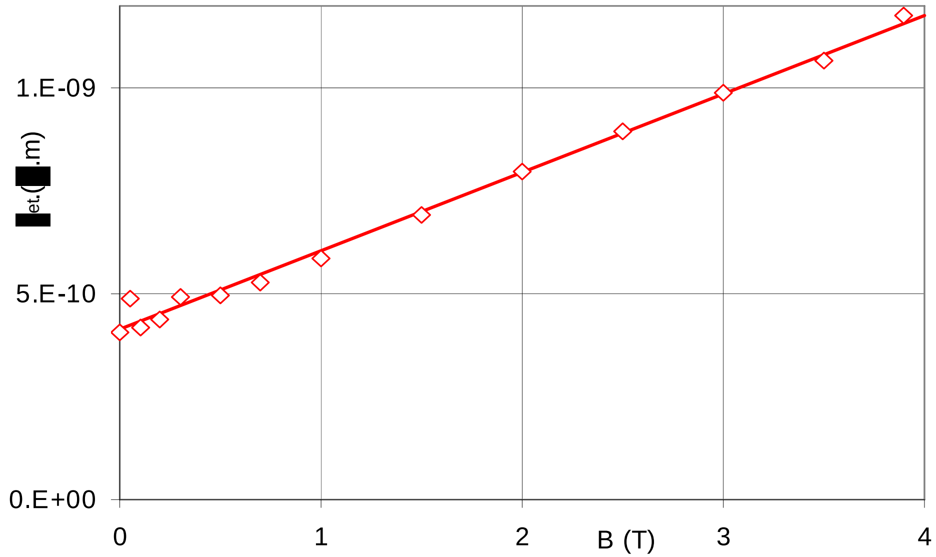
resistance in series for part of current path

Measurement of ρ_t

- measure magnetization loops at different ramp rates B`
- plot M versus B` at chosen fields
- calculate ρ_t as a function of B
  - note the magnetoresistance

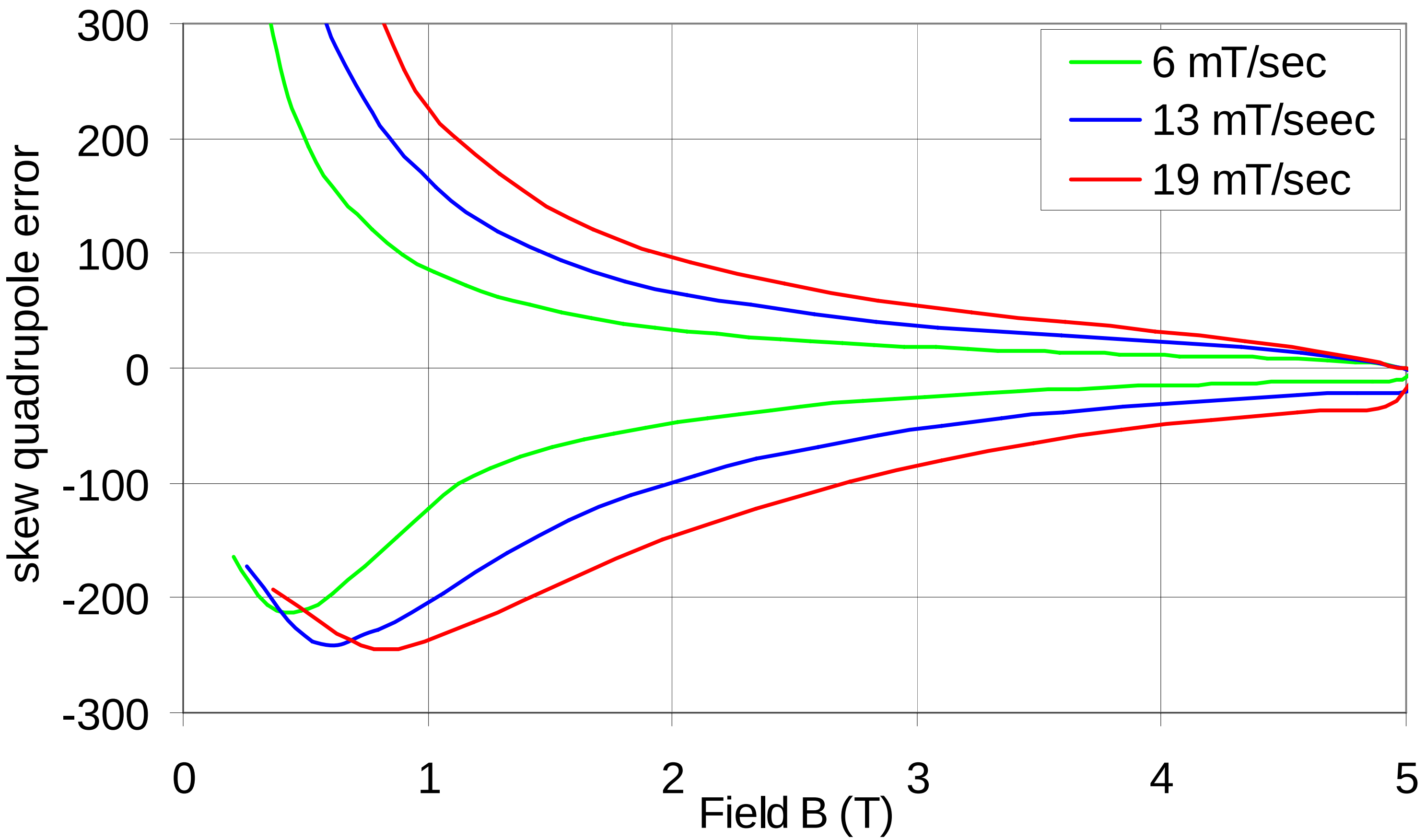






Coupling magnetization and field errors

Coupling magnetization gives a field error which adds to persistent current magnetization and is proportional to ramp rate.



skew quadrupole error in Nb_3Sn dipole which has exceptionally large coupling magnetization (University of Twente)

Magnetization, ac losses and filamentary wires: concluding remarks

- magnetic fields induce persistent screening currents in superconductor
  - which make it look like a magnetic material with a magnetization *M*
- for the technological type 2 superconductors, the magnetization is irreversible and hysteretic, ie it depends on the history
- magnetization ⇒ field errors in the magnet - usually the greatest source of error at injection
- magnetization can go unstable ⇒ flux jumping ⇒ quenches magnet
  - avoid by fine filaments - solved problem
- ac losses may be calculated from the area of the magnetization hysteresis loop
  (remember this is only the work done by the external field, transport current loss is extra)
- magnetization is proportional to filament diameter, so can reduce these problems by making fine filaments - typically 50μm for flux jumping and 5 - 10μm for losses and field quality
- practical conductors are made in the form of composite wires with superconducting filaments embedded in a matrix of copper
- in changing fields the filaments are coupled together through the matrix, thereby losing the benefit of subdivision
- twisting the composite wire reduces coupling
- coupling time constant depends on twist pitch and effective transverse resistivity, which is a function of contact resistance and geometry