X-ray Synchrotron Radiation in Geant4

Vladimir Grichine

e-mail: Vladimir.Grichine@cern.ch

Abstract

The current status of Geant4 X-ray synchrotron radiation process is presented. The energy spectrum, angular distribution and comparison with experimental data are discussed.

1 Introduction

G4SynchrotronRadiation class was implemented in 1998 in the framework of RD44 project. It was discrete electro-magnetic process working with global magnetic field and generating secondary G4Gamma. The implementation was directed mainly to the LHC applications.

Recently it was updated to work with magnetic field set in a volume. Few modifications were introduced for angular distribution as well as polarisation of secondary X-ray photons. The current status of G4SynchrotronRadiation is discussed as well as possible extensions to cover accelerator applications and XSR produced in materials rather than vacuum.

2 X-ray Synchrotron Radiation Process

The theory of X-ray synchrotron radiation (XSR) is well established. The mean number of XSR photons $\bar{N}$ generated by a relativistic electron in unit energy ω range per unit trajectory length x reads:

$$\frac{d^2\bar{N}}{d\omega dx} = \frac{1}{\omega}\frac{d\bar{\Delta}}{d\omega} = \frac{\sqrt{3}}{2\pi}\alpha\frac{\gamma}{R}\frac{1}{\omega_c}\int_{\omega/\omega_c}^{\infty} K_{5/3}(\eta)d\eta,$$

where α is the fine structure constant, γ is the electron Lorentz factor, R is the instantaneous radius of curvature of the trajectory, and ω_c is the characteristic XSR photon energy. $K_{5/3}$ is the modified Bessel function of the third kind. Both R and ω_c depends on $B_\perp$, the component of magnetic field flux density (magnetic induction) perpendicular to the electron momentum. The value of $B_\perp$ is usually measured in Tesla, therefore it is convenient to use the SI system of units:

$$R = \frac{mc\beta\gamma}{eB_\perp}, \quad \omega_c = \frac{3}{2}\frac{e\hbar}{m}\gamma^2 B_\perp,$$

where e and m are the electron charge and mass, c is the speed of light in vacuum, and $\beta^2 = 1 - \gamma^{-2} \sim 1$ for practical cases correspond to $\gamma \gg 1$.

The mean free path of XSR λ_{xsr} reads:

$$\frac{d\bar{N}}{dx} = \frac{1}{\lambda_{xsr}} = \frac{\sqrt{3}}{2\pi}\alpha\frac{\gamma}{R}\int_0^\infty K_{5/3}(\eta)d\eta = \frac{5\alpha}{2\sqrt{3}}\frac{\gamma}{R} = \frac{5\alpha}{2\sqrt{3}}\frac{eB_\perp}{mc\beta}.$$

Practically, for $\gamma \gg 1$:

$$\lambda_{xsr}(cm) \sim 16/B_\perp(T), \quad \bar{N} \simeq 6.2L(\mathrm{m})B_\perp(\mathrm{T}).$$

Therefore XSR is relatively rare process which is convenient to implement as discrete.

The Monte-Carlo generation of XSR energy spectrum requires the calculation of the following double integral ($\xi = \omega/\omega_c$):

$$P_{>\xi} = \frac{3}{5\pi}\int_\xi^\infty d\xi' \int_{\xi'}^\infty K_{5/3}(\eta)d\eta.$$

Usually the expansion of $K_{5/3}$ over Chebyshev polynomials is used. However one can use the integral representation for $K_{5/3}$:

$$K_\nu(\eta) = \int_0^\infty \cosh(\nu t)\exp[-\eta\cosh(t)]dt,$$

allowing one to reduce the number of integrations for $P_{>\xi}$:

$$P_{>\xi} = \int_0^\infty \frac{\cosh\left(\frac{5}{3}t\right)}{\cosh^2(t)} \exp\left[-\xi \cosh(t)\right] dt.$$

The latter integral is calculated numerically by the quadrature Laguerre formula. Calculations indicate that about 50 roots of the Laguerre polynomials are required in order for the accuracy of the integral estimation to be better than 10^{-4}.

The angular distribution of XSR reads:

$$\frac{d^3\bar{N}}{d\xi d\tilde{\psi} dx} = \frac{3\alpha}{4\pi^2}\frac{eB_\perp}{mc}\xi[1+\tilde{\psi}^2]\left\{\frac{\tilde{\psi}^2}{1+\tilde{\psi}^2}K^2_{1/3}(\eta) + K^2_{2/3}(\eta)\right\},$$

$$\xi = \frac{\omega}{\omega_c}, \quad \tilde{\psi} = \gamma\psi, \quad \eta = \frac{\xi}{2}[1+\tilde{\psi}^2]^{3/2},$$

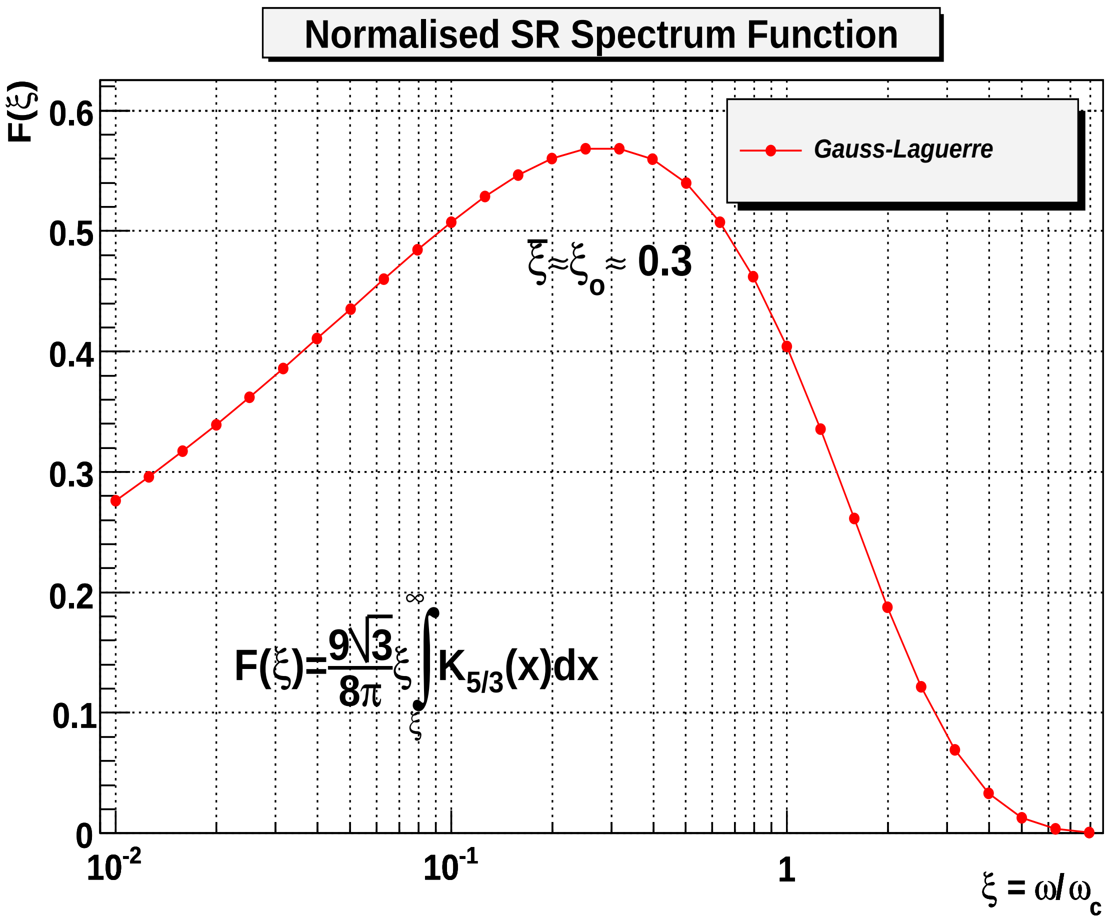
where ψ is the angle between the XSR photon direction and the plane of instantaneous rotation of the electron. XSR photons are emitted in narrow $\sim \gamma^{-1}$ cone around the electron momentum.

3 G4SynchrotronRadiation class

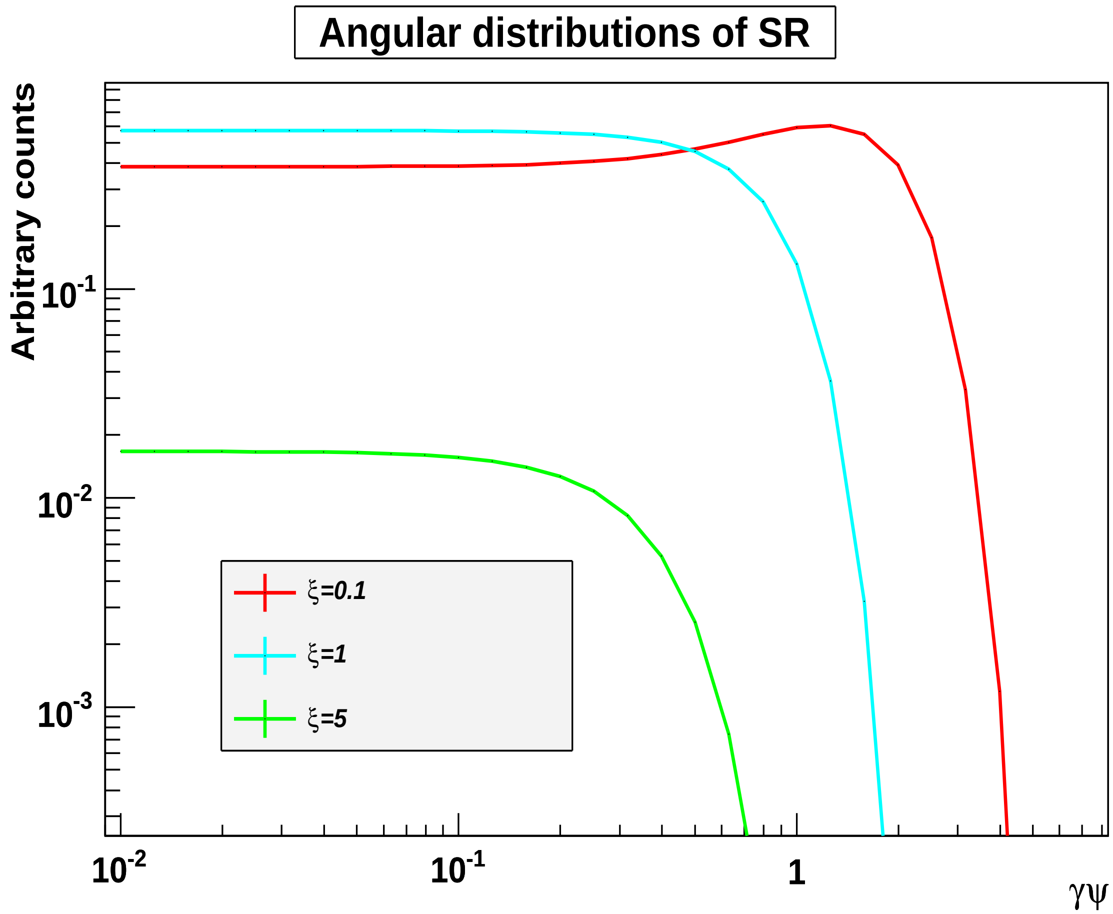
1. Magnetic field can be set in a volume in global frame.
2. Energy spectrum is distributed according to high energy small angle approximation (is applicable for $\gamma > 10^3$):

$$\omega_c(\text{keV}) \simeq 1.74 \cdot 10^{-7} \gamma^2 B_\perp(\text{T}),$$

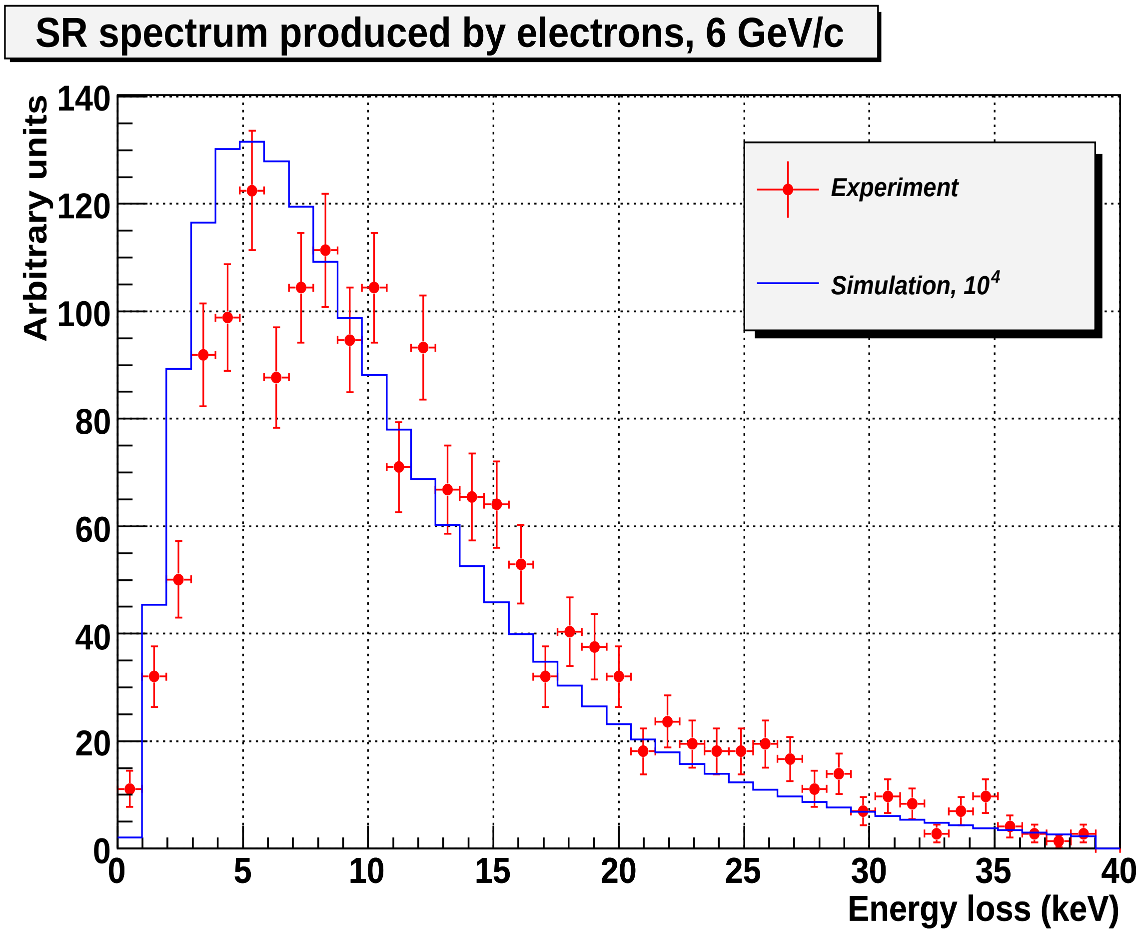
   therefore G4Gamma particles are secondaries (not G4OpticalPhoton ?).
3. Angular distribution is simplified to be flat random in the limits of the reciprocal electron Lorentz factor value.
4. Polarization is set approximately to be linear (perpendicular to $B_\perp$ and SXR photon momentum direction).



The normalized energy spectrum of XSR photons.



The angular distribution of XSR photons in terms of the angle between the photon direction and the plane of instantaneous rotation of the electron.



The XSR energy spectrum produced by electrons with the momentum 6 GeV/c in magnetic field $B = 1$ Tesla. Histogram is GEANT4 simulation, points are ALICE XTR test beam measurements

4 XSR in Materials

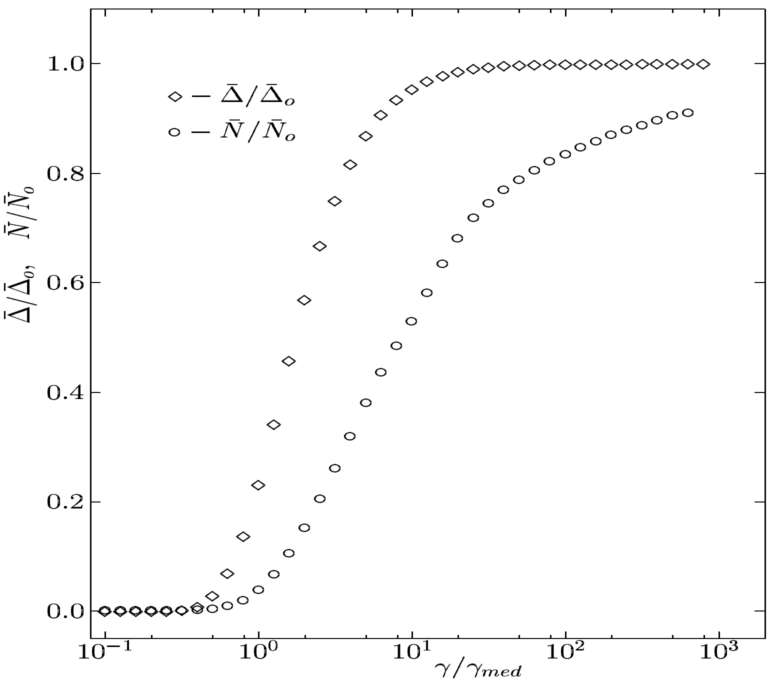
XSR in materials experiences the suppression of the energy and number spectra in the region of low energies. For X-ray photons the dielectric permittivity reads:

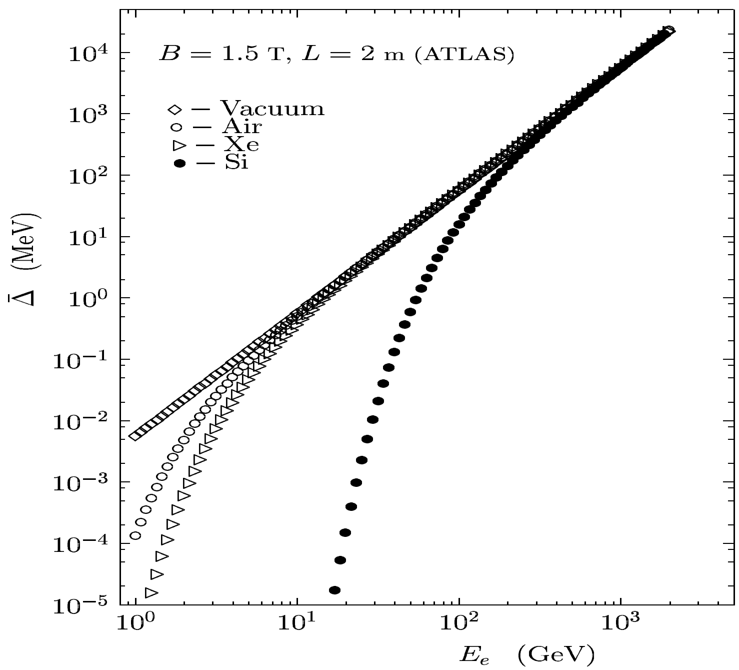
$$\varepsilon \simeq Re(\varepsilon) = 1 - \left(\frac{\omega_p}{\omega}\right)^2, \quad Im(\varepsilon) \ll |1 - Re(\varepsilon)| \ll 1,$$

where ω_p is the plasma energy. Theoretical analysis shows that for small $\gamma < \gamma_{med}$, where γ_{med} reads:

$$\gamma_{med} = \frac{\gamma^2 \omega_p}{\omega_c} = \frac{2}{3}\frac{m}{e\hbar}\frac{\omega_p}{B_\perp} \simeq 5.8 \cdot 10^3 \frac{\omega_p(eV)}{B_\perp(T)},$$

the medium begins to influence more and more on the low energy part of the X-ray synchrotron radiation energy spectrum. The number of emitted XSR photons shows relativistic rise in the broad range $\gamma/\gamma_{med} = 1 - 1000$.


$\diamond - \bar{\Delta}/\bar{\Delta}_o$
$\circ - \bar{N}/\bar{N}_o$
1.0
0.8
0.6
0.4
0.2
0.0
$\bar{\Delta}/\bar{\Delta}_0, \quad \bar{N}/\bar{N}_0$
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
γ/γ_{med}


$B = 1.5$ T, $L = 2$ m (ATLAS)
◇ – Vacuum
○ – Air
▷ – Xe
● – Si
$\bar{\Delta}$ (MeV)
E_e (GeV)
10^{4}
10^{3}
10^{2}
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{0}
10^{1}
10^{2}
10^{3}

5 Conclusions

1. GEANT4 X-ray synchrotron radiation (XSR) process is implemented as discrete electromagnetic process.
2. GEANT4 XSR process shows satisfactory agreement with known experimental data for energy spectrum of emitted quanta.
3. GEANT4 XSR process can be extended to describe the magnetic bremsstrahlung in different materials.