

# Fundamental principles of particle physics

Our description of the fundamental interactions and particles rests on two fundamental structures :

- Quantum Mechanics
- Symmetries

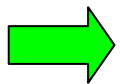
# Symmetries

Central to our description of the fundamental forces :

Relativity - translations and Lorentz transformations

Lie symmetries -  $SU(3) \otimes SU(2) \otimes U(1)$

Copernican principle : “Your system of co-ordinates and units is nothing special”



Physics independent of system choice

# Fundamental principles of particle physics

Quantum Mechanics

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Relativity



Quantum Field theory

## Special relativity

- Space time point  $a^\mu = (ct, x, y, z)$  not invariant under translations
- Space-time vector  $(a + \Delta a)^\mu - a^\mu = \Delta a^\mu = (c\Delta t, \Delta x, \Delta y, \Delta z)$

Invariant under translations ...but not invariant under rotations or boosts

- Einstein postulate : the real invariant distance is

$$(\Delta a^0)^2 - (\Delta a^1)^2 - (\Delta a^2)^2 - (\Delta a^3)^2 = \sum_{\mu, \nu=0}^3 g_{\mu\nu} \Delta a^\mu \Delta a^\nu = \Delta a^\mu \Delta a_\mu = (\Delta a)^2$$

$$g_{\mu\nu} = \text{diag}(+1, -1, -1, -1)$$

- Physics invariant under all transformations that leave all such distances invariant :

Translations and Lorentz transformations

## Lorentz transformations :

$$x^\mu \rightarrow \Lambda_\nu^\mu x^\nu = \sum_{\nu=0}^3 \Lambda_\nu^\mu x^\nu = \tilde{x}^\mu \quad \Rightarrow \quad g_{\mu\nu} \tilde{x}^\mu \tilde{x}^\nu = g_{\mu\nu} x^\mu x^\nu \quad \Rightarrow \quad g_{\mu\nu} \Lambda_\alpha^\mu \Lambda_\beta^\nu = g_{\alpha\beta}$$

*(Summation assumed)*

### Solutions :

- 3 rotations R

$$R_z(\theta) \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \theta & \sin \theta & 0 \\ 0 & -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix} = \begin{pmatrix} ct \\ x \cos \theta + y \sin \theta \\ y \cos \theta - x \sin \theta \\ z \end{pmatrix}$$

## Lorentz transformations :

$$x^\mu \rightarrow \Lambda_\nu^\mu x^\nu = \sum_{\nu=0}^3 \Lambda_\nu^\mu x^\nu = \tilde{x}^\mu \quad \Rightarrow \quad g_{\mu\nu} \tilde{x}^\mu \tilde{x}^\nu = g_{\mu\nu} x^\mu x^\nu \quad \Rightarrow \quad g_{\mu\nu} \Lambda_\alpha^\mu \Lambda_\beta^\nu = g_{\alpha\beta}$$

(Summation assumed)

### Solutions :

- 3 rotations R

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \theta & \sin \theta & 0 \\ 0 & -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

- Space reflection – parity P

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

- 3 boosts B

$$\begin{pmatrix} \cosh \alpha & \sinh \alpha & 0 & 0 \\ \sinh \alpha & \cosh \alpha & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

- Time reflection, time reversal T

$$\begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

- The Lorentz transformations form a group,  $G$  ( $g_1 g_2 \in G$  if  $g_1, g_2 \in G$ )

## Rotations

$$R(\theta) = e^{-i\mathbf{J}\cdot\theta/\hbar}, \quad J_z = i\hbar(x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x}).. \quad \text{Angular momentum operator}$$

(c.f.  $\mathbf{J} = \mathbf{r} \times \mathbf{p}$ )

$$R_z(\theta) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \theta & \sin \theta & 0 \\ 0 & -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$R_z(\theta) = e^{-iJ_z\theta/\hbar} = 1 - iJ_z\theta/\hbar + \frac{1}{2}(-iJ_z\theta/\hbar)\cdot(-iJ_z\theta/\hbar) + \dots$$

$$-iJ_z/\hbar = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

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## Rotations

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(c.f.  $\mathbf{J} = \mathbf{r} \times \mathbf{p}$ )

$$[J_i, J_j] = i\hbar \sum_{k=1}^3 \epsilon_{ijk} J_k$$

$\epsilon_{ijk}$  totally antisymmetric Levi-Civita symbol,

$$\epsilon_{123} = \epsilon_{231} = \epsilon_{312} = +1; \quad \epsilon_{213} = \epsilon_{132} = \epsilon_{321} = -1$$

The  $J_i$  are the “generators” of the group.

Their commutation relations define a “Lie algebra”.



Demonstration that  $R_Z(\theta) = e^{-iJ_z\theta}$

$$R_Z(\theta)\psi(x, y) \equiv R_Z(\theta) \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x' \\ y' \end{pmatrix} \equiv \psi(x', y')$$

For small  $\theta = \varepsilon$ , 
$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} x + \varepsilon y \\ y - \varepsilon x \end{pmatrix}$$

$$\begin{aligned} R_Z(\varepsilon)\psi(x, y) &= \psi(x + \varepsilon y, y - \varepsilon x) \approx \psi(x, y) + \varepsilon \left( y \frac{\partial \psi}{\partial x} - x \frac{\partial \psi}{\partial y} \right) \\ &= (1 - i\varepsilon(xp_y - yp_x))\psi(x, y) \end{aligned}$$

i.e.  $R_Z(\varepsilon) = (1 - i\varepsilon(xp_y - yp_x)) = 1 - i\varepsilon J_z$

Hence

$$R_Z(\theta = n\varepsilon) \approx (1 - i\varepsilon J_z)^n \xrightarrow{n \rightarrow \infty} e^{-iJ_z\theta}$$

## Derivation of the commutation relations of $SO(3)$ ( $SU(2)$ )

$$R_x(\varepsilon)R_y(\eta)R_x(-\varepsilon)R_y(-\eta) \quad \varepsilon, \eta \text{ small (infinitesimal)}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & \varepsilon \\ 0 & -\varepsilon & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & \eta \\ 0 & 1 & 0 \\ -\eta & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -\varepsilon \\ 0 & \varepsilon & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & \eta \\ 0 & 1 & 0 \\ -\eta & 0 & 1 \end{pmatrix}$$

$$\approx \begin{pmatrix} 1 & \varepsilon\eta & 0 \\ -\varepsilon\eta & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = R_z(\varepsilon\eta) \approx (1 - i\varepsilon\eta J_z)$$

$$R_x(\varepsilon)R_y(\eta)R_x(-\varepsilon)R_y(-\eta)$$

$$= (1 - i\varepsilon J_x)(1 - i\varepsilon J_y)(1 + i\varepsilon J_x)(1 + i\varepsilon J_y) = -\varepsilon\eta(J_x J_y - J_y J_x)$$

Equating the two equations implies

$$[J_x, J_y] = iJ_z$$

QED

# Fundamental principles of particle physics

Quantum Mechanics

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Relativity



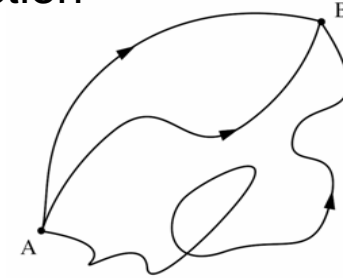
Quantum Field theory

# Relativistic quantum field theory

Fundamental division of physicist's world :

		speed	
		slow	fast
Action (S)	large	Classical Newton	Classical relativity
	small	Classical Quantum mechanics	Quantum Field theory
		 c	

Action



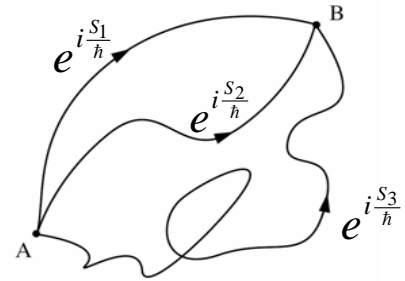
$$S_{classical} = \int_{t_A}^{t_B} (K.E. - P.E.) dt$$

“Principle of Least Action”  
Feynman Lectures in Physics  
Vol II Chapter 19

# Relativistic quantum field theory

Fundamental division of physicist's world :

		speed	
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A c t i o n  (S)	large	Classical Newton	Classical relativity
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		$\xrightarrow{\hspace{2cm}}$ <b>C</b>	



$\hbar$  (QM amplitude  $\propto e^{i(\frac{S}{\hbar})}$ )

## Quantum Mechanics : Quantization of dynamical system of particles

## Quantum Field Theory : Application of QM to dynamical system of fields

Why fields?

- No right to assume that any relativistic process can be explained by single particle since  $E=mc^2$  allows pair creation
- (Relativistic) QM has physical problems. For example it violates causality

Amplitude for free propagation from  $x_0$  to  $x$

$$\begin{aligned} U(t) &= \langle x | e^{-i(p^2/2m)t} | x_0 \rangle = \int d^3 p \langle x | e^{-i(p^2/2m)t} | p \rangle \langle p | x_0 \rangle \\ &= \frac{1}{2\pi^3} \int d^3 p e^{-i(p^2/2m)t} e^{ip(x-x_0)} \\ &= \left( \frac{m}{2\pi i t} \right)^{3/2} e^{im(x-x_0)^2/2t} \dots \text{nonzero for all } x, t \end{aligned}$$

Relativistic case :  $U(t) \propto e^{-m\sqrt{x^2-t^2}} \dots \text{nonzero for all } x, t$

# Quantum Mechanics

$$E - \frac{p^2}{2m} = 0$$

Classical – non relativistic

$$i\hbar \frac{\partial \phi}{\partial t} + \frac{\hbar^2}{2m} \nabla^2 \phi = 0$$

Quantum Mechanical : Schrodinger eq

# Quantum Mechanics

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Quantum Mechanical : Schrodinger eq

$$E^2 - \mathbf{p}^2 = m^2$$

Classical – relativistic

$$-\frac{\partial^2 \phi}{\partial t^2} + \nabla^2 \phi = m^2 \phi$$

Quantum Mechanical - relativistic



- **Relativistic QM - The Klein Gordon equation (1926)**

Scalar particle (field) (J=0) :  $\phi(\mathbf{x})$

$$E^2 = \mathbf{p}^2 + m^2 \quad \Rightarrow \quad -\frac{\partial^2 \phi}{\partial t^2} + \nabla^2 \phi = m^2 \phi \quad (\text{natural units})$$

Energy eigenvalues  $E = \pm(\mathbf{p}^2 + m^2)^{1/2}$  ???

1927 Dirac tried to eliminate negative solutions by writing a relativistic equation linear in E (a theory of fermions)

1934 Pauli and Weisskopf revived KG equation with  $E < 0$  solutions as  $E > 0$  solutions for particles of opposite charge (antiparticles). Unlike Dirac's hole theory this interpretation is applicable to bosons (integer spin) as well as to fermions (half integer spin).

As we shall see the antiparticle states make the field theory causal

