

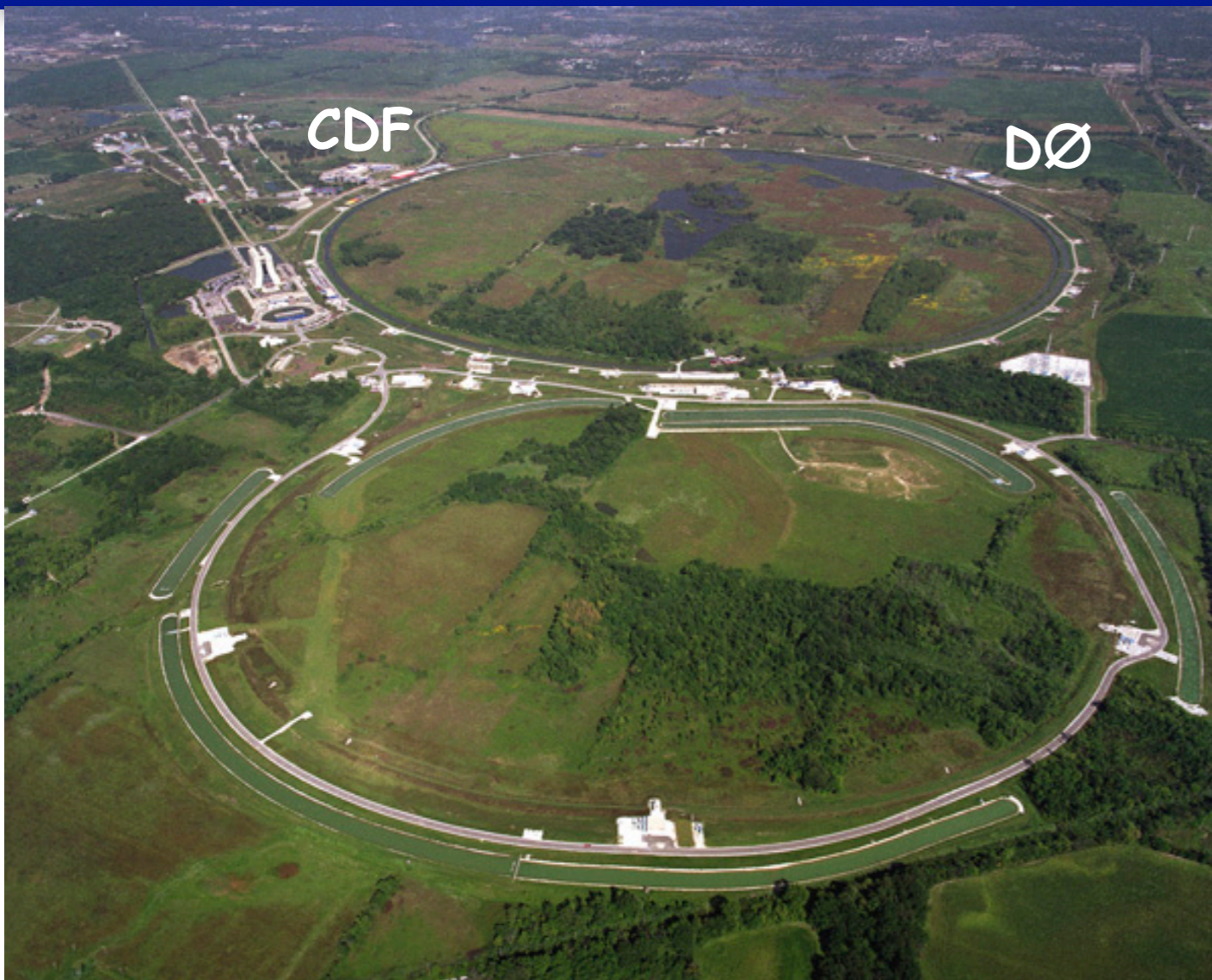
# Measurement of the Top Quark Mass with the Matrix Element Method at the Tevatron Run II

- Outline:**
- The Tevatron and its Detectors in Run II
  - Top Quark Pair Production and Decay at Tevatron
  - The Top Quark Mass
  - The Matrix Element Method
  - Applications of the Matrix Element Method to Run II Data
  - Summary



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The World's only top quark factory to date!

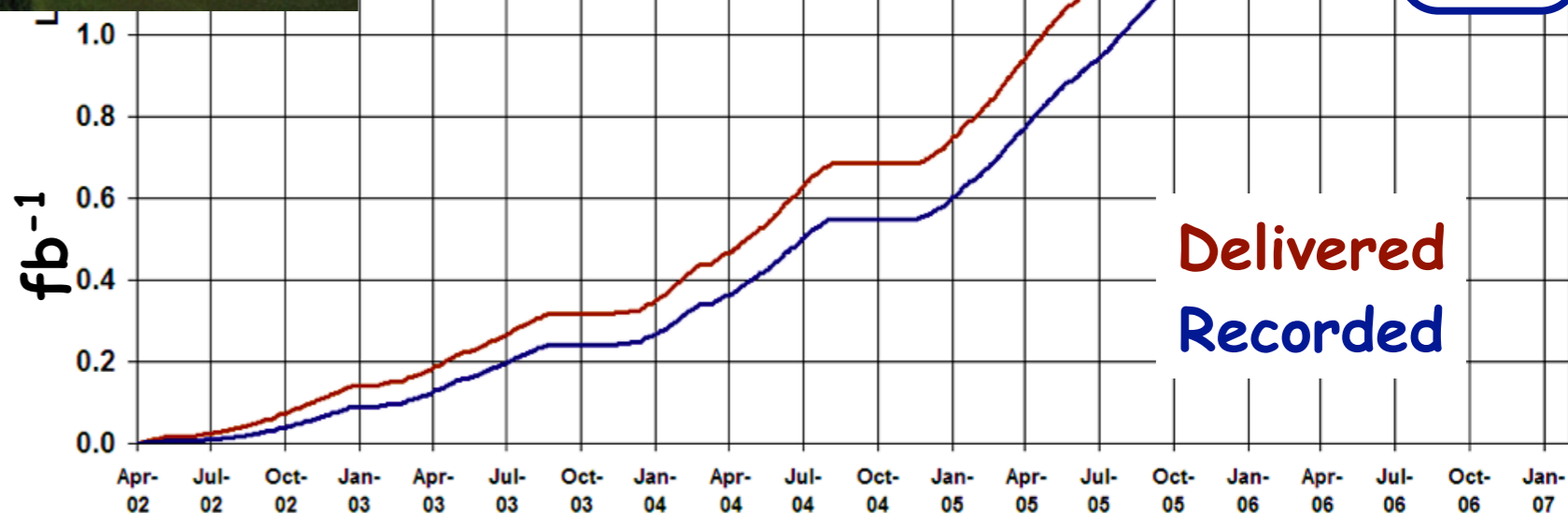
## Run II Integrated Luminosity (DØ)

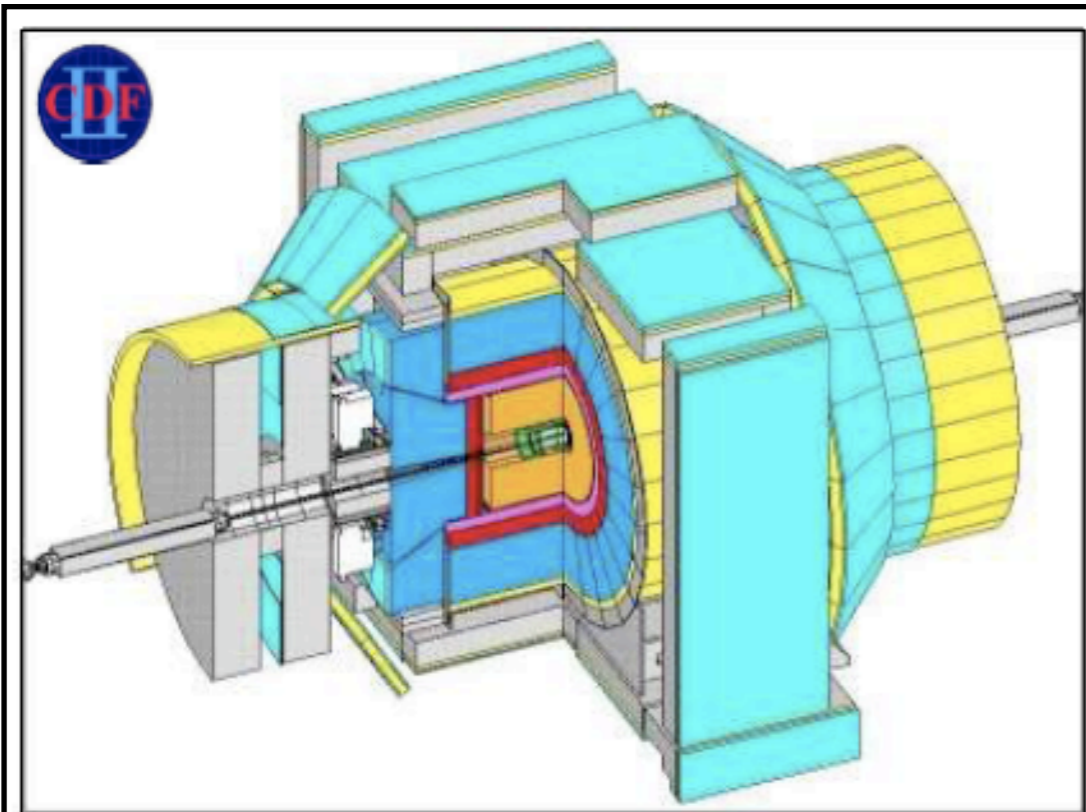
19 April 2002 - 21 January 2007

2.38

2.00

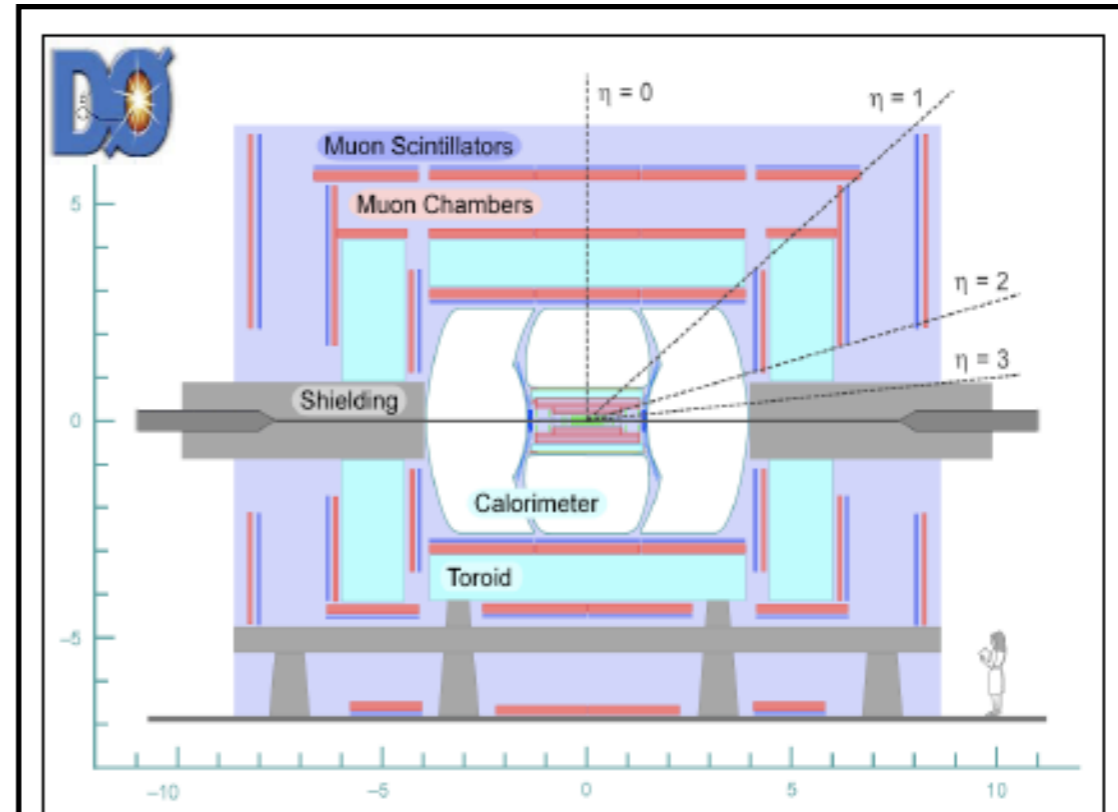
p-pbar collisions  
at  $\sqrt{s} = 1.96$  TeV  
bunch-crossing: 396ns





## CDF Run II Upgrades:

- New Silicon Vertex Detector (SVX) and faster tracking drift chamber (COT)
- New scintillating-tile end-plug calorimeters
- Increased  $\eta\phi$  coverage for muon detectors
- New Scintillator Time-Of-Flight system



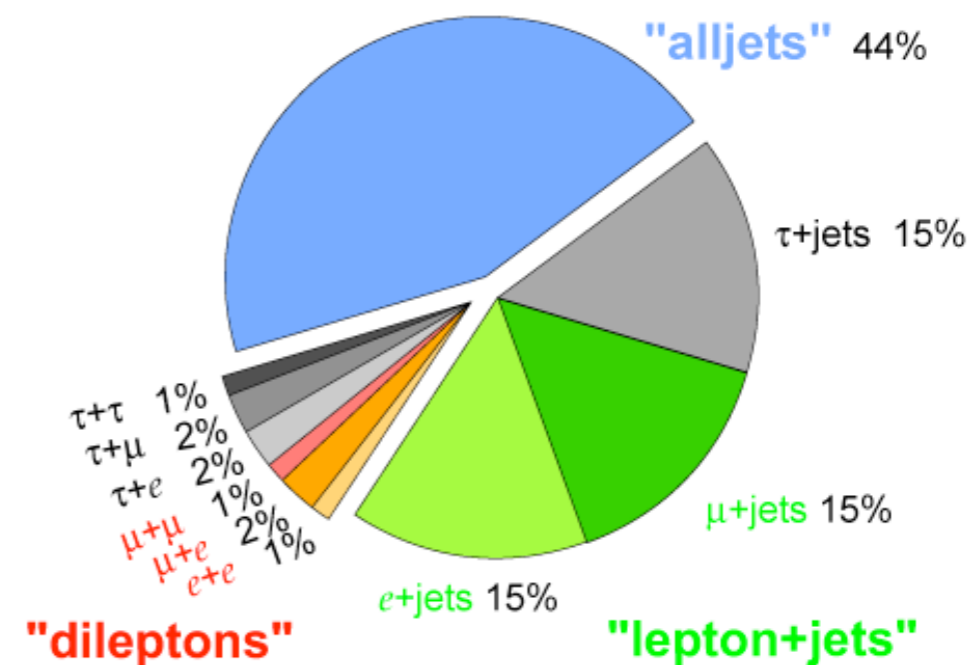
## DØ Run II Upgrades:

- New silicon (SMT) and Fiber (CFT) trackers, placed in new 2T Solenoid
- Calorimeters supplemented with Preshower detectors
- Significantly improved Muon System

Both detectors underwent major DAQ/Trigger upgrades w.r.t to Run I to cope with the reduced bunch-crossing rate of 396 ns (Run I: 2.4  $\mu$ s, LHC: 25 ns)

- In p-pbar collisions at  $\sqrt{s} = 1.96$  TeV, top quarks are primarily produced in pairs
  - ★ Standard Model cross-section  $\sim 7$  pb
  - ★  $\sim 85\%$  via quark annihilation
  - ★  $\sim 15\%$  via gluon fusion
- top decays almost exclusively to  $Wb$  (SM)
- "Leptons" here refers to Electrons or Muons

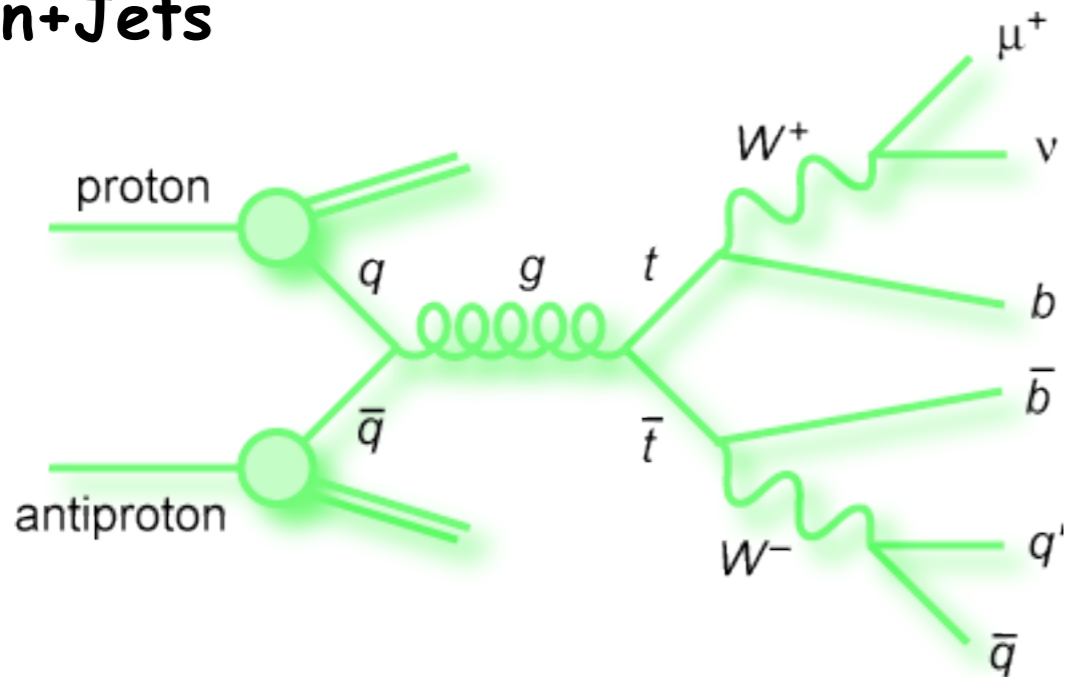
Top Pair Branching Fractions



	All-Jets	Lepton+Jets	Dilepton
BR	$\sim 44\%$	$\sim 30\%$	$\sim 5\%$
S/B	1:1000 to 1:4	1:4 to 11:1 varies w/ #b-tags	$\sim 2:1$
Backgrounds	Jet Production	W+jets fake leptons	Z+jets WW, WZ

# Lepton+Jets and Dilepton Final-States

## Lepton+Jets



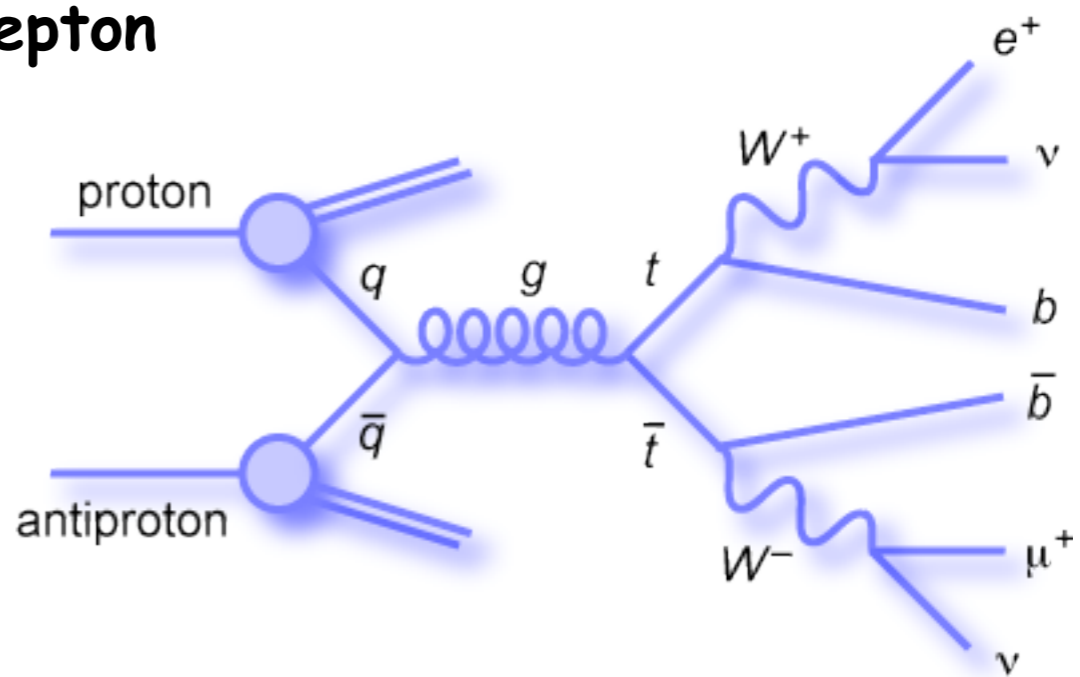
### Signature:

- Four [or more] Calorimeter Jets
- Exactly one Isolated Energetic Lepton (e or  $\mu$ )
- Significant Missing Transverse Energy  $> 20\text{GeV}$
- $p_T > 20\text{GeV}$  required for jets/lepton
- 24 possible jet/parton assignments  $\rightarrow$  use life-time tagging information

### Backgrounds:

- $W$ +jets production
- Instrumental background due to fake leptons (Jet Production, "QCD")

## Dilepton



### Signature:

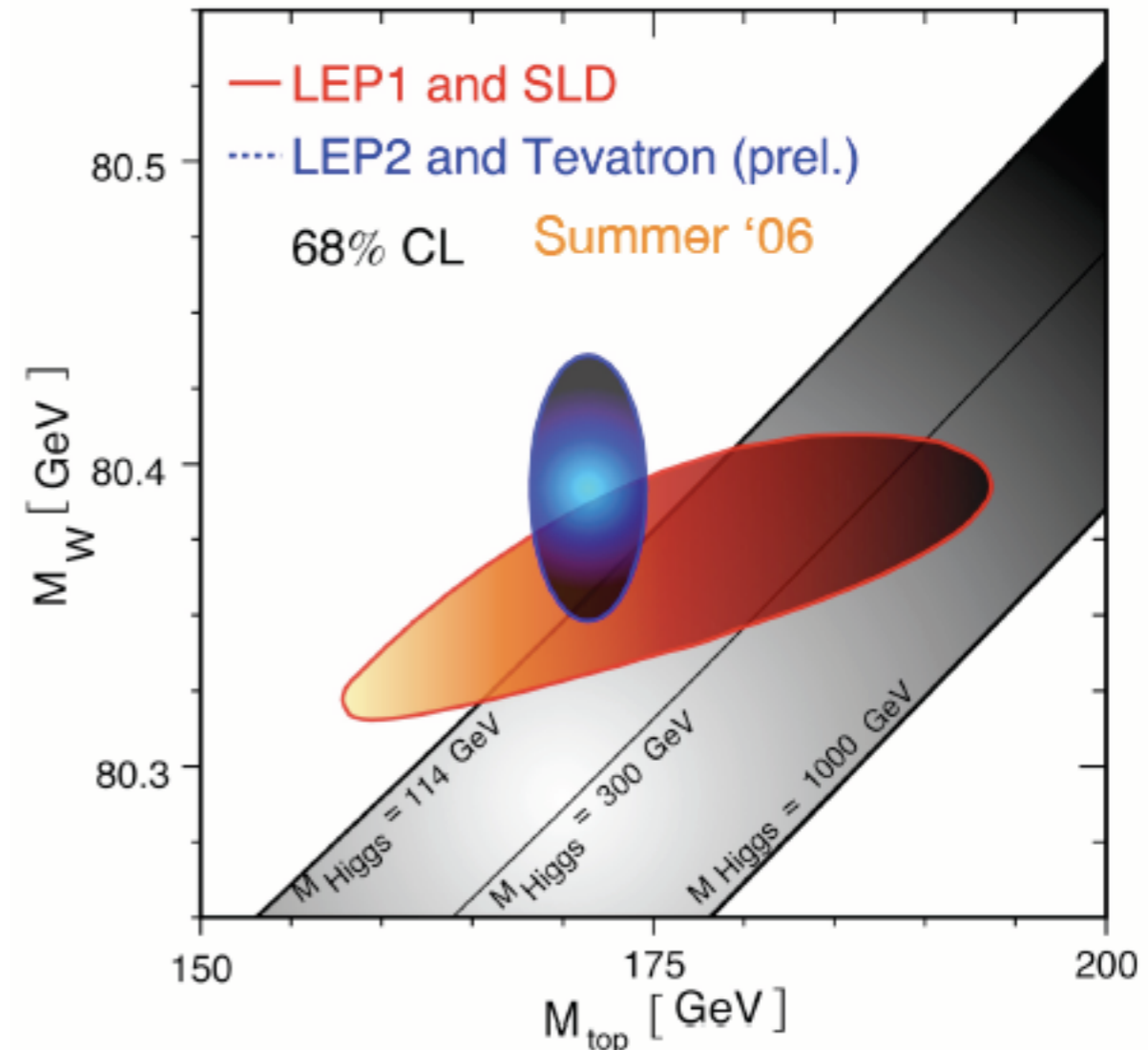
- Two [or more] Calorimeter Jets
- Exactly two Isolated Energetic Leptons (e or  $\mu$ )
- Significant Missing Transverse Energy  $> 20\text{GeV}$
- $p_T > 20\text{GeV}$  required for jets/leptons
- 2 possible jet/parton assignments; life-time tagging information to increase S/B

### Backgrounds:

- $WW, Z$ +jets production
- Instrumental background due to fake leptons (WZ production)

- The top quark mass is a free parameter of the Standard Model
- The top quark is by far the heaviest of the six known quarks
- Its suspiciously high mass suggests a special role of the top quark in the Standard Model yet to be revealed
- Precision measurements of the top quark and  $W$  boson masses constrain the mass of the Higgs boson via radiative corrections
- With  $4\text{-}8\text{fb}^{-1}$  of Tevatron data:  $\delta m_t \sim 1.5\text{ GeV}$  (CDF and DØ combined)

Tevatron top quark mass measurements will be relevant for many years, even after LHC turnon

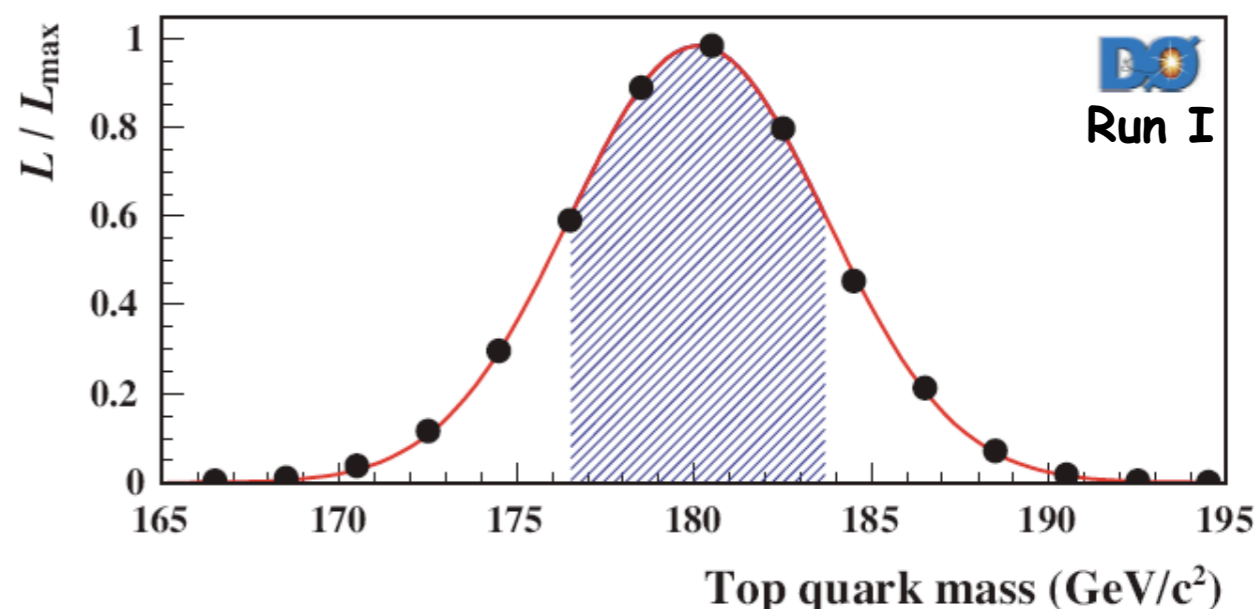


- Developed during Run I, Application to D0 Lepton+Jets dataset yielded most precise Tevatron Run I result:

$$m_{\text{top}} = 180 \pm 5.3 \text{ GeV}$$

$$(\pm 3.6 \text{ (stat.)} \pm 3.9 \text{ (syst.) GeV})$$

Nature 429, 638 (2004)



- Applied to 370 pb<sup>-1</sup> of D0 Run II Data (Lepton+Jets) to yield most precise D0 result
  - ★ FERMILAB-THESIS 2005-46 (320 pb<sup>-1</sup>, topological analysis)
  - ★ Phys.Rev. D74, 092005, 2006 (370 pb<sup>-1</sup>, topological + b-tagging)
- Application to 940 pb<sup>-1</sup> CDF Run II Data (Lepton+Jets) yields world's best measurement
- Application to 1.03 fb<sup>-1</sup> of CDF Run II Data (Dilepton) yields world's best measurement in the Dilepton channel
  - ★ Phys.Rev.Lett. 96,152002, 2006



# Basics of the ME Method



- Use each event's full kinematic information to calculate its probability to originate from  $t$ - $t$ bar production, as a function of the top mass  $m_t$ .
- Calculate its probability to be produced via the background process ( $W$ + $jjjj$ ) accordingly, and combine both to an event probability  $P_{\text{evt}}$ :

$$P_{\text{evt}}(\mathbf{x}; m_t) = f_{\text{sgn}} P_{\text{sgn}}(\mathbf{x}; m_t) + (1 - f_{\text{sgn}}) P_{\text{bkg}}(\mathbf{x})$$

$f_{\text{sgn}}$ : signal fraction  
 $\mathbf{x}$  : event's kinematic variables

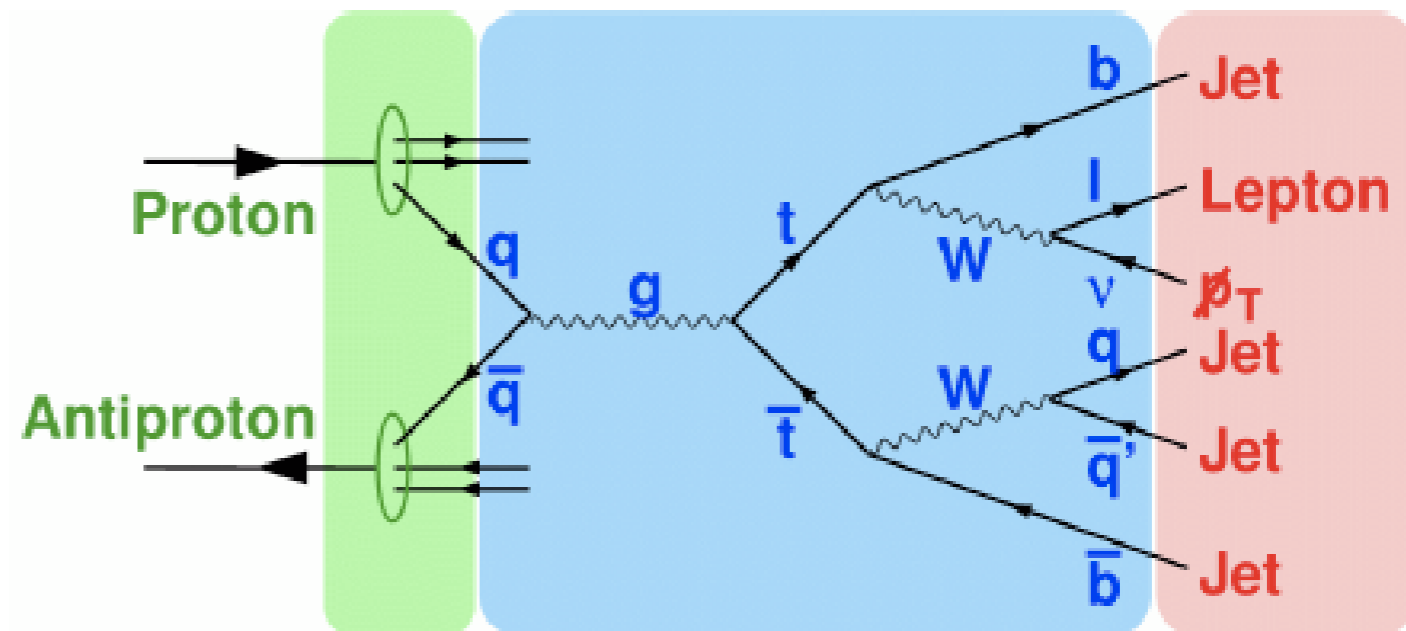
- Combine Event Probabilities of all  $n$  events in the dataset into a log likelihood:

$$-\ln L(\mathbf{x}_1, \dots, \mathbf{x}_n; m_t, f_{\text{sgn}}) = \sum_i \ln P_{\text{evt}}(\mathbf{x}_i; m_t, f_{\text{sgn}})$$

- Measure top quark mass by minimizing log likelihood w.r.t.  $m_t$  and  $f_{\text{sgn}}$ .
- **Achieve optimal use of statistical information by treating events individually: well measured events contribute more than poorly measured events.**
- The instrumental background from misidentified leptons in All-Jets events is not explicitly modeled, expected to be  $Wjjjj$ -like, the difference QCD/ $Wjjjj$  is treated as a systematic uncertainty



- Integration over parton phasespace
- Assume all angles (jets, lepton) to be well measured, as well as the energy of the electron
- Parametrize Detector-Resolution of jets and muons using Monte Carlo (transfer function  $W(x,y)$ )
- Calculate remaining 5(6)-dimensional integral using MC techniques (VEGAS)



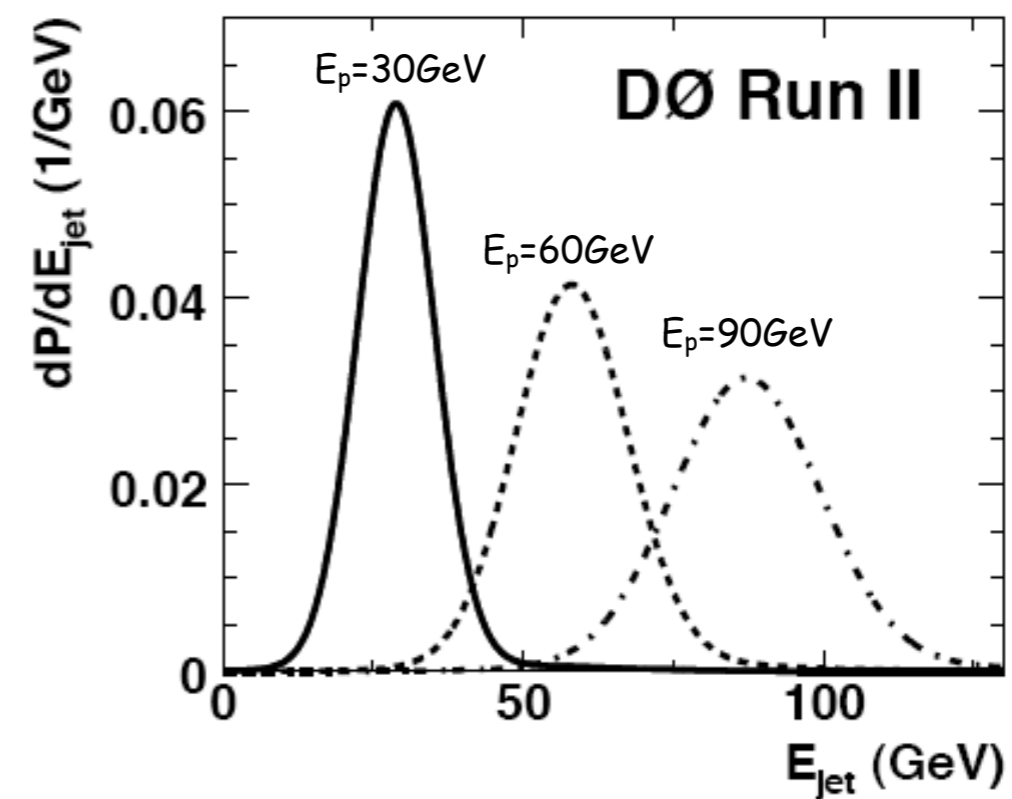
$$\mathcal{P}_{t\bar{t}}(\vec{x}, m_t) = \underbrace{\frac{1}{\sigma_{t\bar{t}}(m_t)}}_{\text{Normalisation}} \int \underbrace{dp_q dp_{\bar{q}} f(p_q) f(p_{\bar{q}})}_{\text{PDFs}} \underbrace{d\sigma_{t\bar{t}}(\vec{y}, m_t)}_{\substack{\text{diff. xsec} \\ \text{MATRIX ELEMENT}}} \underbrace{W(\vec{x}, \vec{y})}_{\text{det. resolutions}}$$

- **Consider all 12 relevant jet-parton assignments**
  - ★ 24 possible combinations, exclude permutations where the two hadronic W daughters are exchanged
- Presence of the escaping neutrino creates a quadratic ambiguity in the kinematic solution of each event: **consider both neutrino solutions.**

- Transfer functions describe the detector resolution of jets and muons:  $W(x,y)$  yields the Probability of a **parton state  $y$**  to be reconstructed as **detector state  $x$**
- $x$  and  $y$  are the respective kinematic variables; since angles are assumed to be well measured, transfer functions need to be derived for jet and muon energies.
- **Jet** Parametrization:  $W(E_{\text{jet}}, E_{\text{parton}}) = F(E_{\text{jet}} - E_{\text{parton}}) = F(\delta)$ :

$$F(\delta) = \frac{1}{\sqrt{2\pi}(p_2 + p_3 p_5)} \left[ \exp \frac{-(\delta - p_1)^2}{2p_2^2} + p_3 \exp \frac{-(\delta - p_4)^2}{2p_5^2} \right]$$

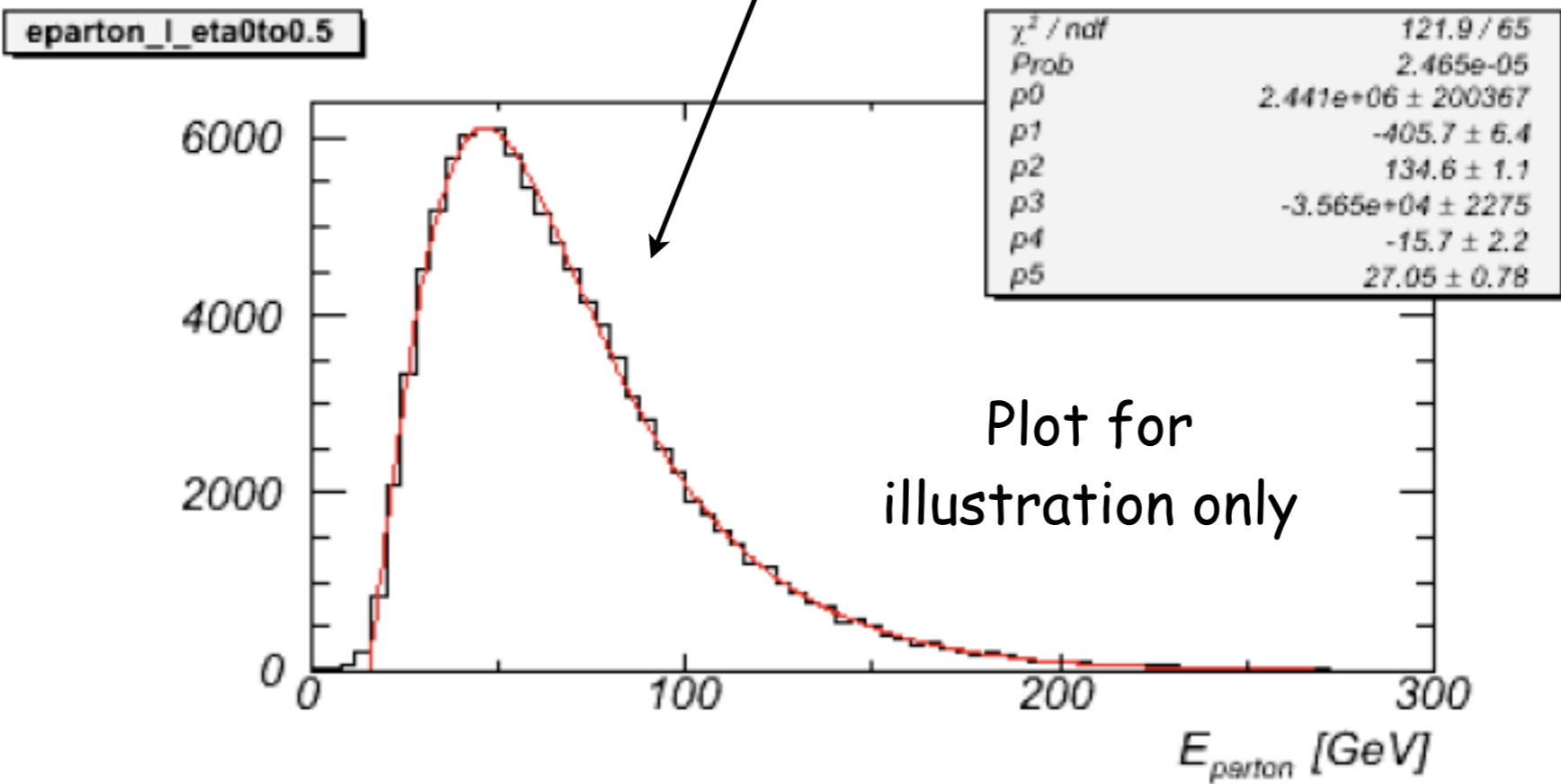
- Assume  $p_i = a_i + b_i E_{\text{parton}}$  and derive the 10 parameters with a likelihood fit from  $t$ - $t$ bar Monte Carlo Events for light,  $b$ -, and  $b(\rightarrow\mu)$  - jets
- **Muons**:  $1/p_T$  parametrized as Gaussian with  $|\eta|$  - dependent width

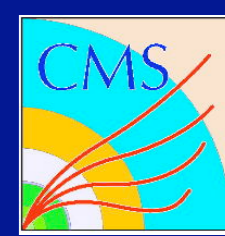


# Transfer Function Cross-Check

- To cross-check the derived parameters, compare MC jet energies with the prediction from the transfer functions
- The transfer function prediction  $H(\delta E)$  is computed by integration over  $E_p$ :

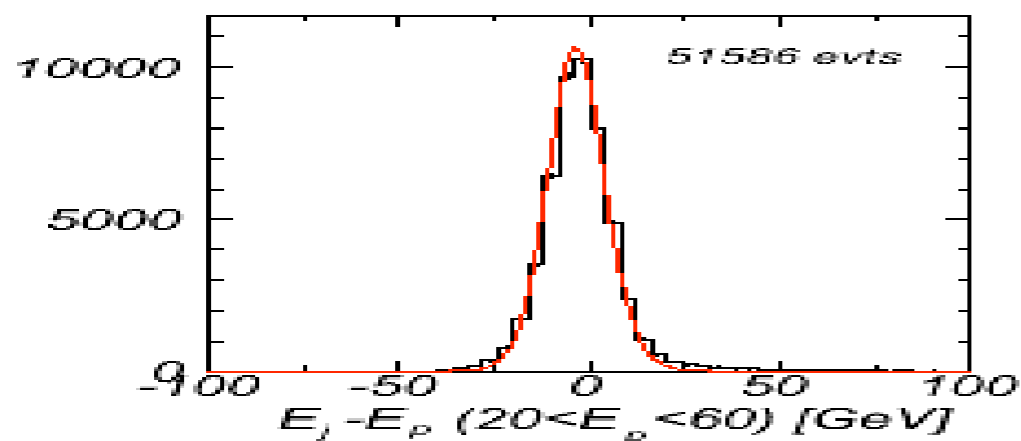
$$H(\delta E) = \int_{E_p^{min}}^{E_p^{max}} dE_p n(E_p) W(E_p, E_p + \delta E)$$



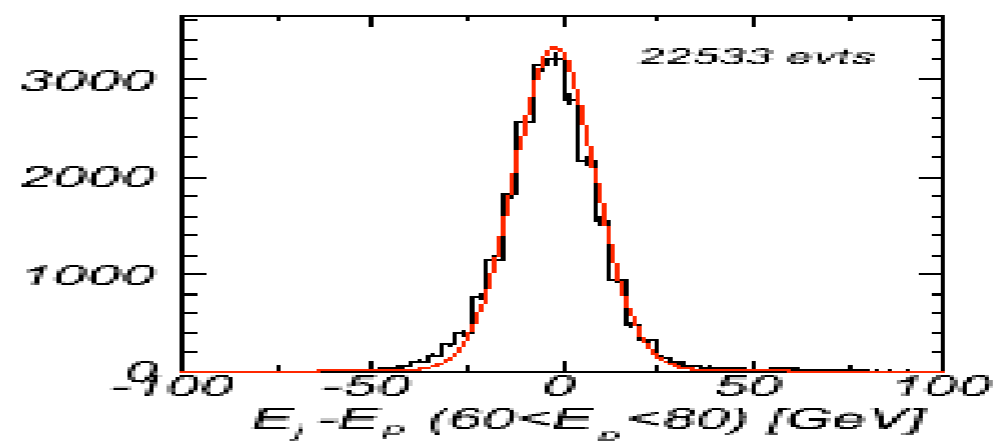


# Transfer Function Cross-Check (2)

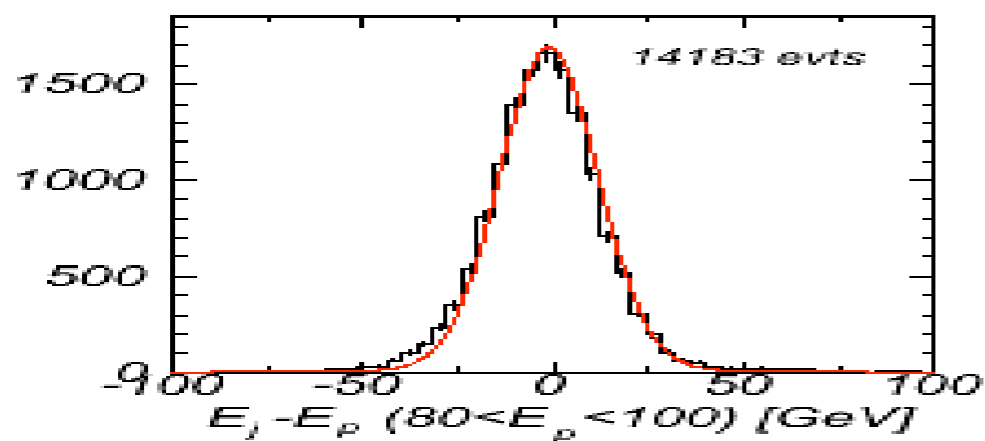
deltae\_l\_eta0to0.5\_ep20to60



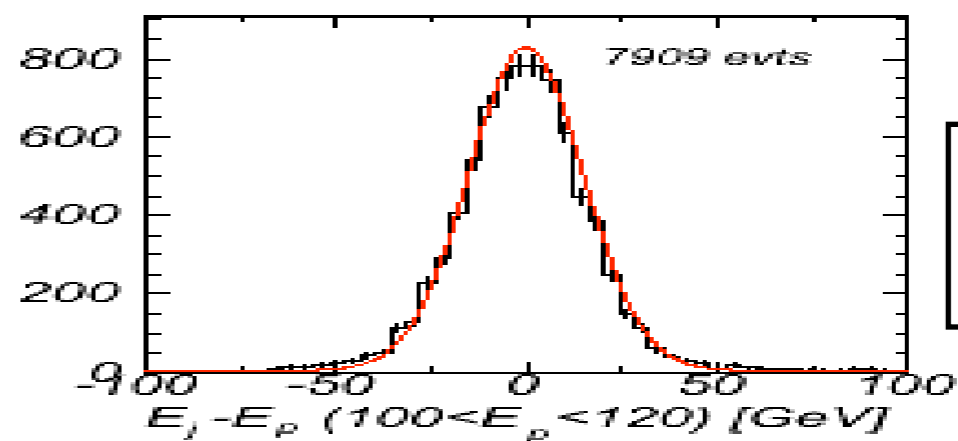
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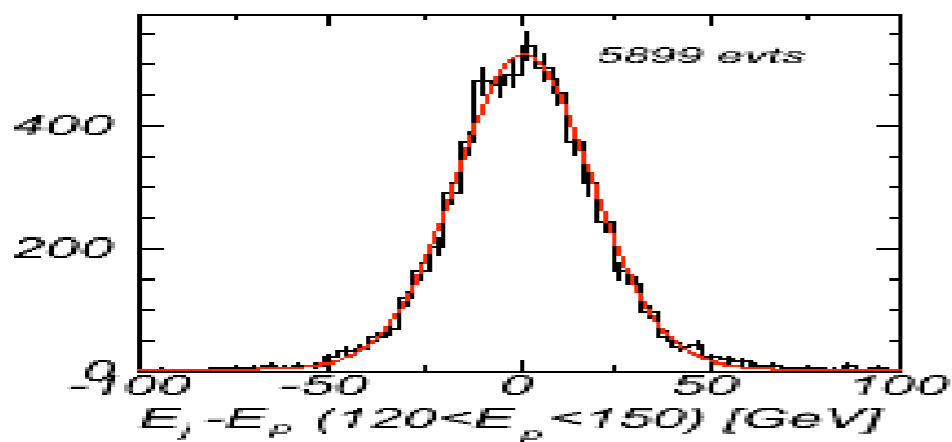
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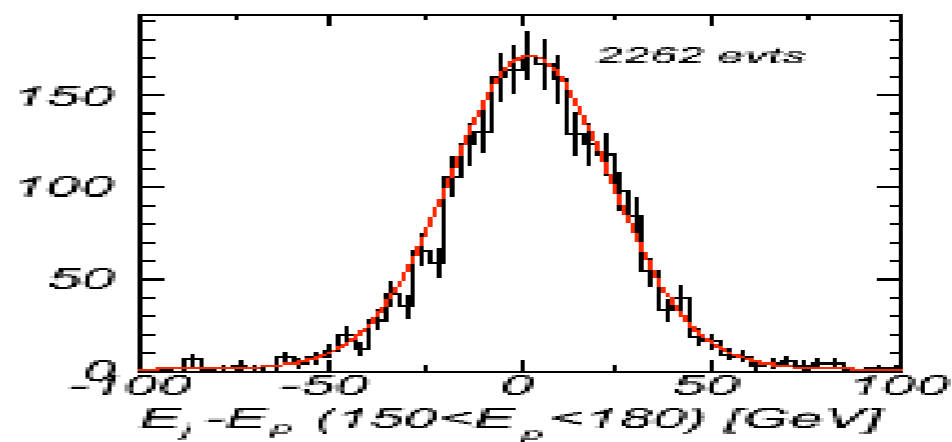
deltae\_l\_eta0to0.5\_ep100to120



deltae\_l\_eta0to0.5\_ep120to150



deltae\_l\_eta0to0.5\_ep150to180



Plots for Illustration only

Monte Carlo  
TF Prediction

$E_{jet} - E_{parton}$



# Jet Energy Scale



- The Jet Energy Scale is by far the dominant systematic uncertainty for a top mass measurement
- The Jet Energy Scale for D0 RunII is derived independently from photon+jets events as a function of  $p_T$  and  $\eta$ .
- If this calibration is off by a **global scale factor "JES"**, than this global factor can be introduced in the ME likelihood function as an additional parameter
  - ★ sensitivity comes from hadronic W decay in Lepton+Jets events: variation of the W mass value translates to a variation of the global JES scale factor
  - ★ Define "JES" such that it is 1.0 if the Jet Energy Scale calibration function is spot on
  - ★ It must be considered that the shape of the calibration might be off as well. "Residual JES uncertainty" (Turns out to be small).
  - ★ Minimization of the extended log likelihood

$$-\ln L(x_1, \dots, x_n; m_t, \mathbf{JES}, f_{sgn}) = \sum_i \ln P_{evt}(x_i; m_t, \mathbf{JES}, f_{sgn})$$

yields the **statistical + JES error** of the measurement!

- ★ The "JES" variables is introduced to the event probability via the Transfer Function:

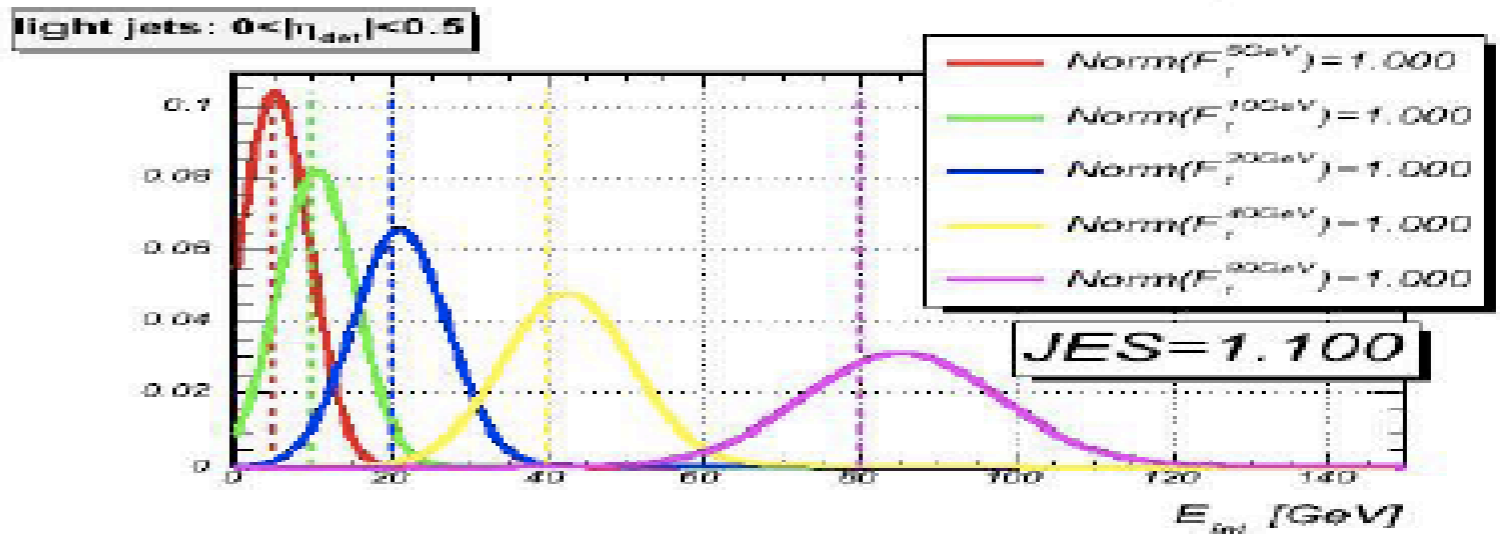
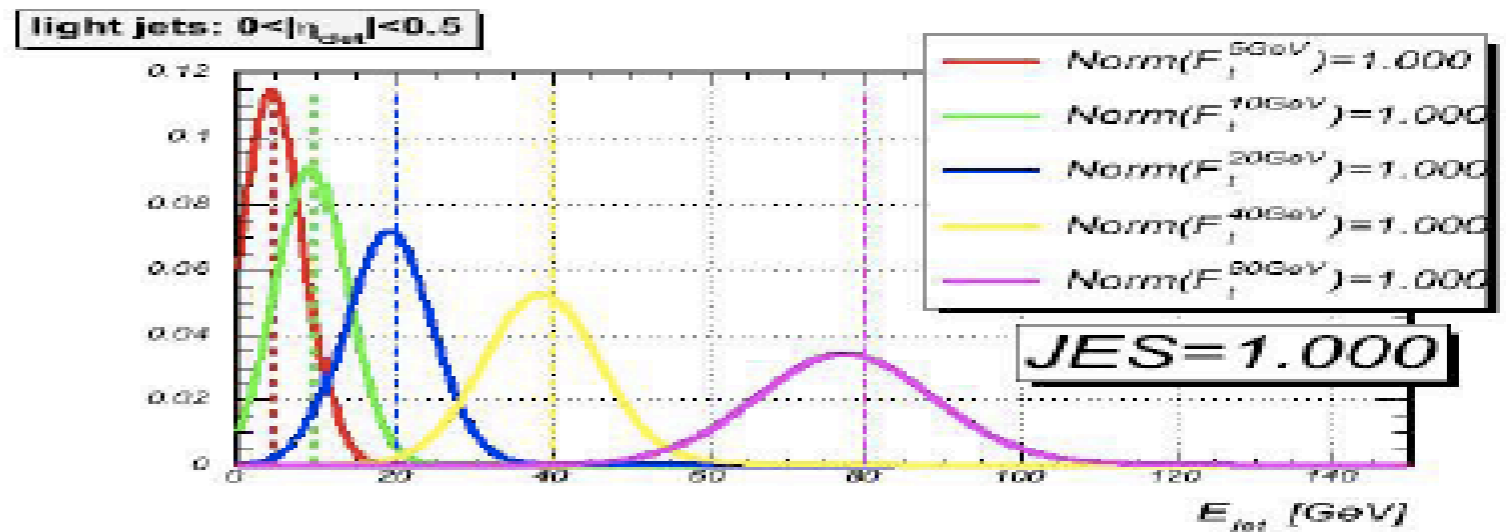
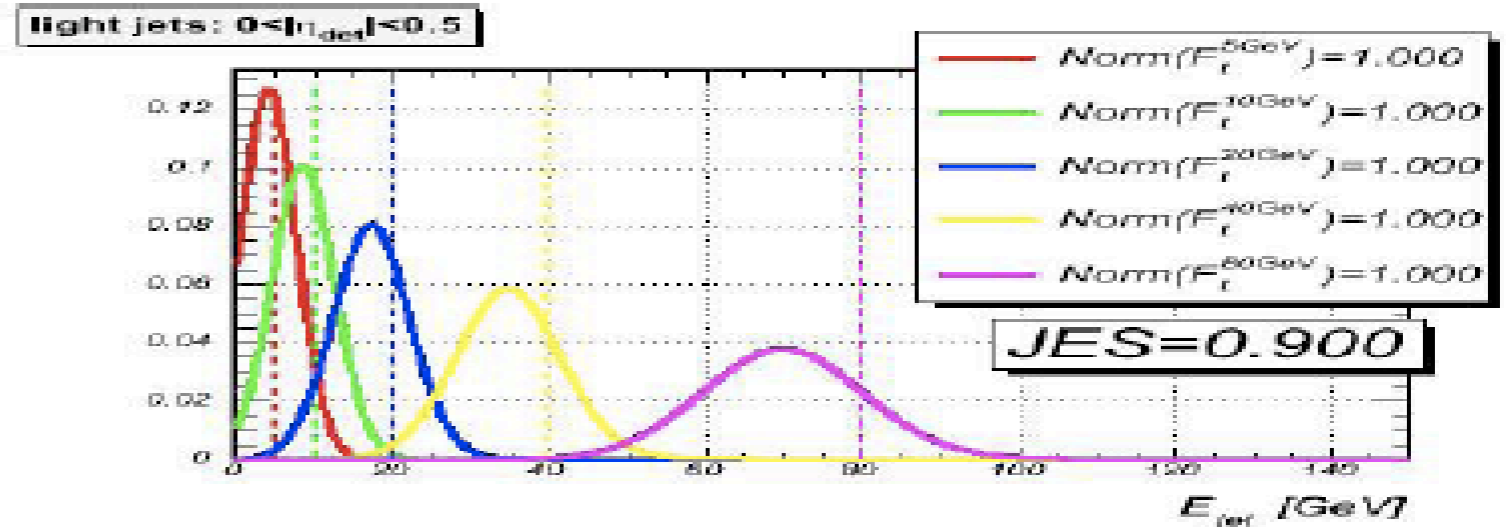
$$W(E_j, E_p; \mathbf{JES}) = W(E_j / \mathbf{JES} - E_p) / \mathbf{JES}$$

- Illustration of how the JES global scale parameter is absorbed by the Transfer Function Parameters
- For  $JES < 1.0$ , the most likely  $E_{jet}$  for a fixed  $E_{parton}$  decreases, the Gaussian "moves to the left"
- For  $JES > 1.0$ , the most likely  $E_{jet}$  for a fixed  $E_{parton}$  increases, the Gaussian "moves to the right"
- To convince yourself of the overall factor  $1/JES$ , consider

$$\int W(E_j, E_p) dE_j = 1$$

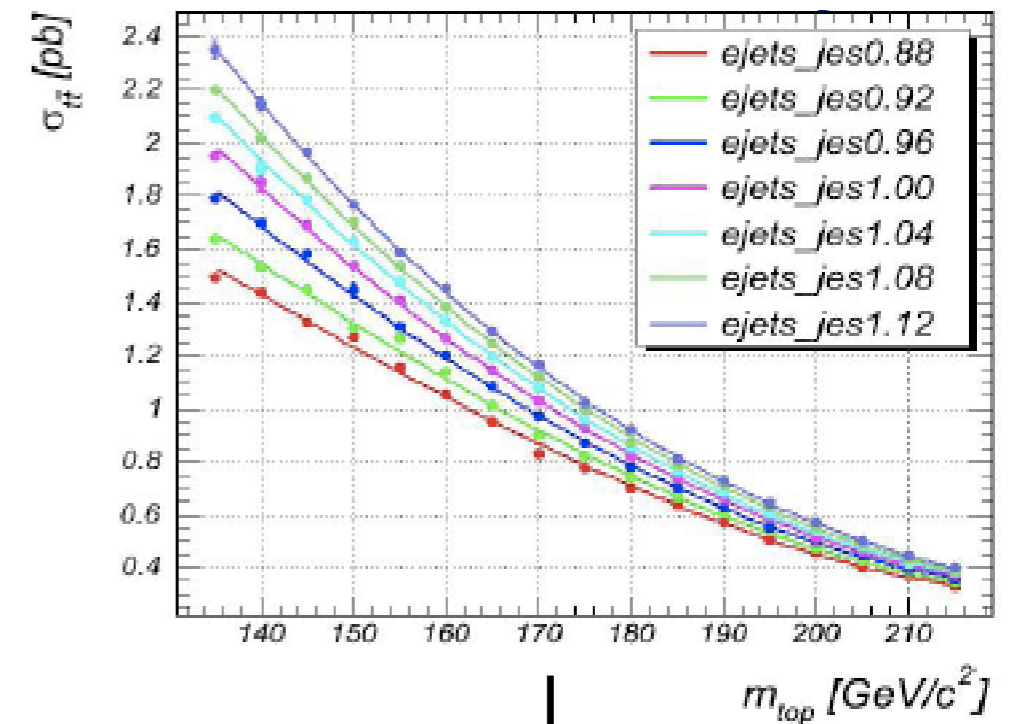
and

$$\int f(ax) dx = 1/a \int f(u) du, u=ax$$

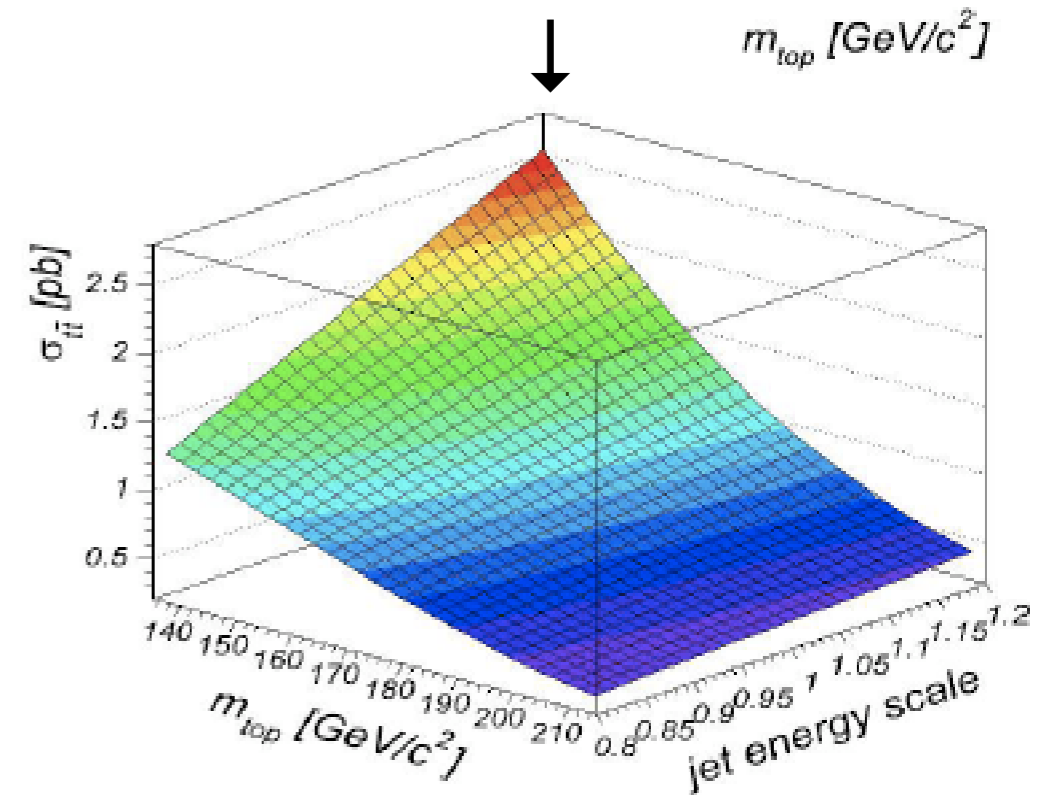
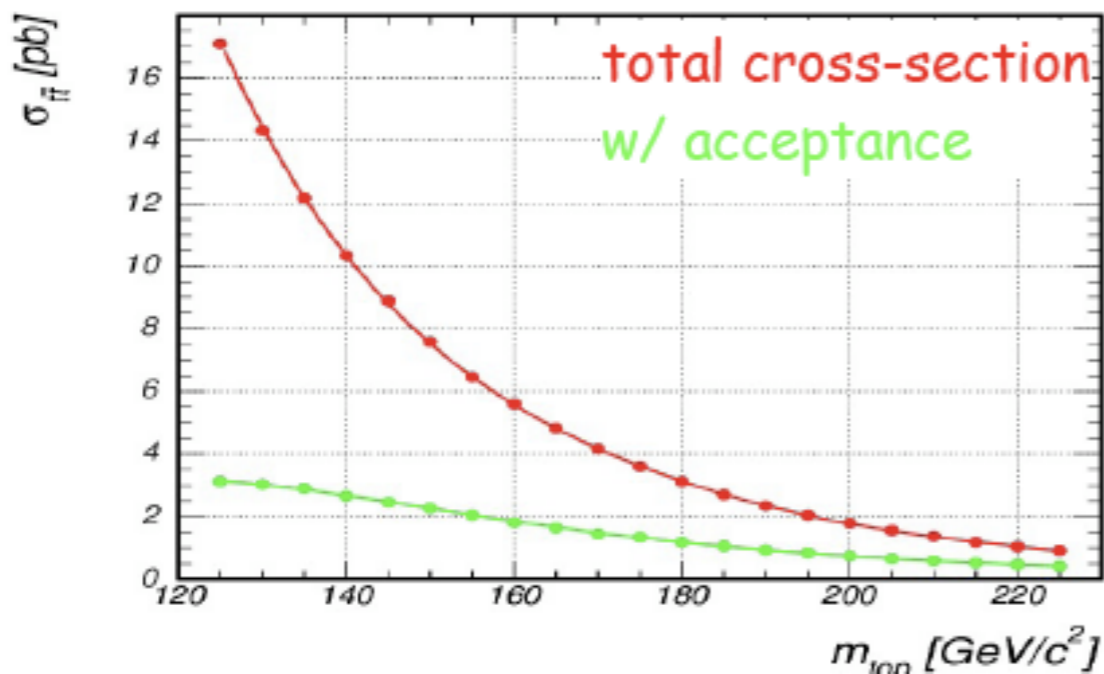


- Normalization = cross-section of the LO Matrix Element, **taking the event selection efficiency into account!**
- Monte Carlo Integration over Parton Phasespace, plus Jet/Muon Energies:  $16 + 4 + 1 = 21$  dim. Integral.
- Transfer Function provide  $E_{parton}/E_{jet}$  mapping during integration: relation between kinematic selection and parton phasespace, take kinematic selection into account!
- The normalization is (via the kinematic selection cuts!) also a function of the global Jet Energy scale Factor "JES", **2D Parametrization**

$t\bar{t} \rightarrow l\nu q\bar{q}b\bar{b}$ : cross section



$t\bar{t} \rightarrow l\nu q\bar{q}b\bar{b}$ : cross section





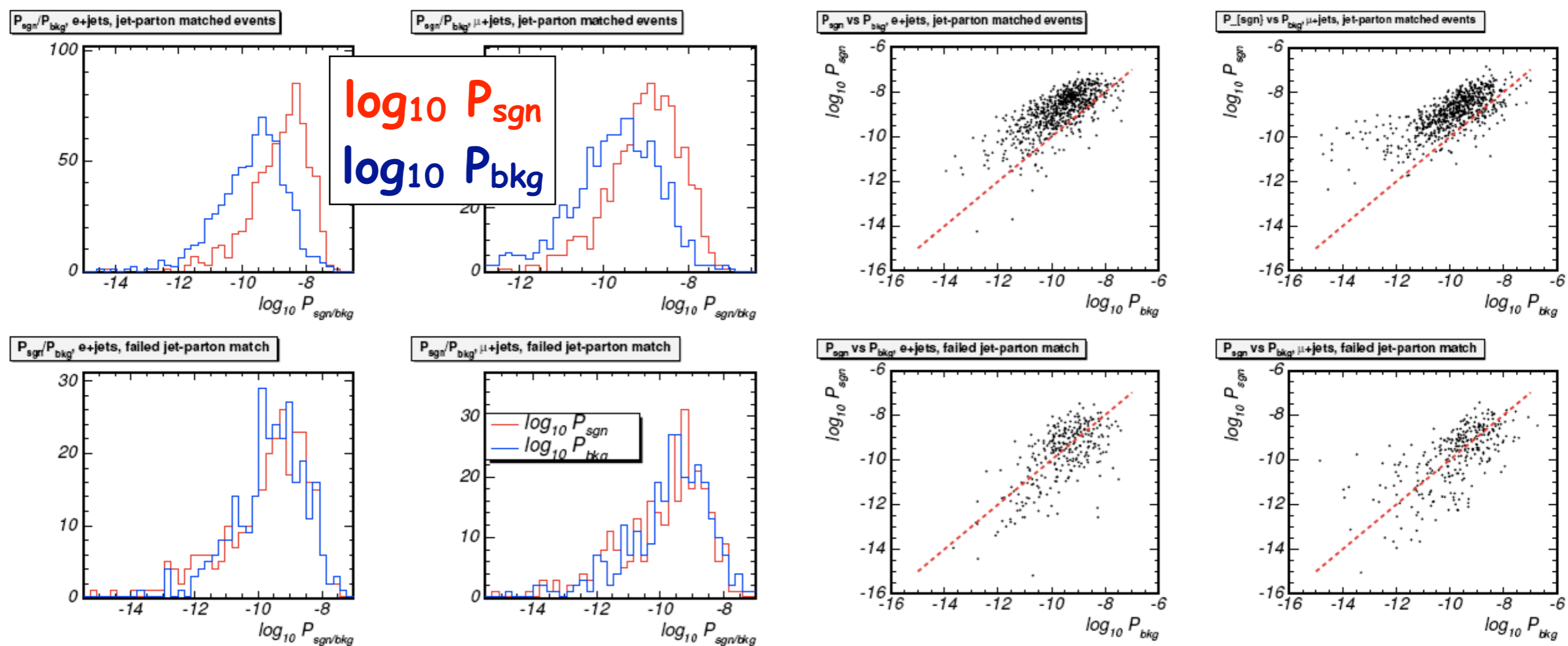
# $P_{bkg}$ Normalization



- The correct normalization of the Background ( $W+jjjj$ ) Probability ensures an unbiased estimate of the sample composition in the fit ( $f_{sgn}$ ).
- Crucial! Systematic over-estimation leads to top-mass bias, under-estimation to non-optimal use of the statistical information of the top events in the sample!
- This Normalization is not derived by calculating the cross-section integral; instead, an iterative calibration procedure is applied to MC events such that the fit produces the correct  $f_{sgn}$  estimate on average for the expected signal fraction in the dataset
- As demonstrated on the next slide,  $t$ - $t$ bar events affected by Radiation don't behave "signal-like": the calculated  $P_{sgn}$  is smaller than the calculated  $P_{bkg}$ .
- We identify this class of events in Monte Carlo by not being able to match all reconstructed jets to the partons from the LO-ME. (Consequently, "good events" are labeled "**jet-parton matched events**")
- By only using jet-parton matched events in the  $P_{bkg}$  calibration, the "bad" events are effectively treated as background, significantly improving the bias and pull of the likelihood fit.



"Bad" t-tbar events bias the mass estimate and increase the pull



These effects can be minimized by appropriate  $P_{bkg}$  normalization

- b-tagging can be incorporated in the analysis via the kinematic selection: requiring at least one jet to be b-tagged significantly enhances the signal fraction (CDF)
- Optimal use of the statistical information is achieved if events with 0, 1, or 2 b-tags are treated as individual samples (w.r.t.  $f_{\text{sgn}}!$ ) and then combined:

$$L = \prod_{0\text{tags}} P_{\text{evt}}^{0\text{tag}}(x; m_{\text{top}}, JES) \prod_{1\text{tag}} P_{\text{evt}}^{1\text{tag}}(x; m_{\text{top}}, JES) \prod_{2\text{tag}} P_{\text{evt}}^{2\text{tag}}(x; m_{\text{top}}, JES)$$

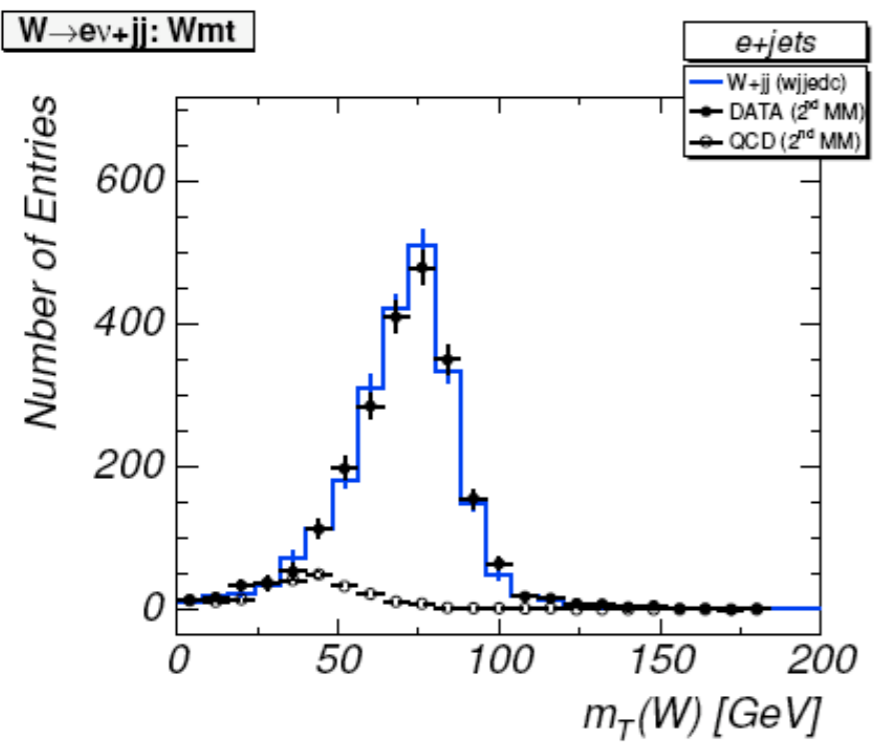
- Moreover, all possible jet parton assignments can be weighted according to the b-tagging information for further statistical gain
  - ★ A permutation with a b-tag corresponding to a b quark gets a high weight
  - ★ A permutation with a b-tag corresponding to a light quark gets a low weight (>0, charm!)
- For the 370 pb<sup>-1</sup> Lepton-Jets sample, this technique amounts to ...
  - ★ ... a **20% improvement in statistical sensitivity** w.r.t. the topological analysis
  - ★ ... a 30% improvement in statistical sensitivity w.r.t. a ">=1 b-tag" selection requirement

# The Data Sample

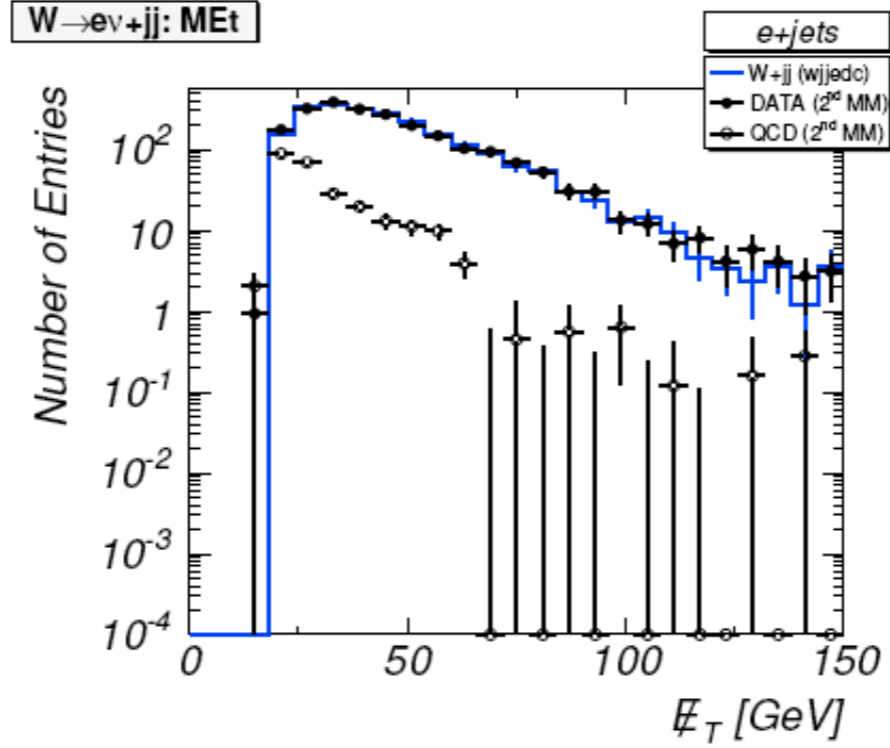
- Since the method is calibrated with Monte Carlo events, it is of course crucial to ensure that the recorded data sample is indeed accurately described by our simulation
- Therefore, we select  $W+n$ -jets events and compare them to the Monte Carlo for jet multiplicities  $n=1,2,3$ , and  $\geq 4$  (the signal bin). The **instrumental background is subtracted** with a special technique from the data before comparison to MC.
- Good agreement for all jet multiplicities for all relevant kinematic quantities leaves us with the necessary confidence that  $W$ +jets production is well described by our simulation, and that we can quantify the contribution from instrumental background

## Examples from $W(\rightarrow ev)+jj$

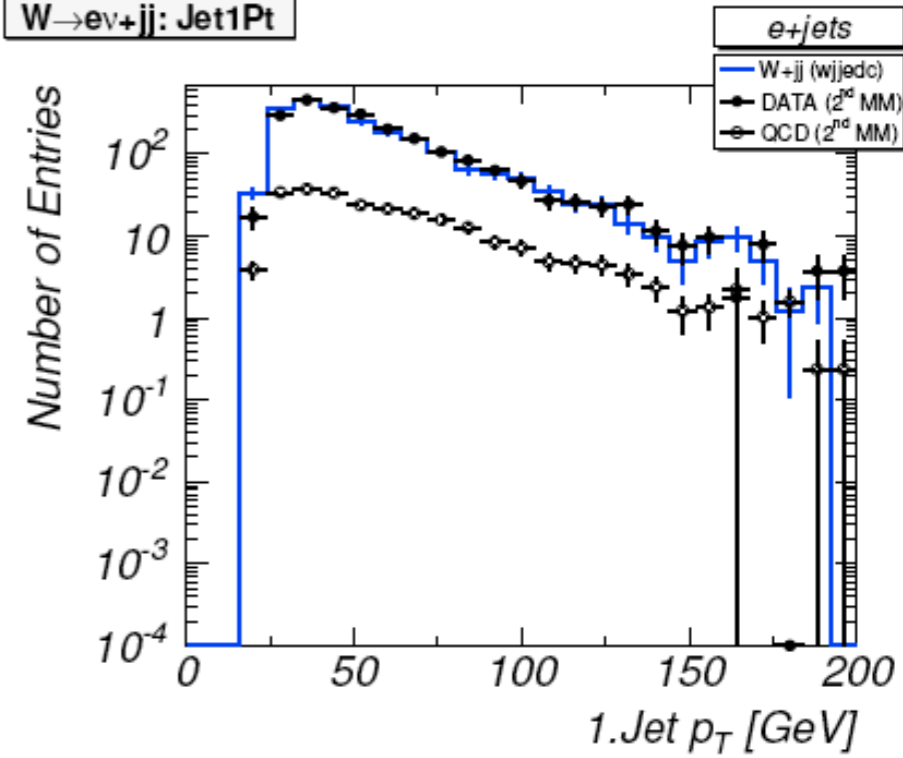
### Transverse W mass

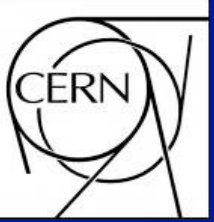


### Missing $E_T$



### Leading Jet $p_T$





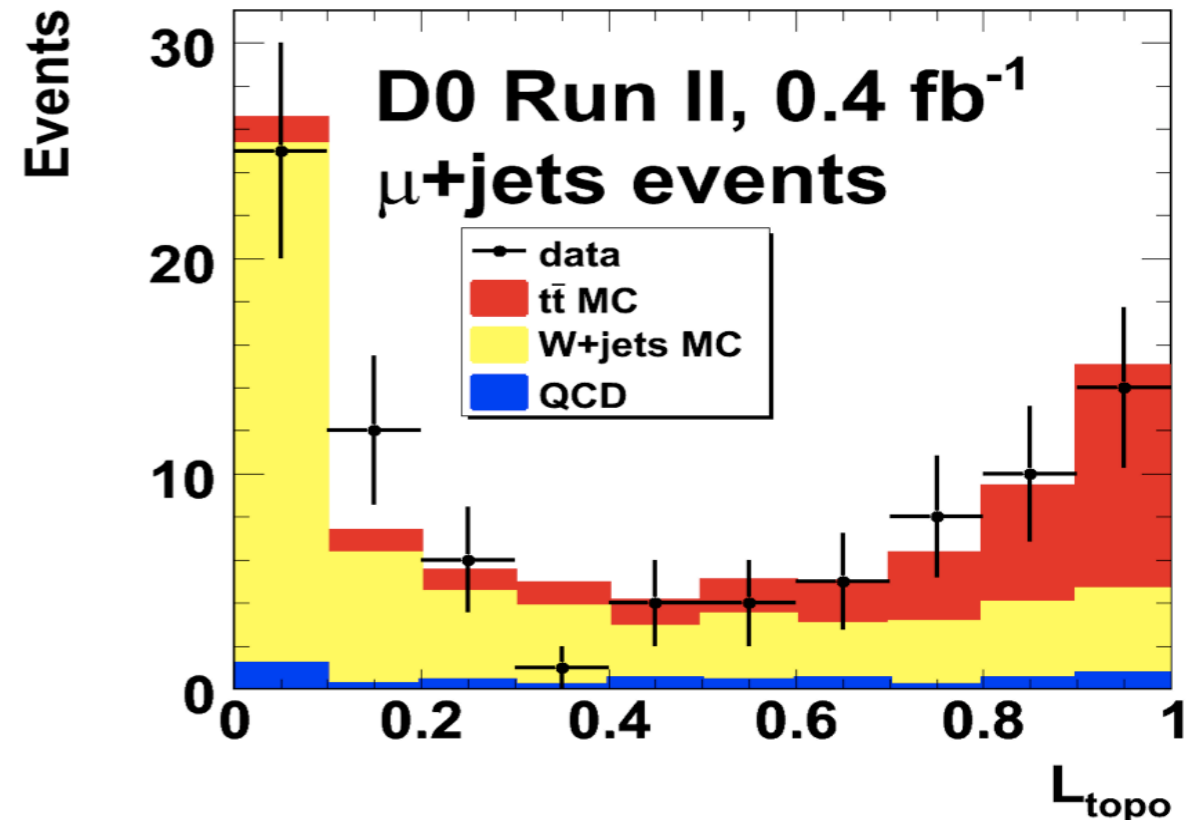
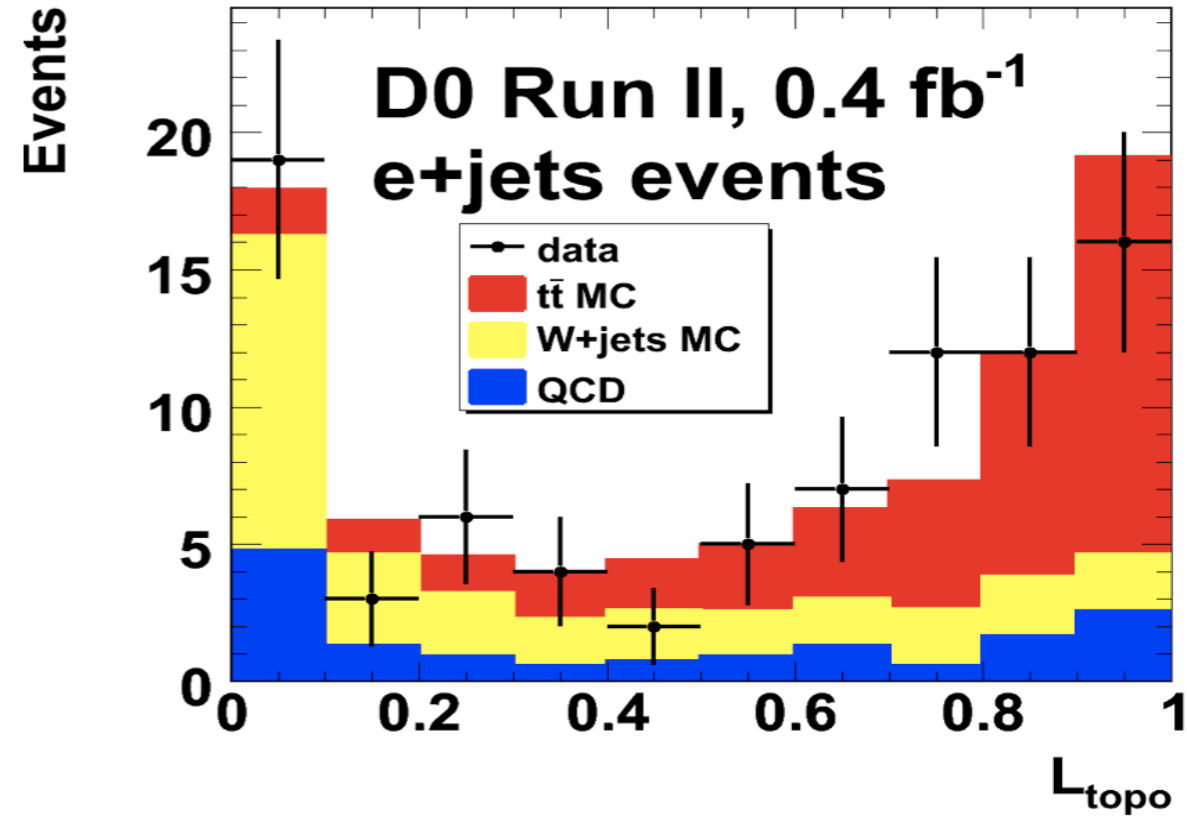
# Data Sample Composition



- Before the  $P_{bkg}$ -Normalization is performed and MC ensembles for the calibration of the method are drawn, the sample composition of the  $370 \text{ pb}^{-1}$  is estimated
- Use a template-fit method based on a topological likelihood, developed for the D0  $t$ - $t$ -bar cross-section measurement in the Lepton+Jets dataset

channel	$N_{\text{evts}}$	$f_{\text{topo}}^{\text{topo}}$	$N_{\text{topo}}^{\text{topo}}$	$f_{\text{QCD}}^{\text{topo}}$
$e$ +jets	86	$47.2^{+10.9}_{-10.6} \%$	$40.6^{+9.4}_{-9.1}$	$17.6^{+2.4}_{-2.2} \%$
$\mu$ +jets	89	$29.0^{+9.7}_{-9.1} \%$	$25.8^{+8.6}_{-8.1}$	$5.1^{+0.9}_{-0.8} \%$
$\ell$ +jets	175	$37.9^{+7.3}_{-7.0} \%$	$66.4^{+12.7}_{-12.2}$	$11.3 \pm 1.2 \%$

- Note that these estimates don't enter the measurement directly as a constraint, but are merely used to calibrate the method with realistically drawn ensembles!

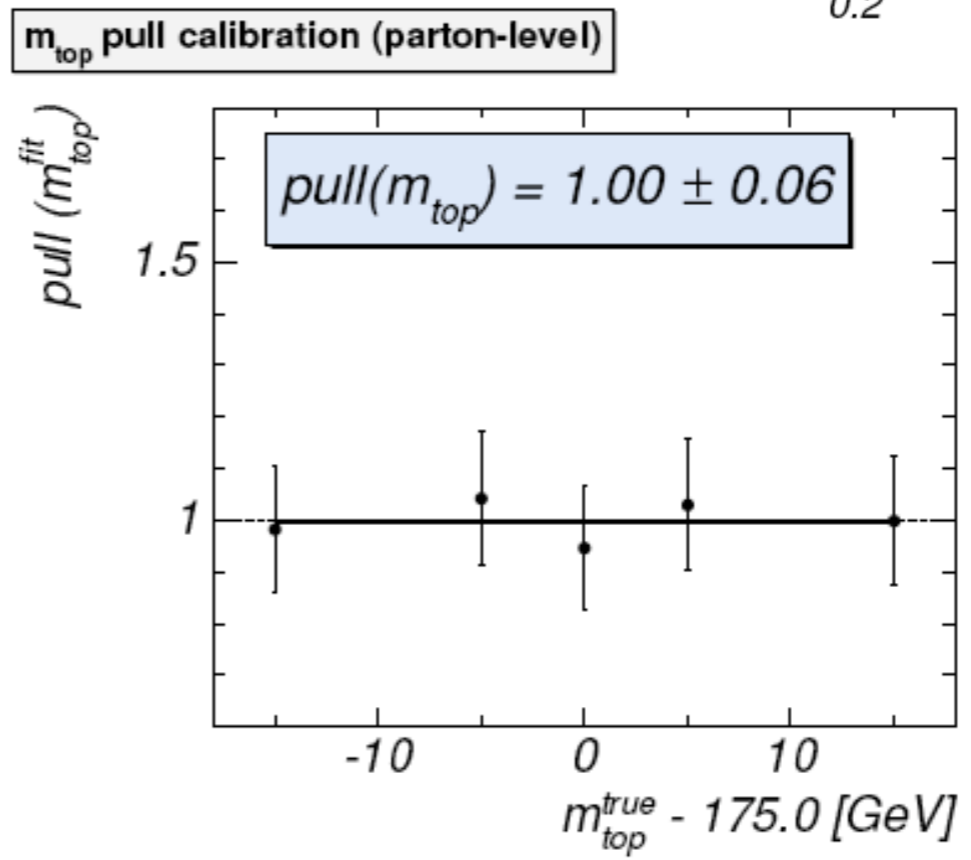
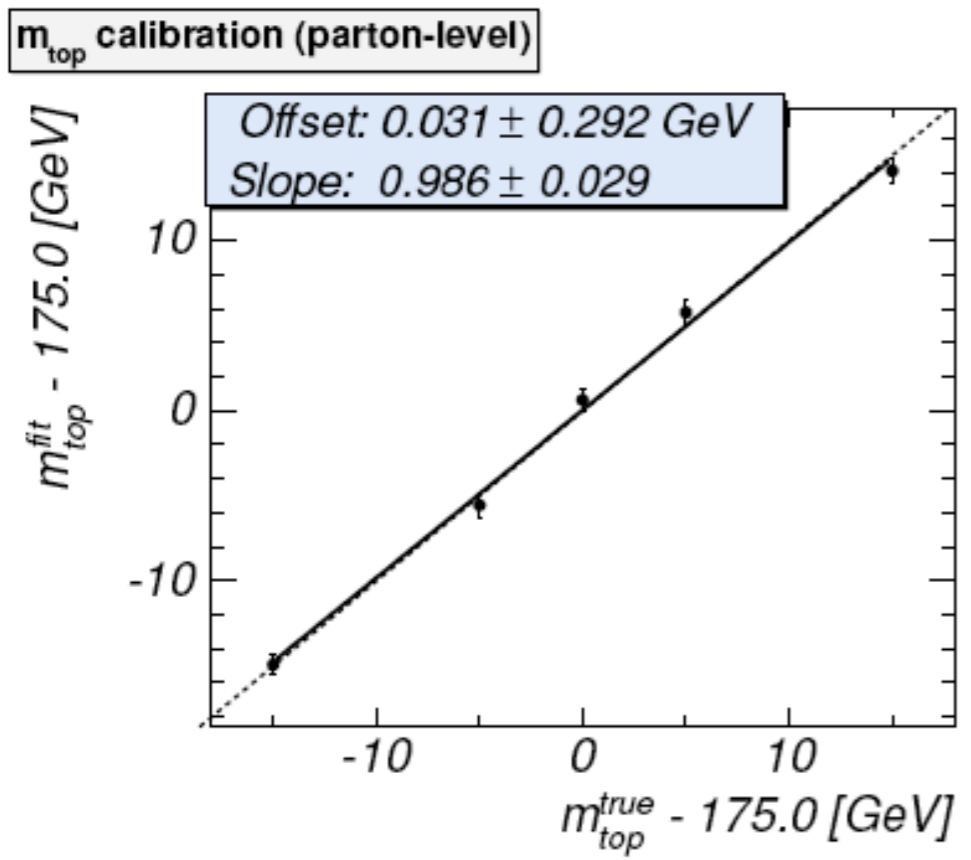
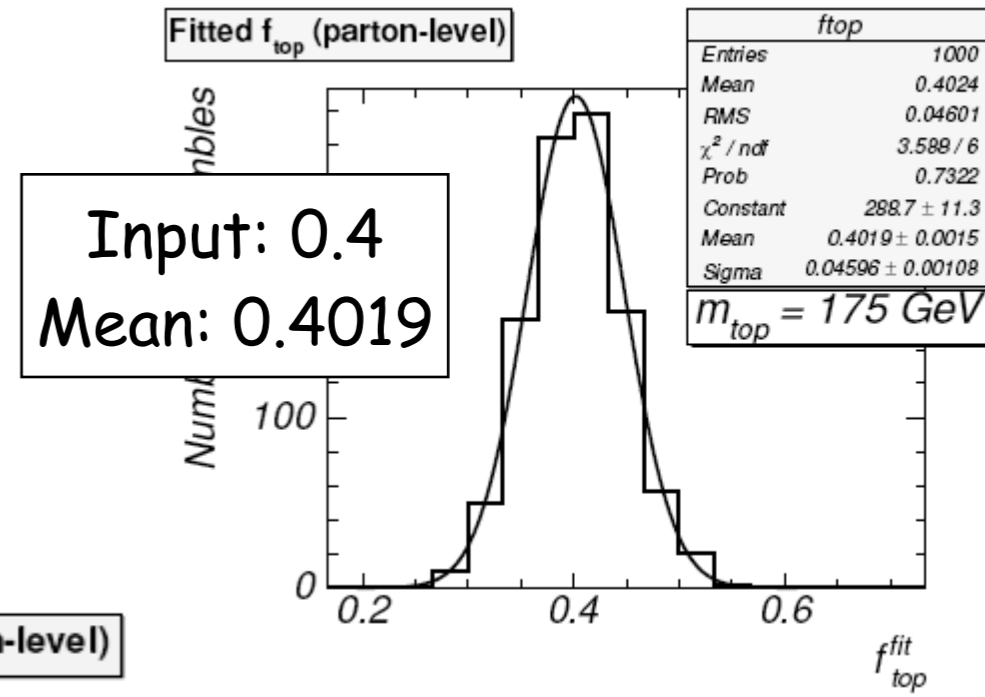




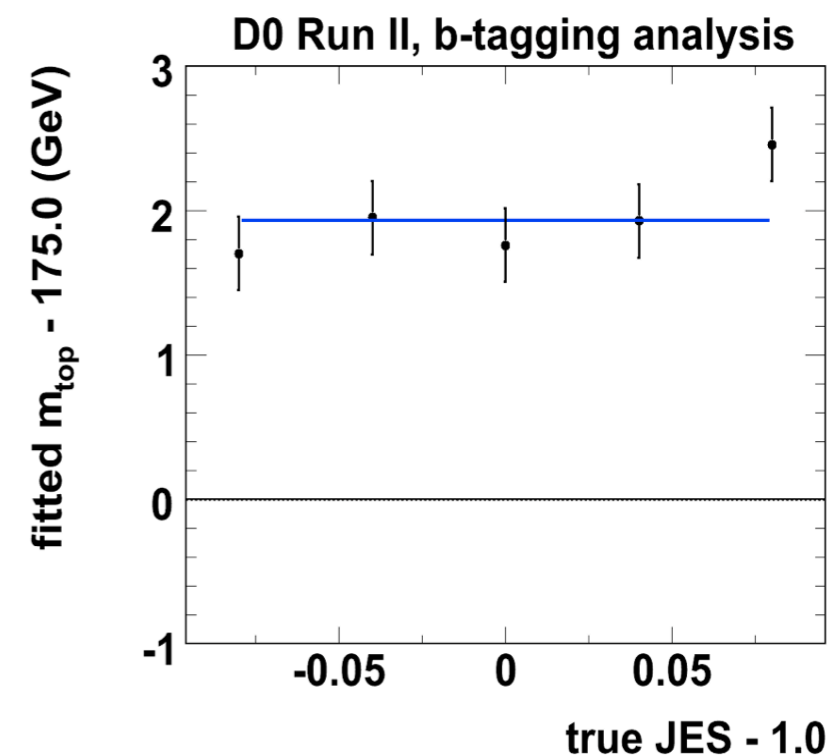
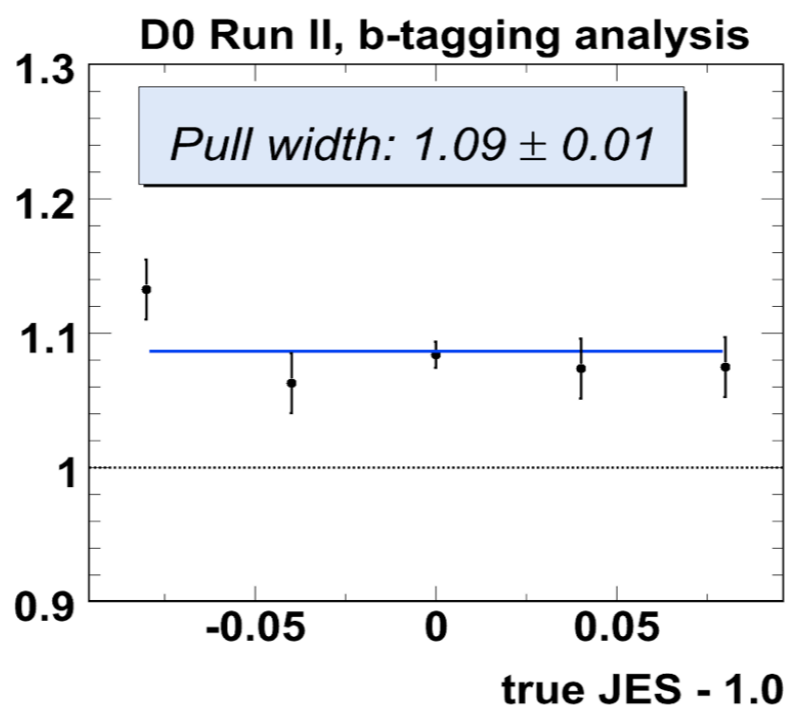
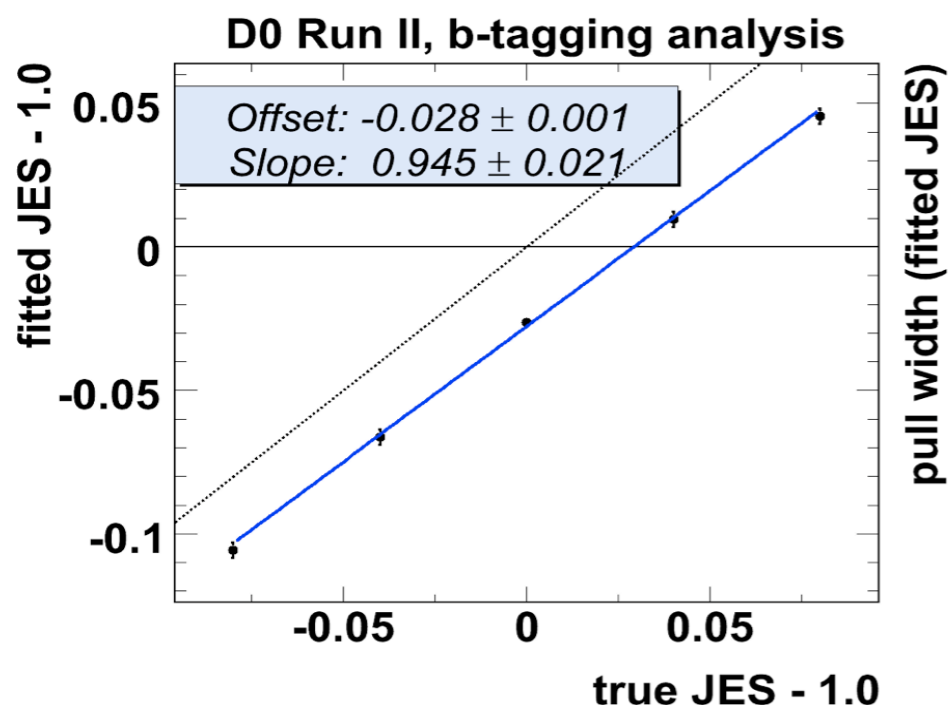
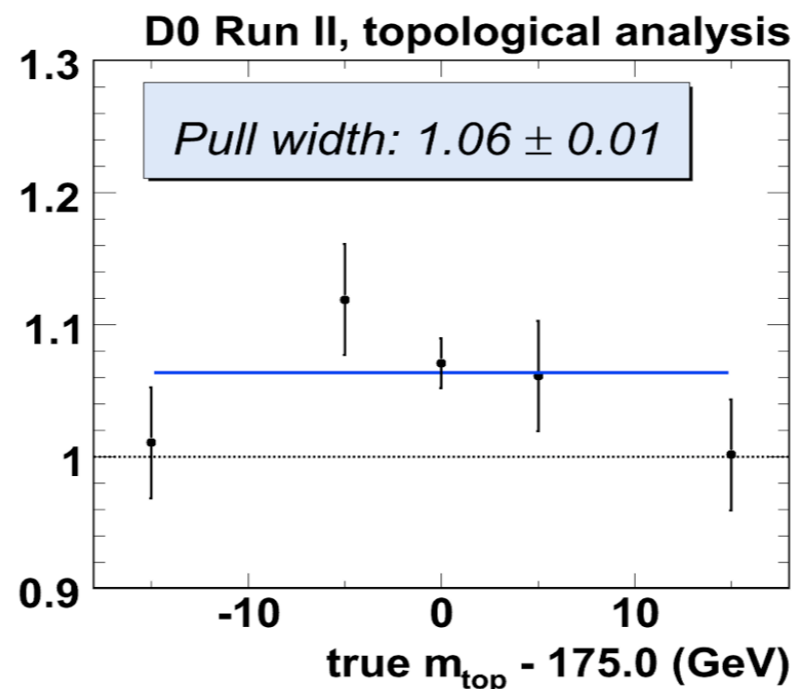
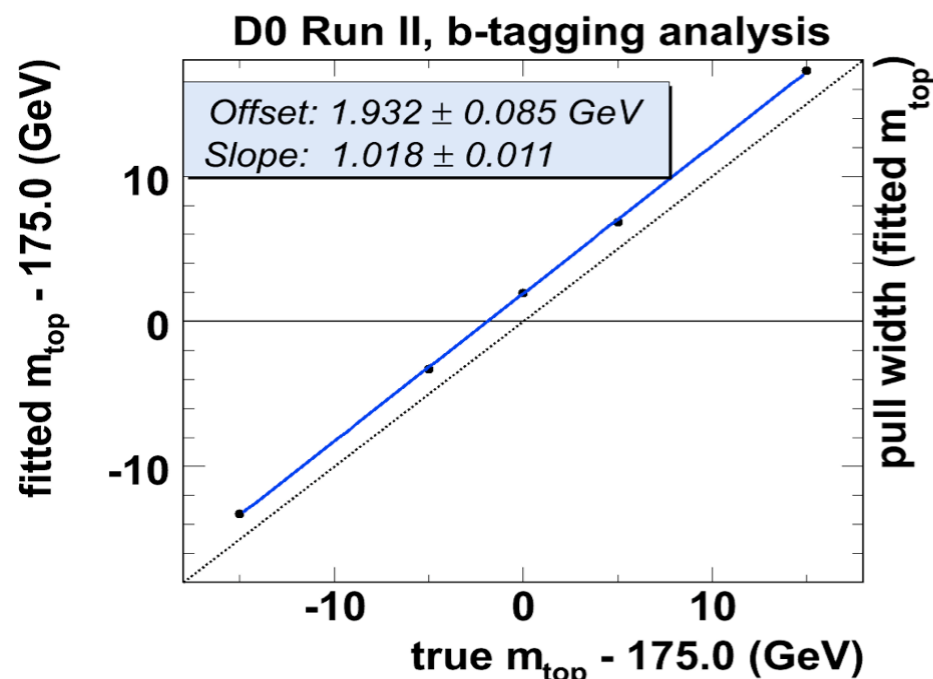
# MC Calibration: Parton-Level



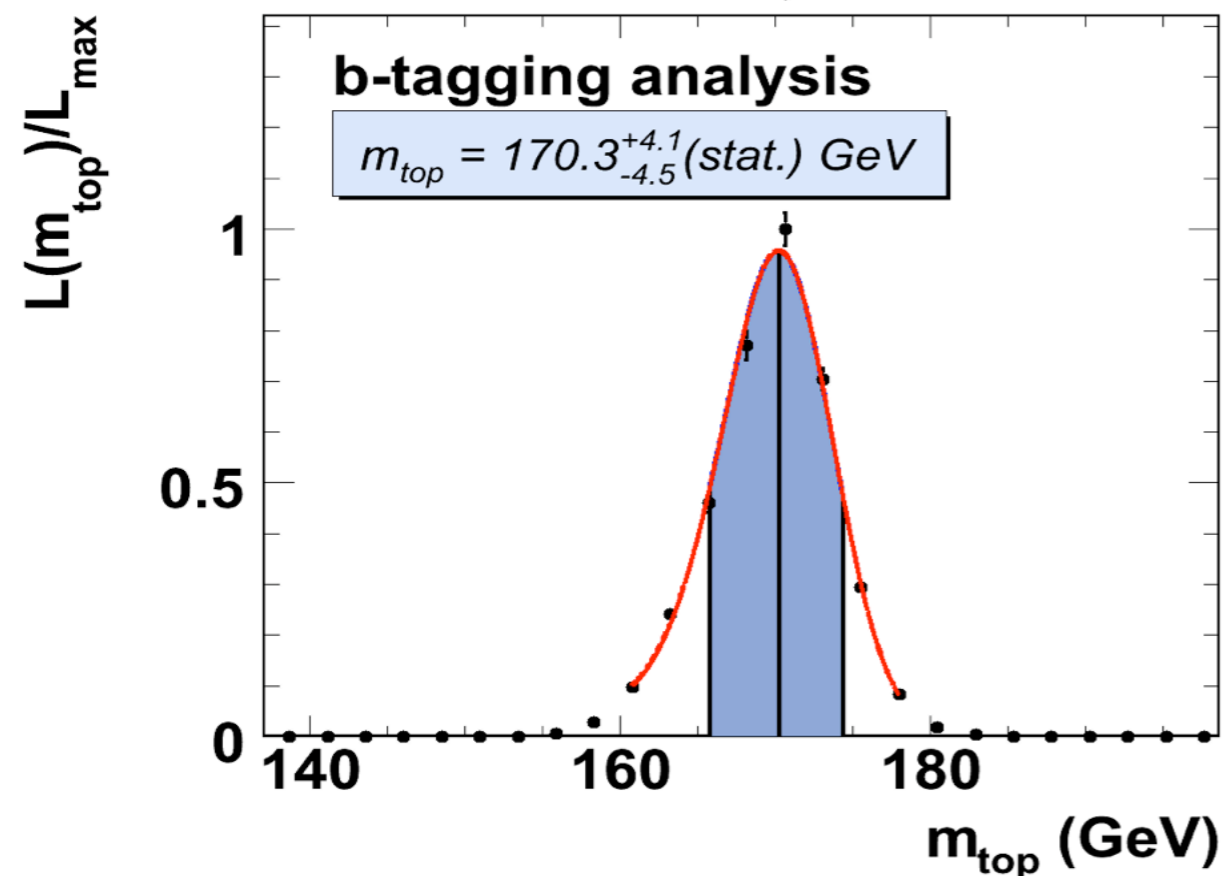
- First, the method is tested in Ensemble-Tests using Parton-Level events: generated with LO ME (MadGraph), energies smeared according to transfer-function parameters
- The idealized assumptions are therefore all valid:
  - ★ Leading-Order production, no radiation
  - ★ jet and lepton angles well measured
  - ★ Resolution parametrization exact



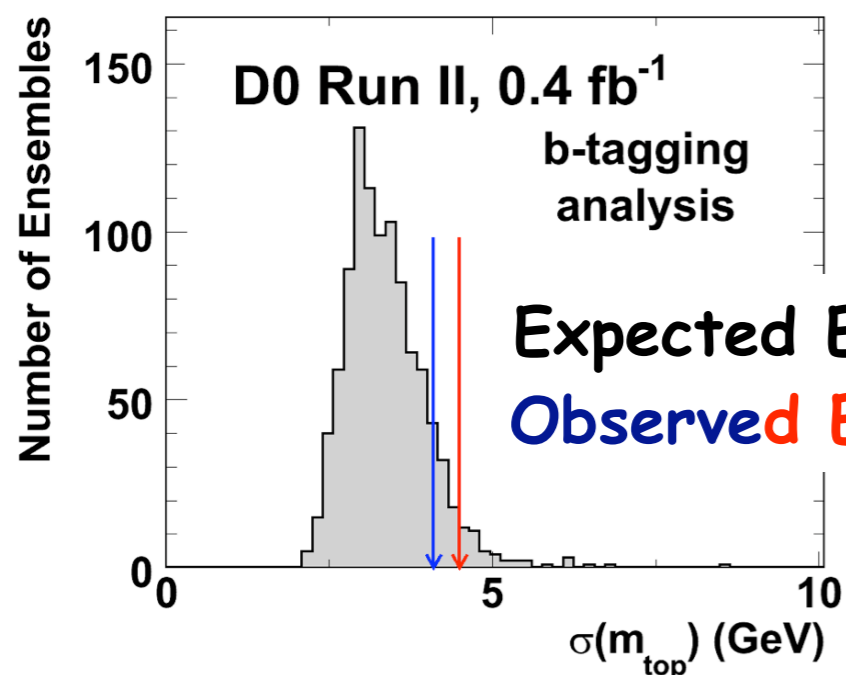
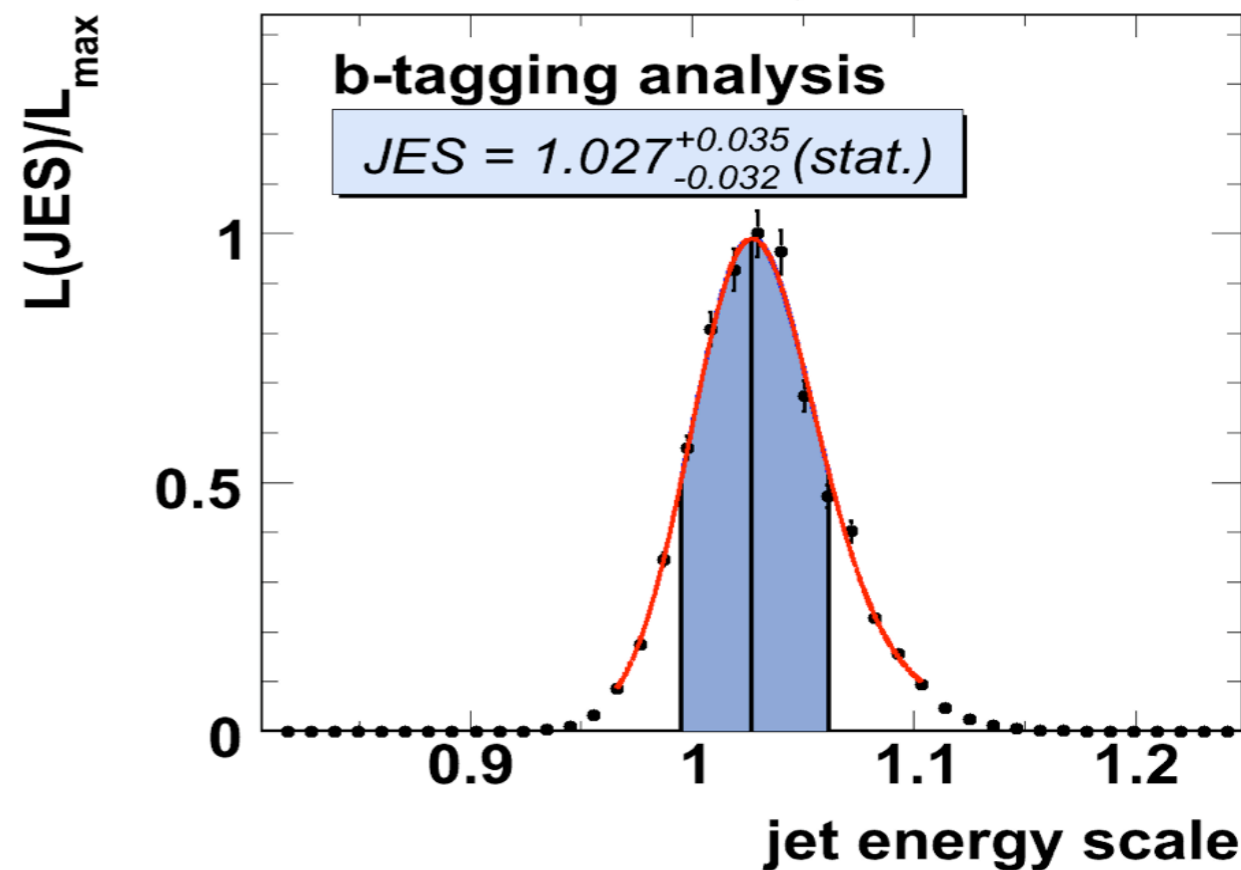
- The calibration for the data measurement is derived from Monte Carlo events processed with the full D0 simulation chain
- The data result is corrected for the observed bias and the pull deviation from 1.0



D0 Run II, 0.4 fb<sup>-1</sup>



D0 Run II, 0.4 fb<sup>-1</sup>

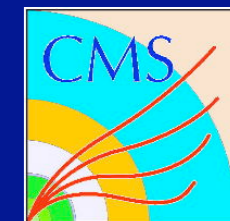


$m_{\top} = 170.6^{+4.0}_{-4.7} (stat.+JES) \pm 1.4 (syst.) \text{ GeV}$

Most precise D0 Top Mass Measurement



# Systematics: D0 Lepton+Jets



The Jet Energy Scale uncertainty is included in the error derived from the 2D log-likelihood minimization, its contribution is  $\sim 3.0$  GeV

	Systematic	GeV
<b>physics modeling</b>	signal modeling	$\pm 0.46$
	background modeling	$\pm 0.32$
	PDF Uncertainty	$\pm 0.07$
	b fragmentation	$\pm 0.71$
	b/c semileptonic decays	$\pm 0.07$
<b>detector modeling</b>	Residual Jet Energy Scale	$\pm 0.25$
	b response	$\pm 0.80$
	trigger	$\pm 0.08$
	b-tagging	$\pm 0.24$
<b>method</b>	signal fraction	$\pm 0.15$
	QCD contamination	$\pm 0.29$
	MC Calibration	$\pm 0.48$
	<b>TOTAL</b>	<b><math>\pm 1.4</math></b>



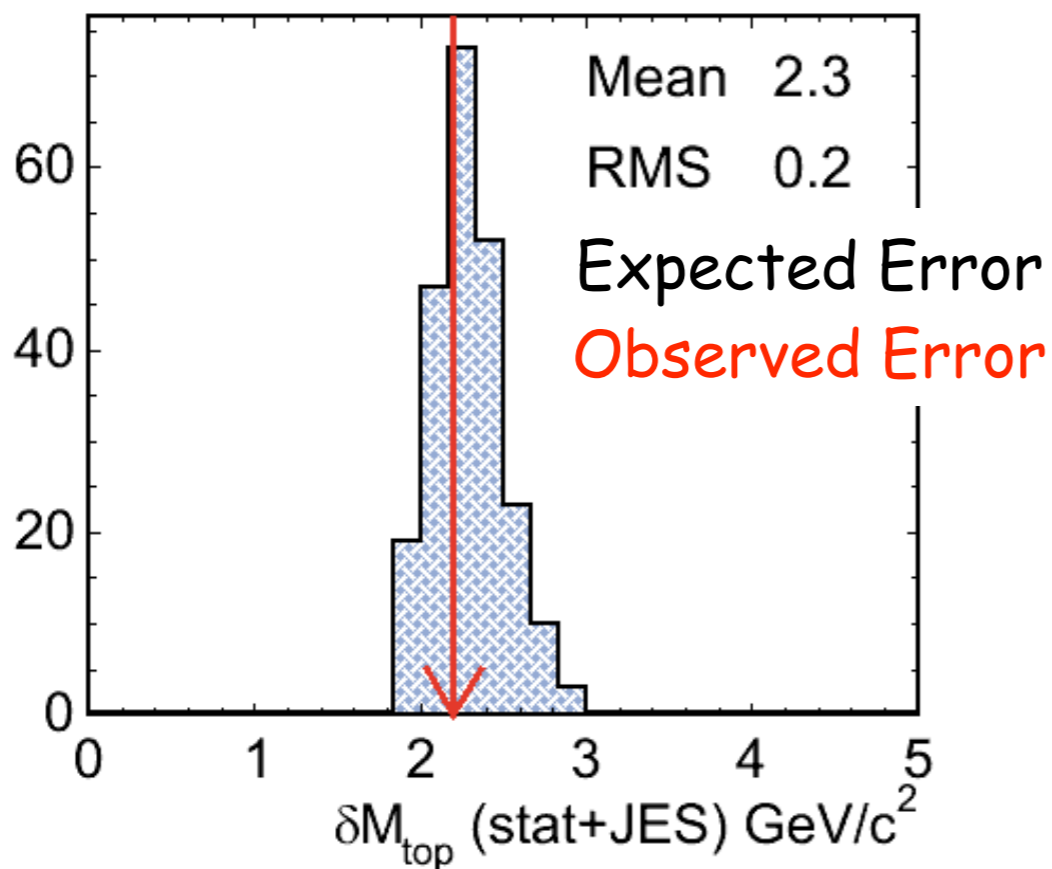


# Results: CDF Lepton+Jets

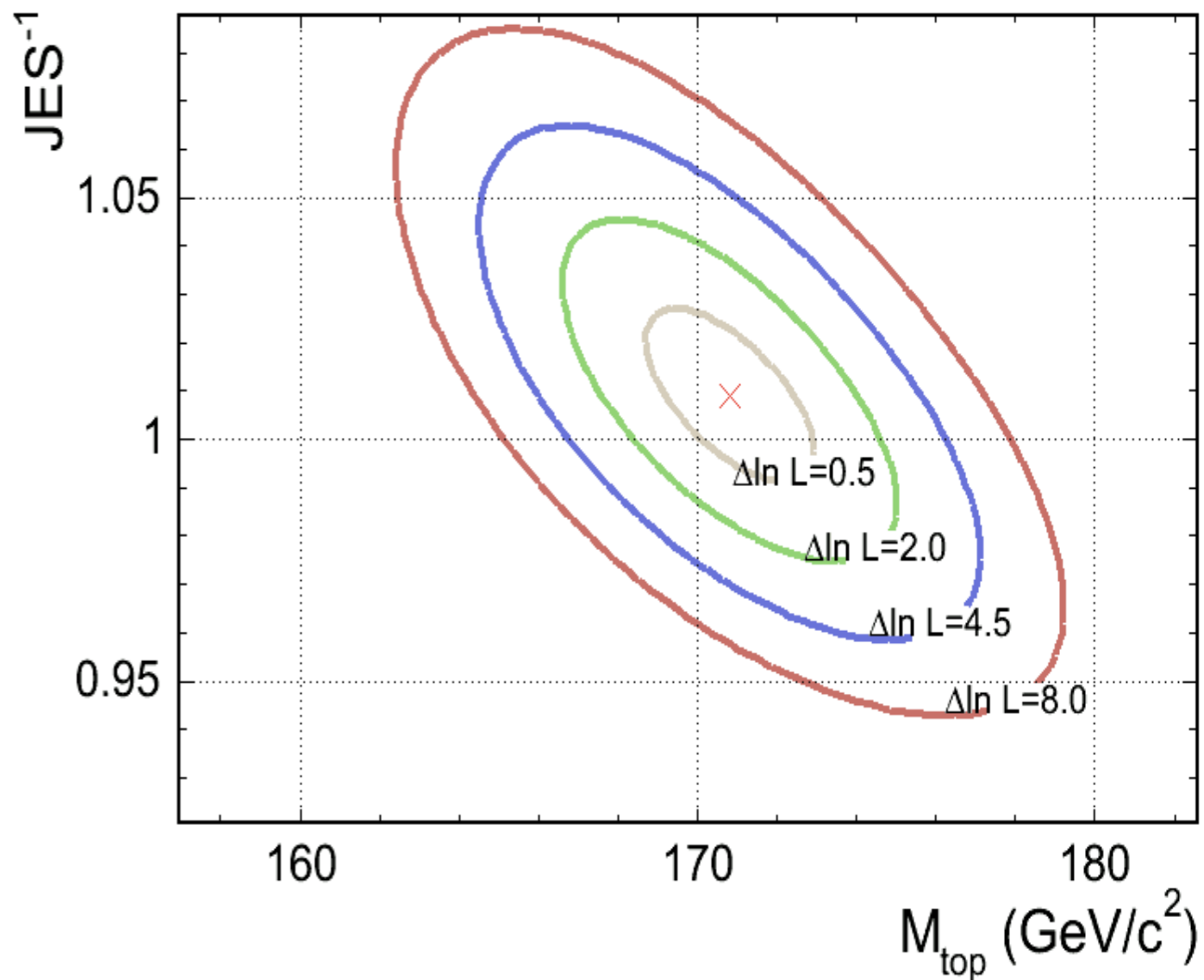
$$m_t = 170.9 \pm 2.2 \text{ (stat.+JES)} \pm 1.4 \text{ (syst.) GeV}$$

940 pb<sup>-1</sup>

CDF Run II Preliminary (940 pb<sup>-1</sup>)

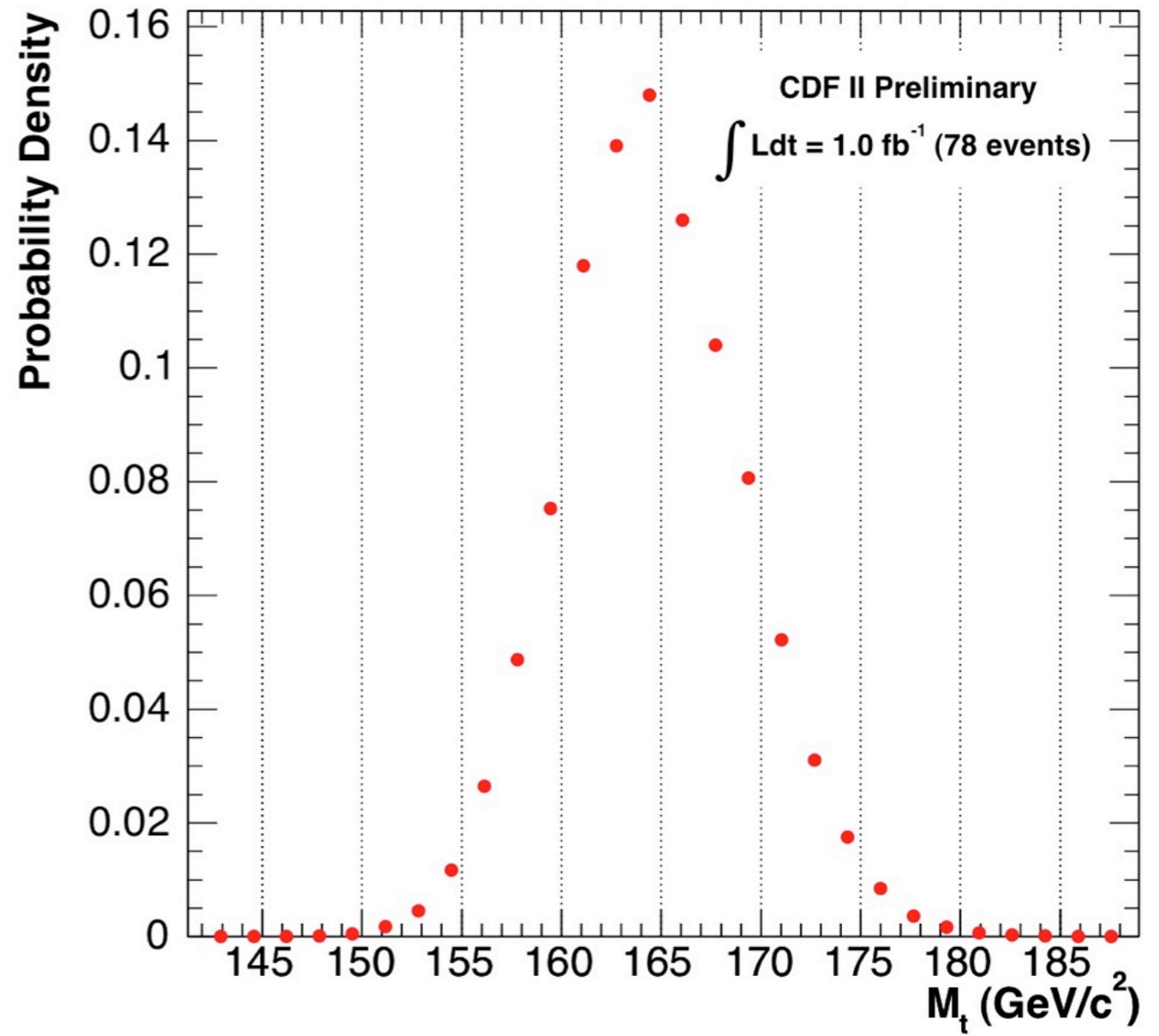
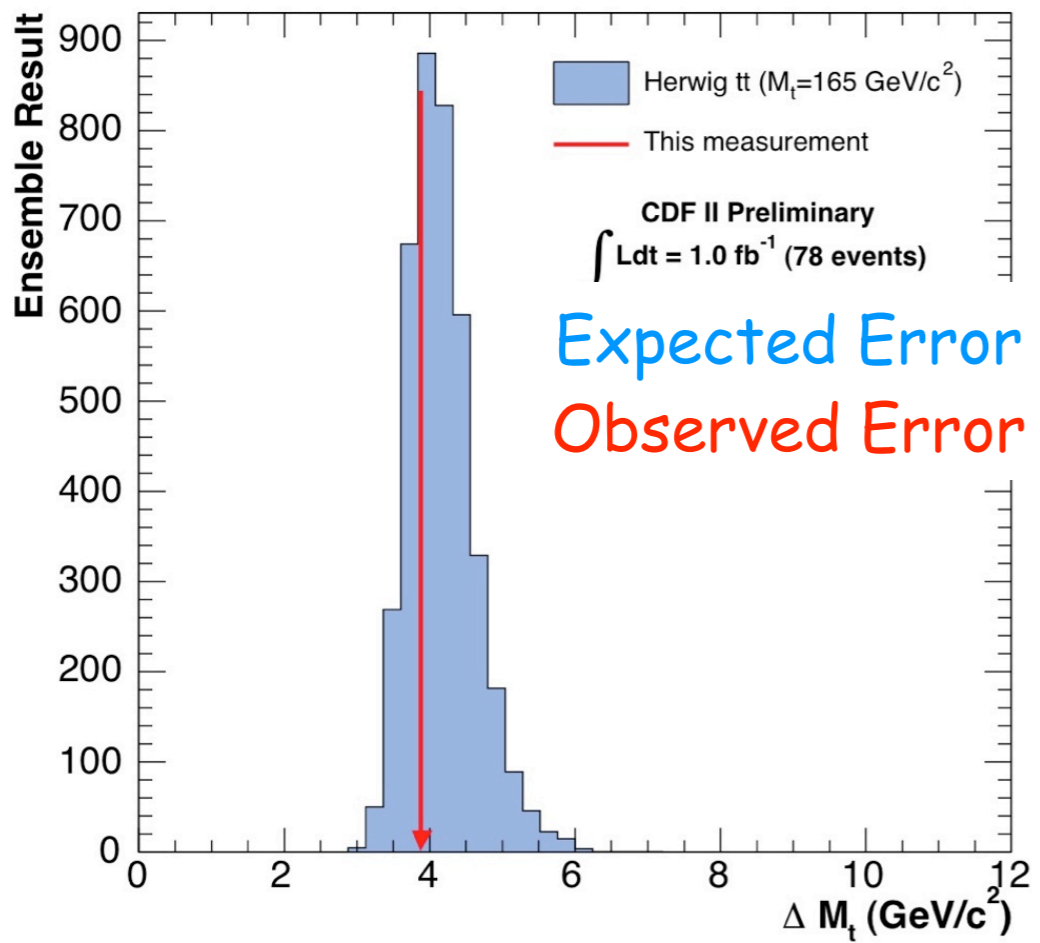


CDF Preliminary 940 pb<sup>-1</sup>



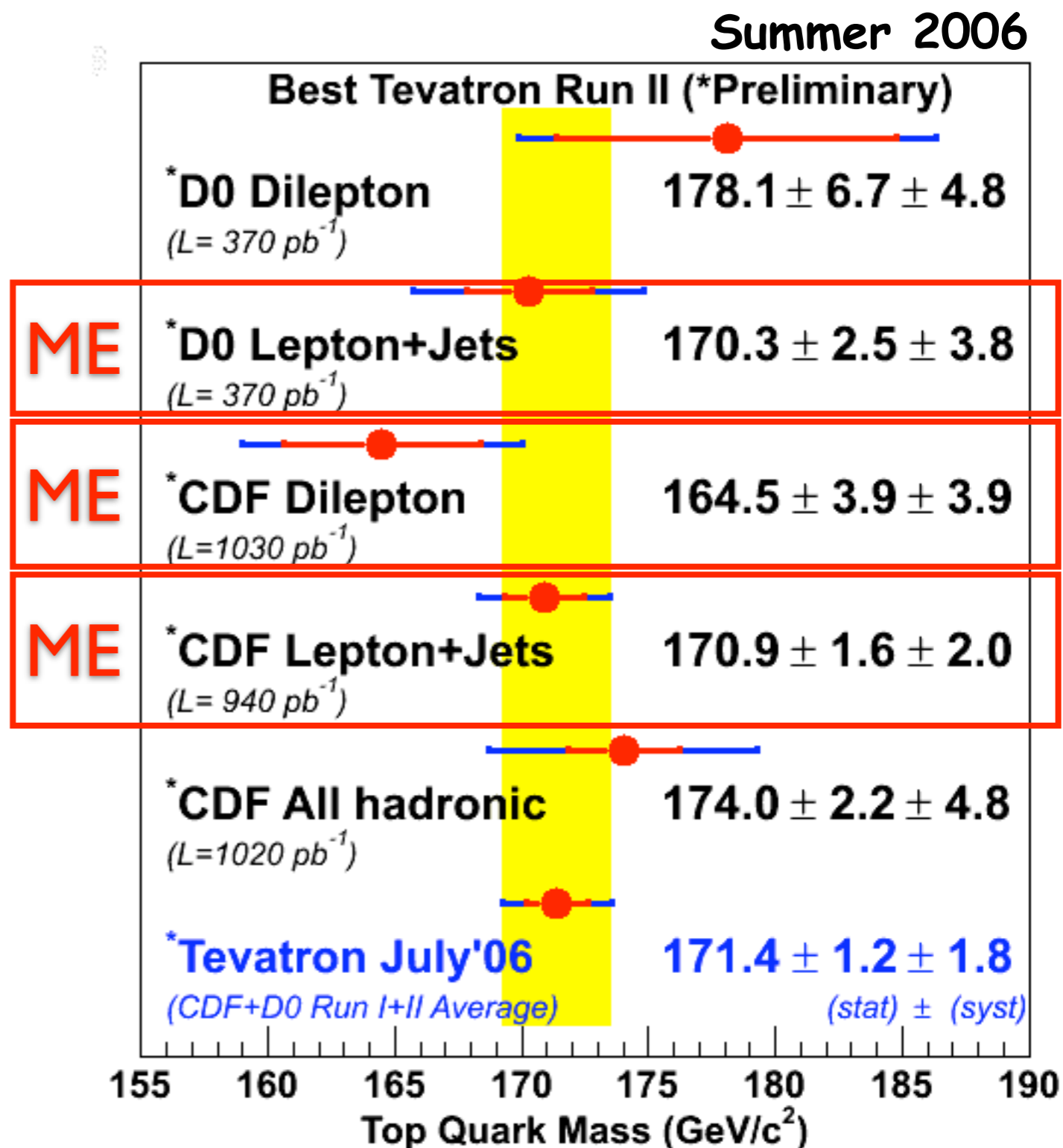
**World's single most precise measurement of the top mass!**

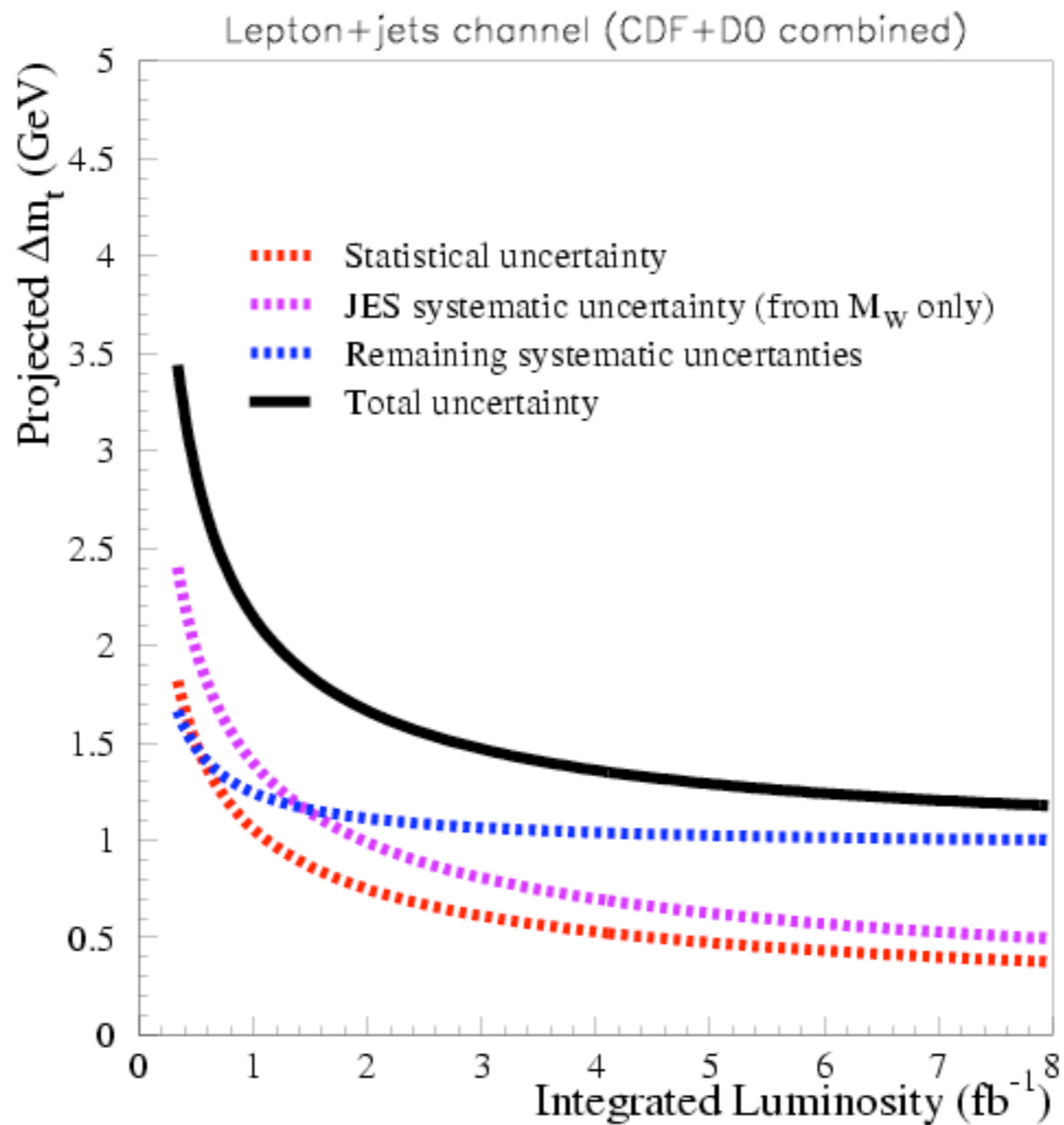
$$m_t = 164.5 \pm 3.9 \text{ (stat.)} \pm 3.9 \text{ (syst.) GeV}$$



**World's single most precise measurement of  $m_t$  in dilepton channel!**

- RunII ME Analyses play a key role in the striking precision of the current world average!
- Already limited by systematic uncertainty
- Precision Measurement:  $\delta M_t \sim 1.3\%$
- For the 4-8 fb<sup>-1</sup> future, a total uncertainty of  $\sim 1.5$  GeV is projected
- The JES uncertainty is going to improve with rising statistics as well, thanks to The In-Situ calibration technique





- The Matrix Element Method has been developed at D0 during Run I and yielded the most precise measurement of the top mass in Run I
- It is again successfully applied in Run II by both CDF and D0:

$m_t = 170.6^{+4.0}_{-4.7} \text{ (stat.+JES)} \pm 1.4 \text{ (syst.) GeV}$	D0, Lepton+jets, 370 pb <sup>-1</sup>
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$m_t = 170.9 \pm 2.2 \text{ (stat.+JES)} \pm 1.4 \text{ (syst.) GeV}$	CDF, Lepton+jets, 940 pb <sup>-1</sup>
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$m_t = 164.5 \pm 3.9 \text{ (stat.)} \quad \pm 3.9 \text{ (syst.) GeV}$	CDF, Dilepton, 1030 pb <sup>-1</sup>
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- Thanks in part to these and future applications of the method, combined with other methods developed by the Tevatron collaborations, the Tevatron total error on the top mass is projected to be as low as  $\sim 1.5 \text{ GeV}$
- It will not amount a small challenge for the LHC experiments to further improve the top mass precision