Recent Developments in APFEL

[V. Bertone, *et al.*, Comput. Phys. Commun. 185, 1647 (2014)]

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xFitter external meeting

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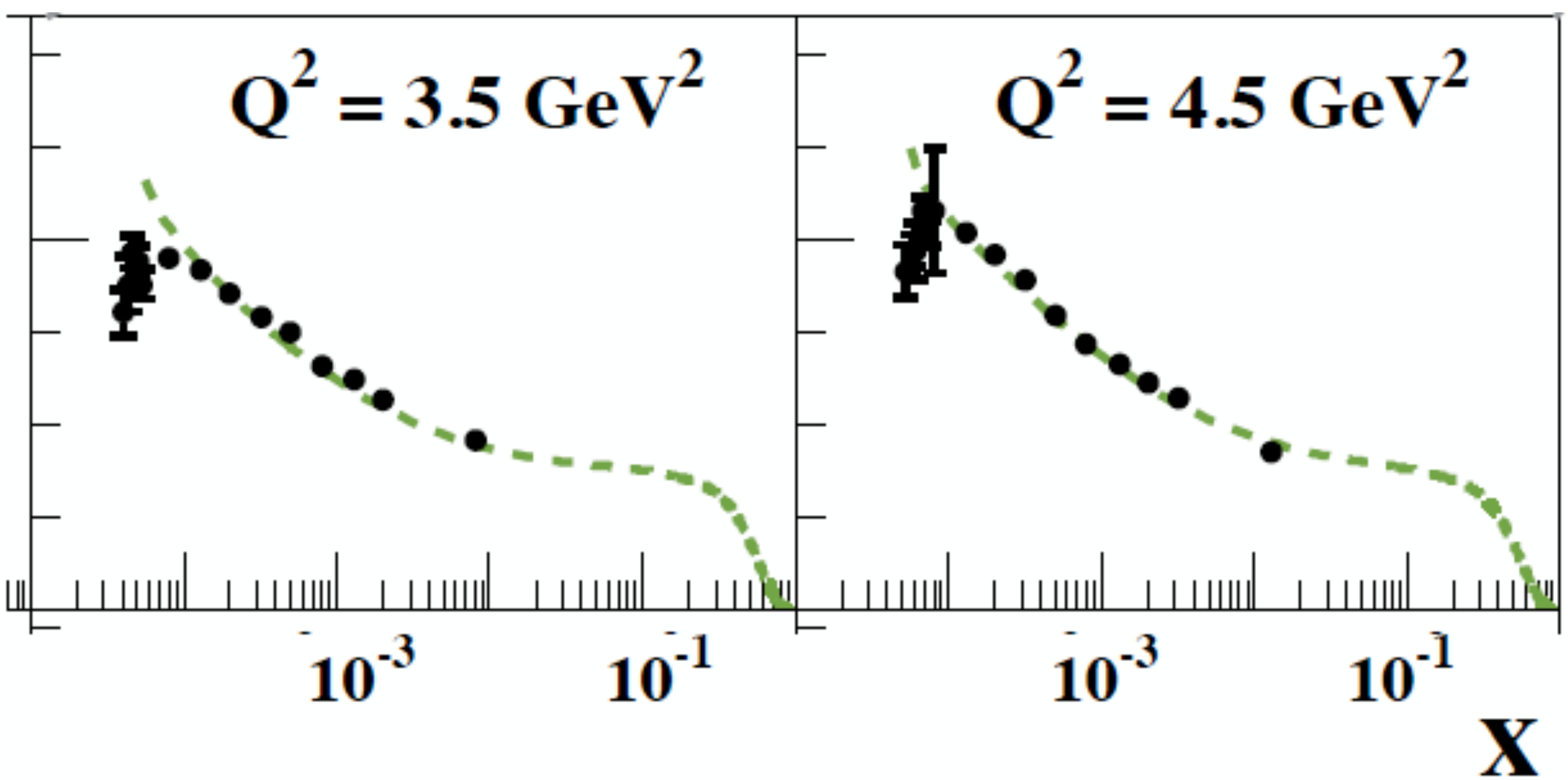
APFEL in a Nutshell

- APFEL is a **public** library for the computation of collinear PDF evolution and DIS structure functions:
  - up to NNLO in QCD combined to QED corrections up to NLO.
  - FFN and VFN schemes.
  - Pole and $\overline{\text{MS}}$ heavy-quark masses.
  - fast computation of DIS NC and CC observables in different mass schemes (ZM-VFNS, FFNS and FONLL).
  - Interfaces to FORTRAN, C/C++ and Python.
  - Web interface available on http://apfel.mi.infn.it.
  - available from http://apfel.hepforge.org/.
- Interfaced to **xFitter**.
- Used for the next generation of the **NNPDF** fits (including FFs).

Small-x Resummation

- **Tension** between fixed-order predictions and data in the small-x region reached by HERA:



From Eram Rizvi talk at QCD@LHC14

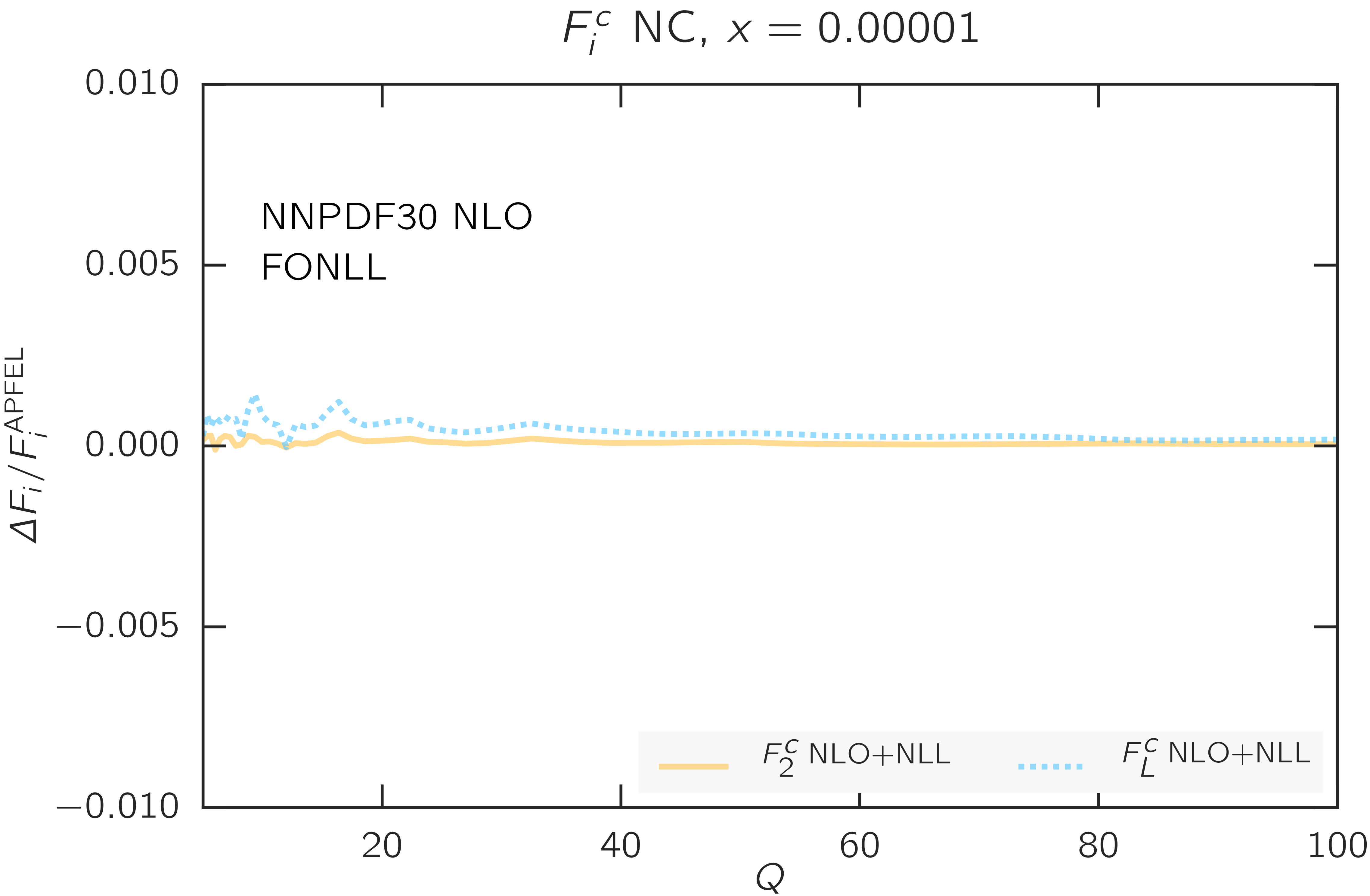
- A similar effect was observed some time ago in the NNPDF framework by F. Caola *et al.* [arXiv:1007.5405].

- Suggestion of the need for **small-x resummation**.

Small-x Resummation

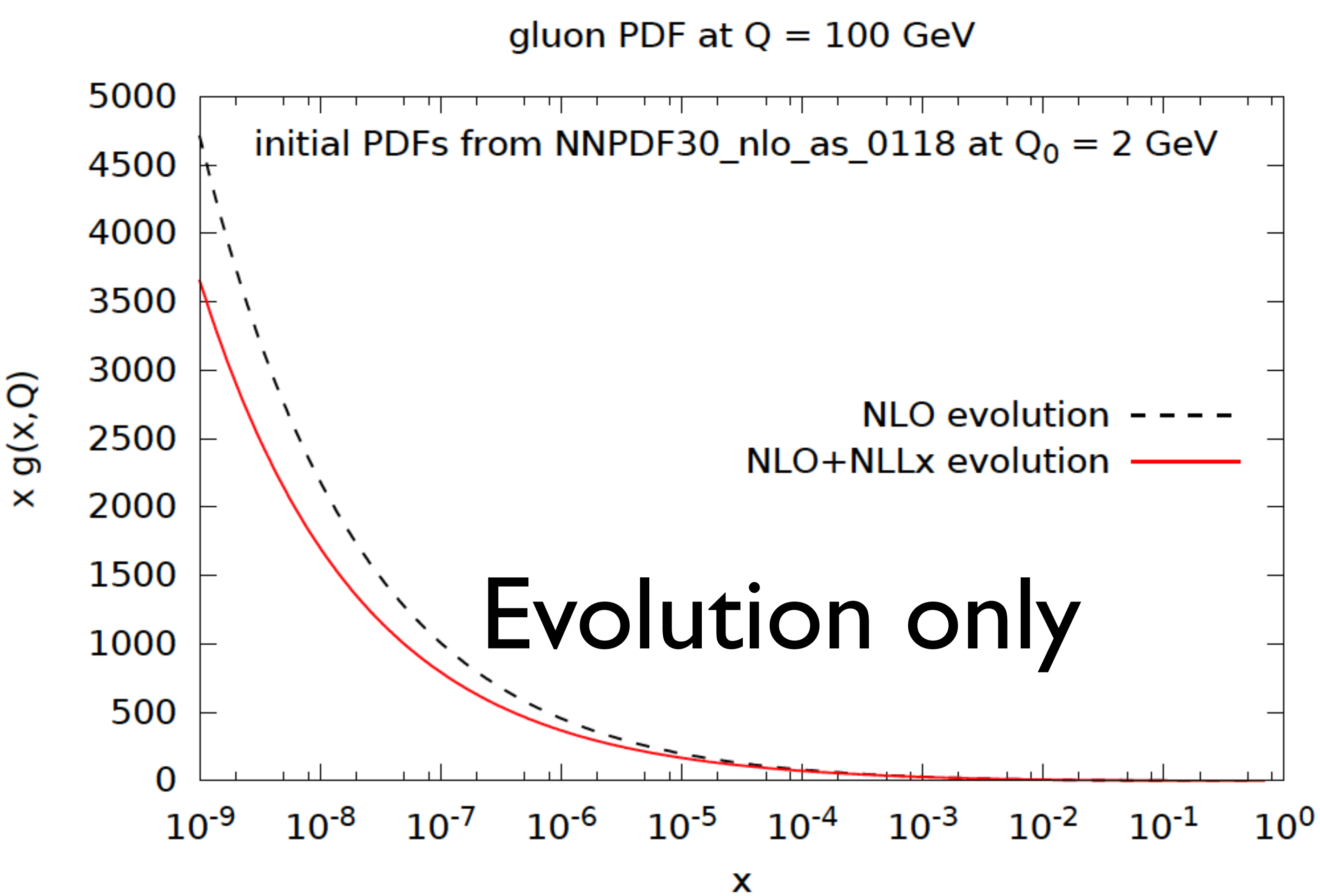
- The **HELL** code [arXiv:1607.02153] has been interfaced to APFEL:
  - based on the ABF formalism (e.g. see [hep-ph/9501231]).
  - Small-x **resummed splitting functions** up to **NLL** accuracy,
  - Small-x **resummed DIS coefficient functions** up to **NLL**:
    - massless,
    - massive.
  - **Resummed matching conditions**.
- It is now possible to compute structure functions in the FONLL GM-VFNS scheme including small-x resummation up to NLL.
- Forthcoming PDF fits in NNPDF.

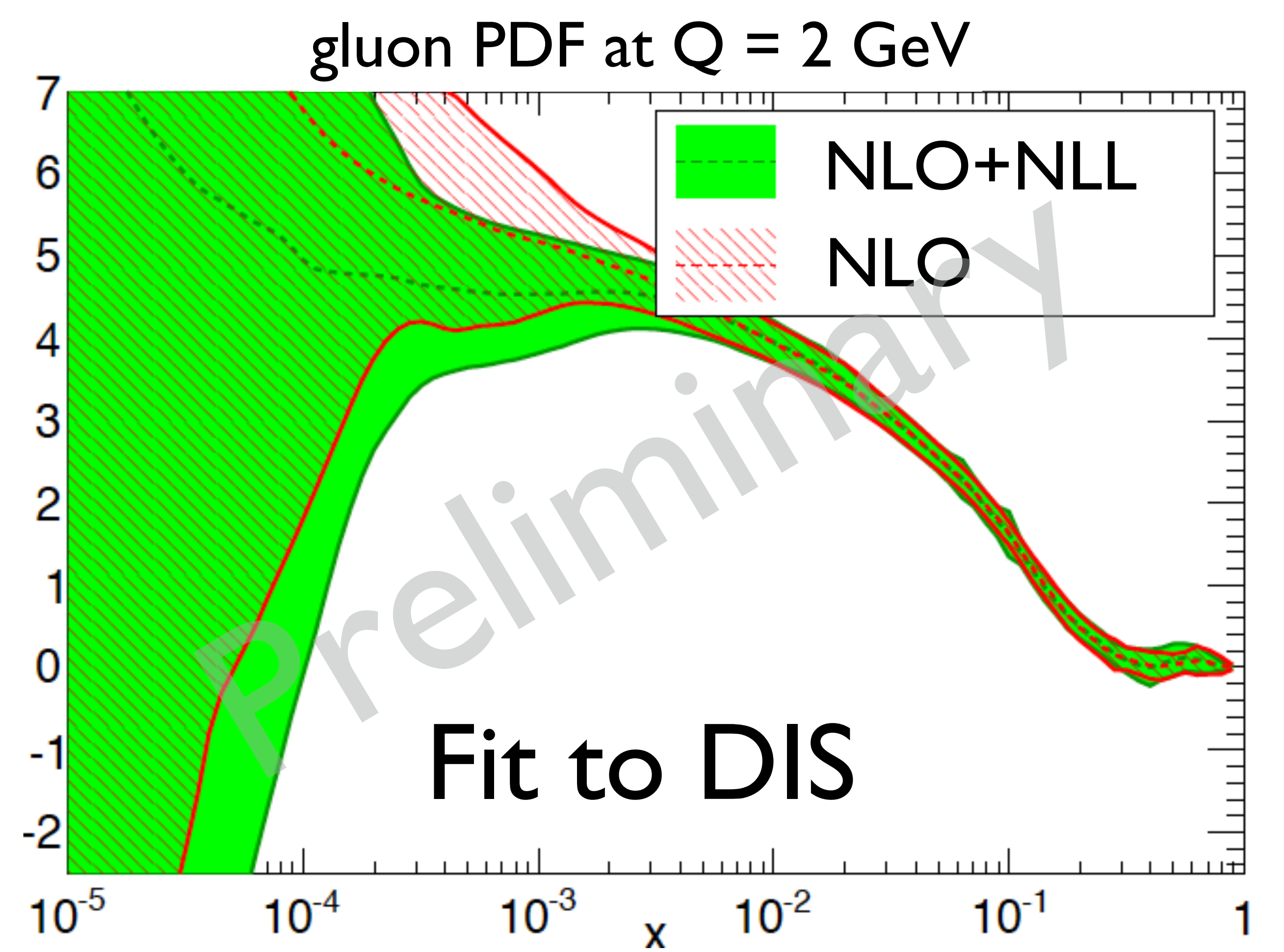
Small-x Resummation



- Excellent agreement with `MassiveDIS` [Thanks to L. Rottoli]

Small-x Resummation





- Resummed evolution leads to a **suppression** of the **gluon PDF** at small values of x as compared to fixed order.
- **Compensation** when also resummed **coefficient functions** are introduced ⇒ effect on the small-x gluon PDF at the level of 1-σ.
- Other PDFs mostly unchanged.

NLO QCD+QED Corrections
Evolution

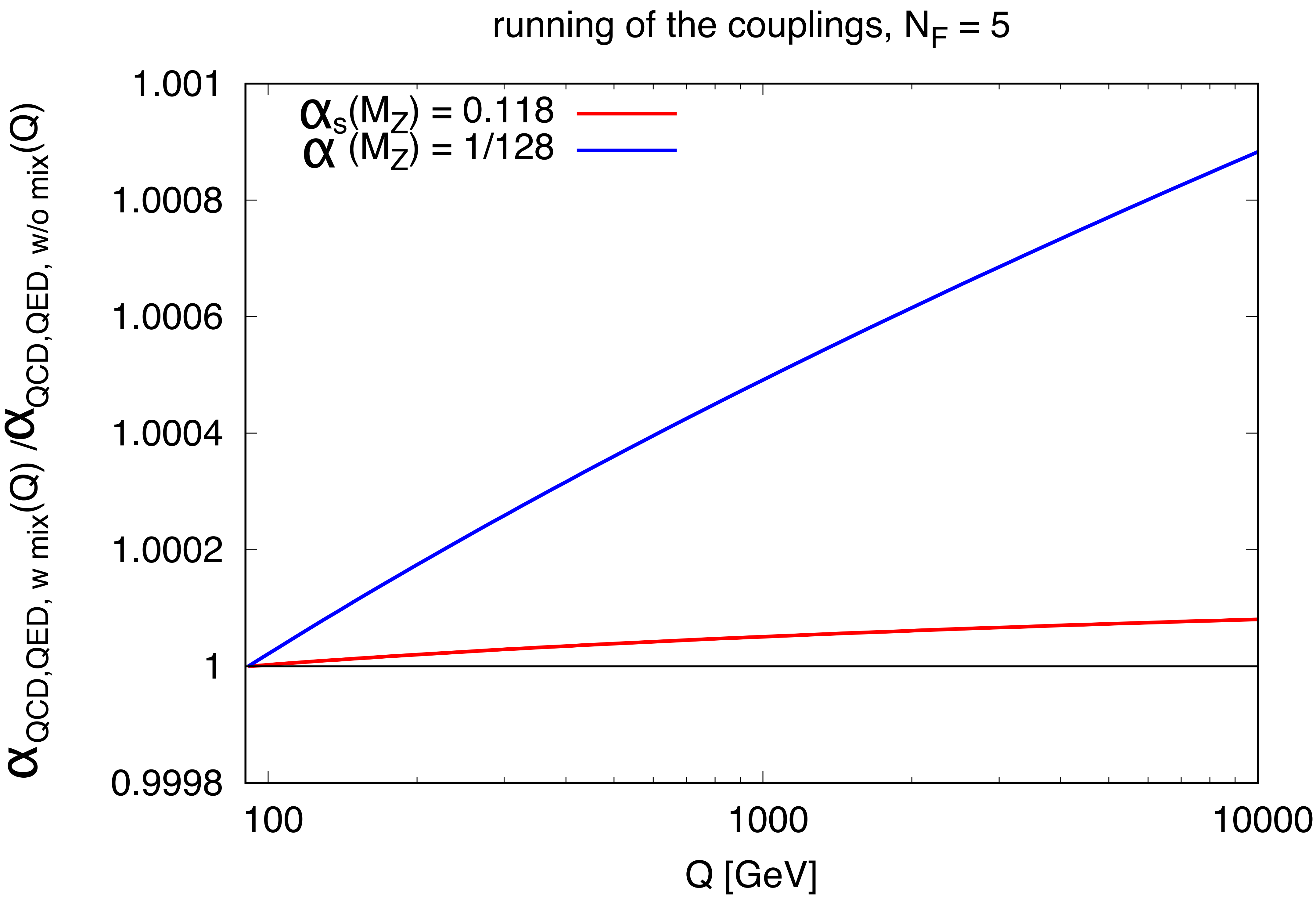
In order to implement the full NLO QCD+QED corrections in the DGLAP evolution two main steps are required:

1. Implementing the $O(\alpha_s^2\alpha)$, $O(\alpha^3)$, $O(\alpha^2\alpha_s)$ corrections to the **β-functions**:

    - running of α_s and α is coupled $\Rightarrow$ solve of a coupled ODE,

    - Numerical tests have shown that such terms lead to differences of $O(10^{-4})$ for α_s and $O(10^{-3})$ for α $\Rightarrow$ **unneeded complication**.

NLO QCD+QED Corrections
Coupling Evolution



- Mixed terms in the β-functions lead to negligible effects.

NLO QCD+QED Corrections
Evolution

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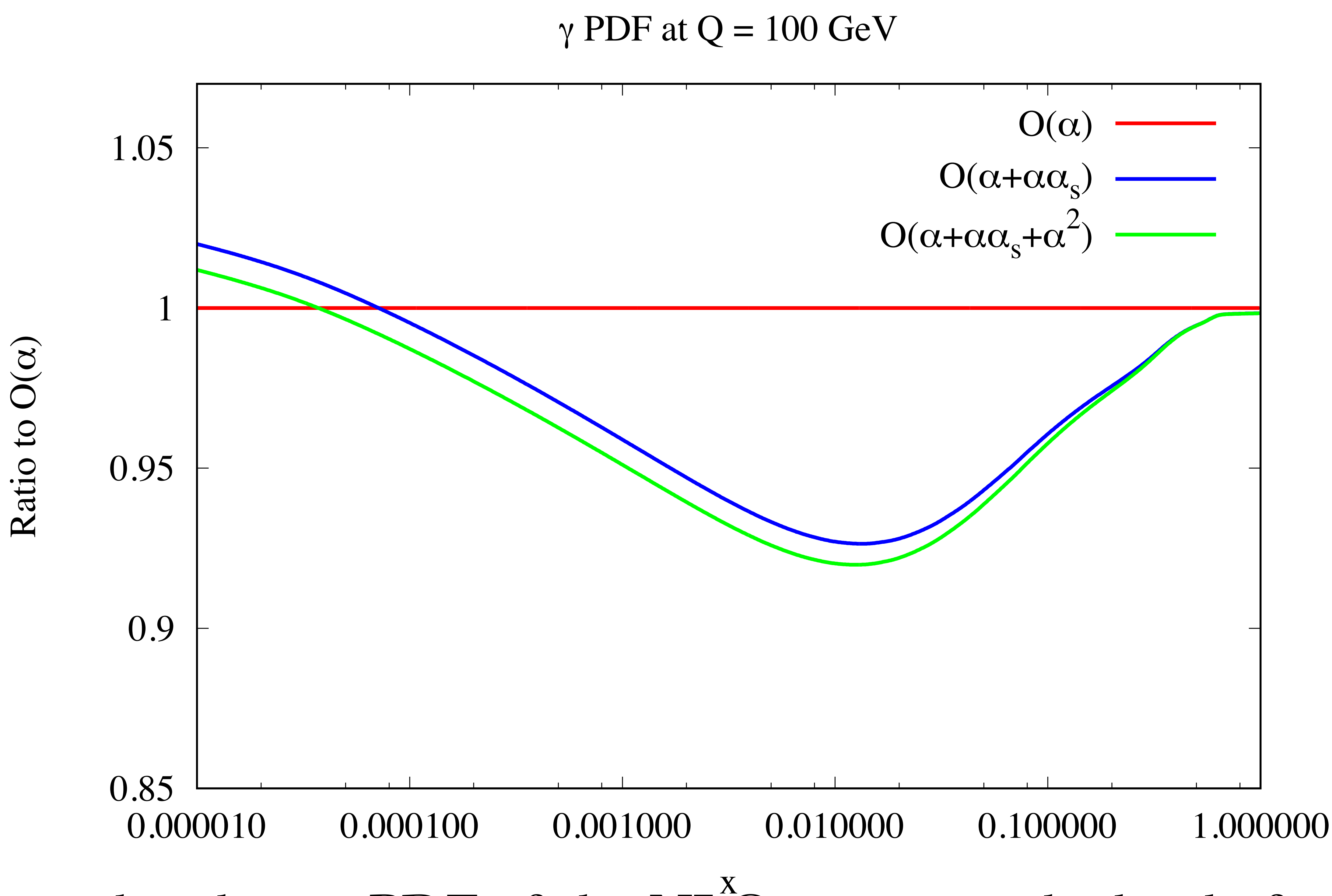
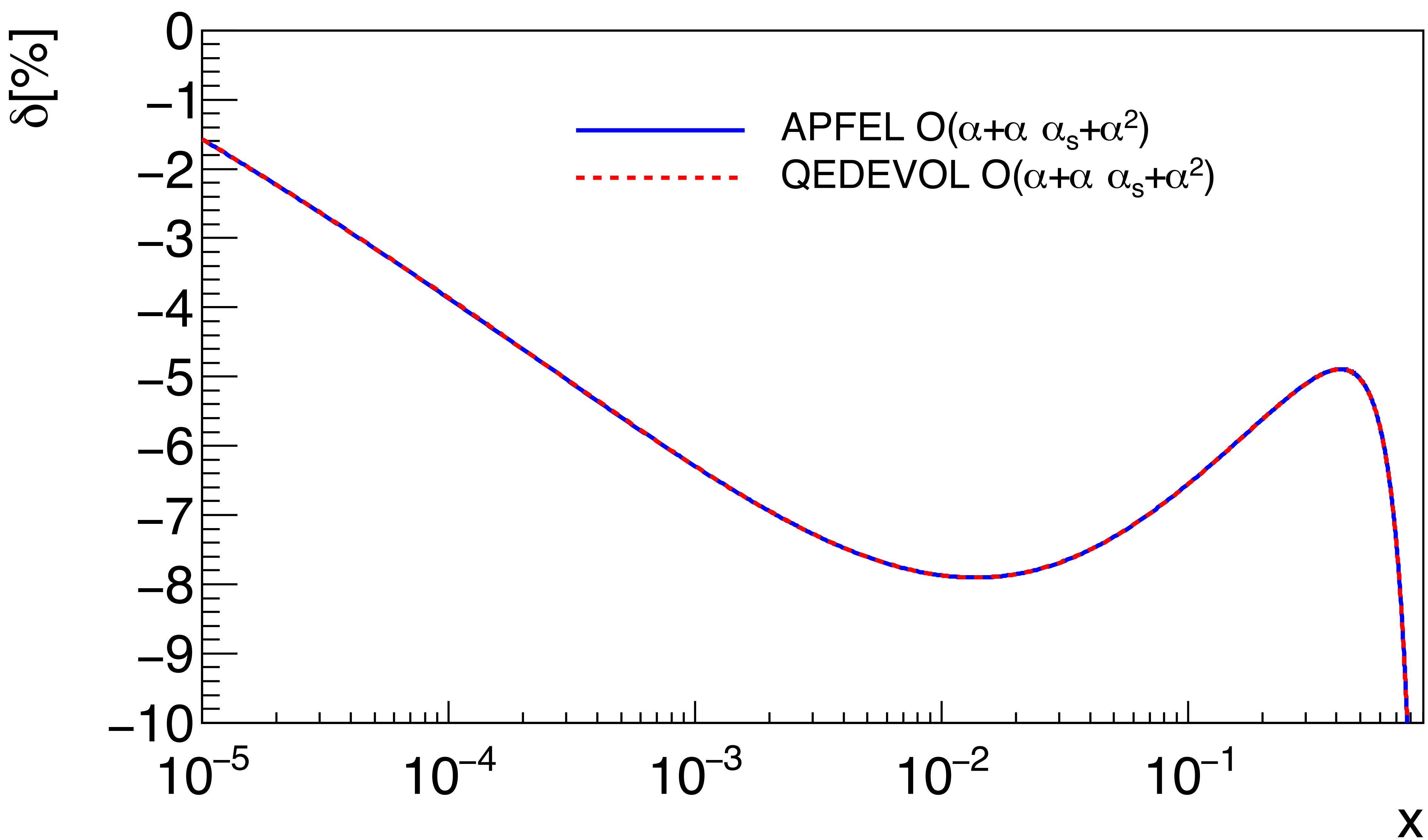
   - Numerical tests have shown that such terms lead to differences of $O(10^{-4})$ for α_s and $O(10^{-3})$ for α $\Rightarrow$ **unneeded complication**.
2. Implementing the $O(\alpha_s\alpha)$ and the $O(\alpha^2)$ corrections to the DGLAP **splitting functions** on top of the $O(\alpha)$ ones:
   - complication of the flavour structure due to the presence of terms promotional to e_q^2 and e_q^4 that break the isospin symmetry,
   - need for a more optimal evolution basis as compared to pure QCD.

NLO QCD+QED Corrections
DGLAP Evolution



- Effect on the photon PDF of the NLO corrns. at the level of 5%.

NLO QCD+QED Corrections
Benchmark against QEDEVOL

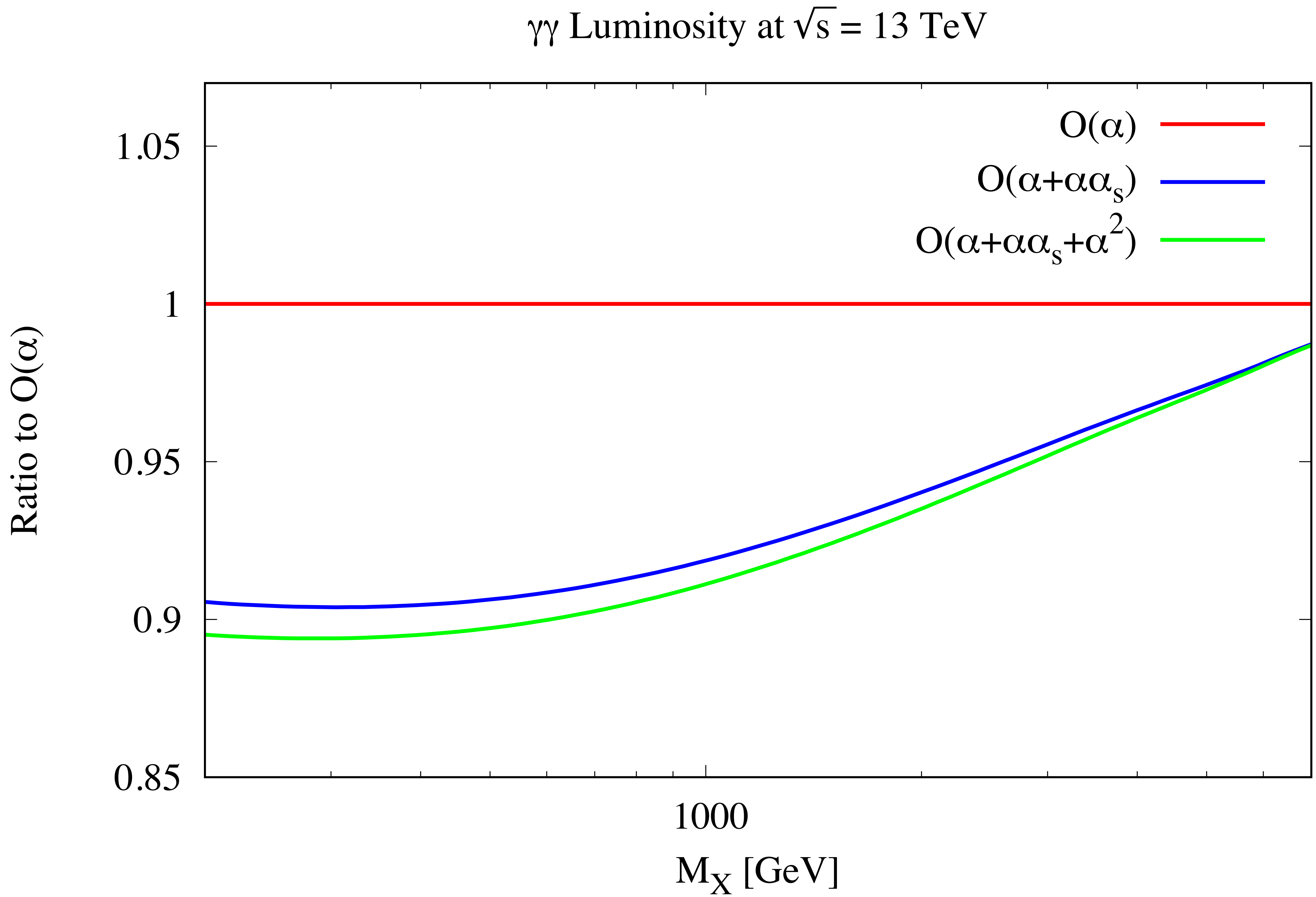


[Thanks to Renat Sadykov]

- Perfect agreement between APFEL and QEDEVOL.

NLO QCD+QED Corrections

Photon Luminosity

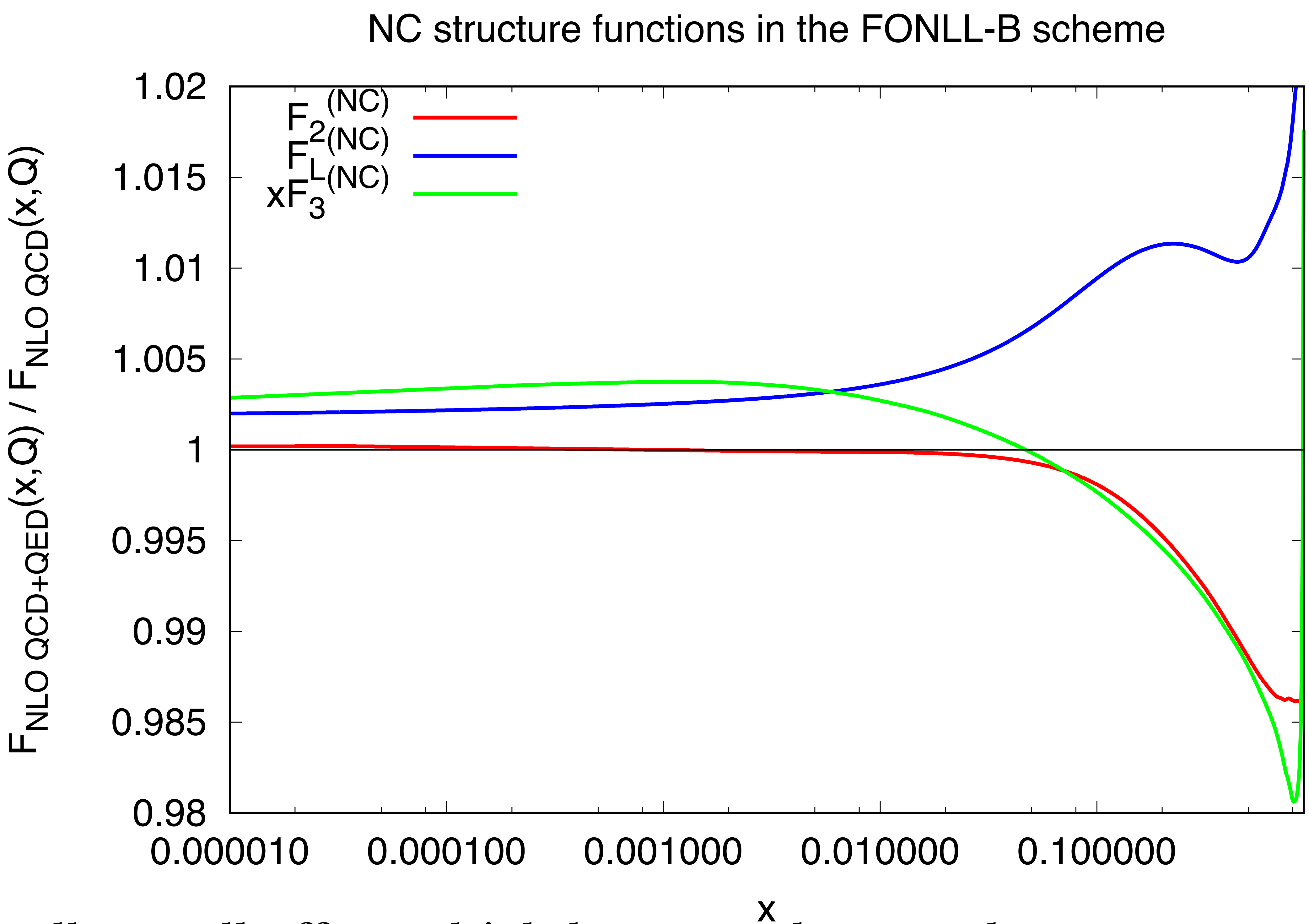


- More sizeable effect on the γγ luminosity.

NLO QCD+QED Corrections
DIS Structure Functions

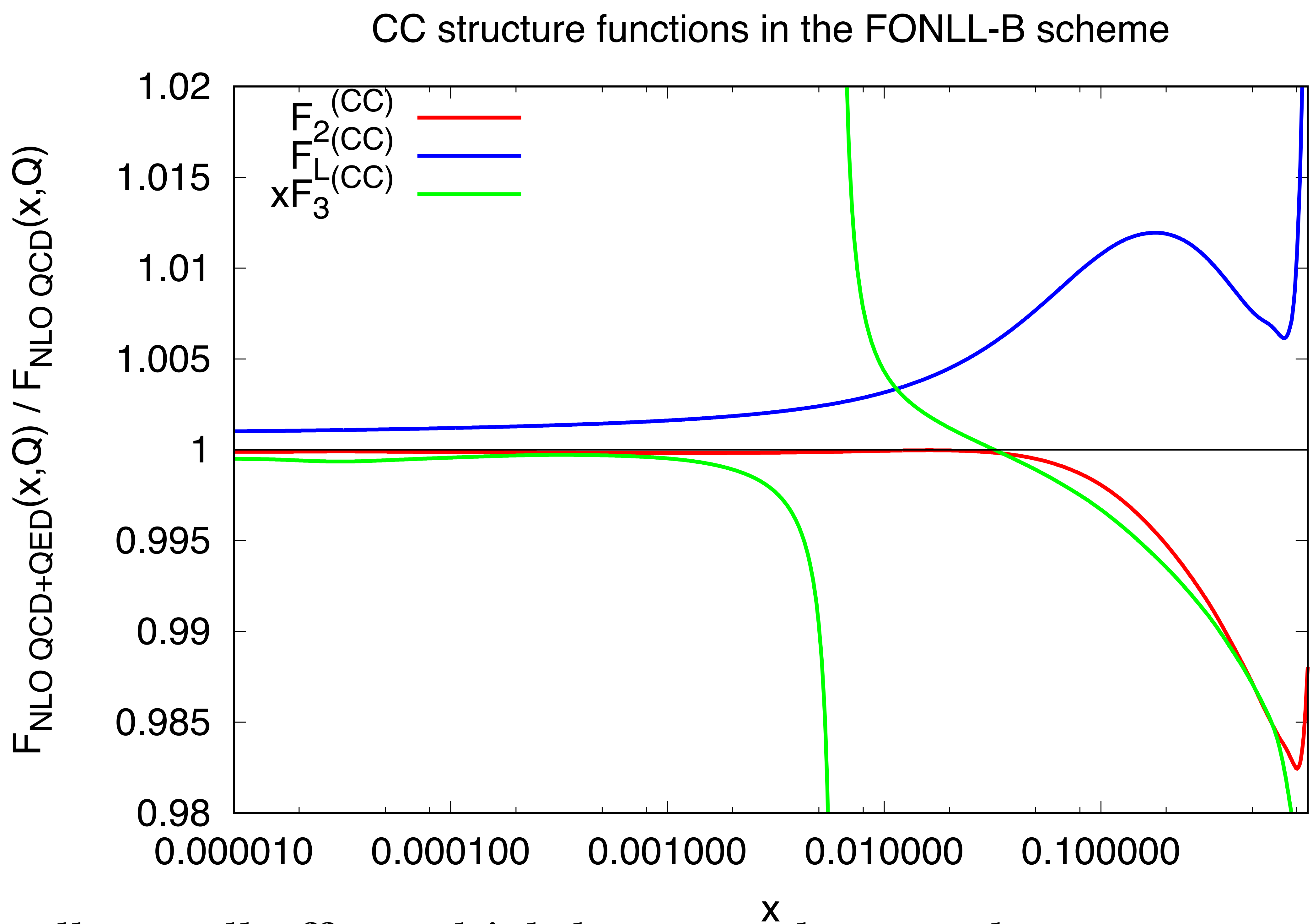
- While at LO in QED no corrections to the DIS structure functions are required ($\gamma^* q \to q$ itself is the LO), at NLO in QED $O(\alpha)$ corrections need to be taken into account:
    - **new diagrams**: $\gamma^*\gamma \to q\overline{q}$ and $\gamma^* q \to q\gamma$,
    - easily derivable from the corresponding QCD diagrams.
- The additional diagrams offer a **direct handle** on the photon PDF in DIS observables:
    - at LO in QED the photon PDF was entirely driven by the evolution.
- Small contribution proportional to $\alpha\gamma \sim O(\alpha^2)$ but can be relevant in some kinematic regions:
    - typically at large x and large Q^2.

NLO QCD+QED Corrections
DIS Structure Functions (NC)



- Generally small effect which becomes large at large x.

NLO QCD+QED Corrections
DIS Structure Functions (CC)

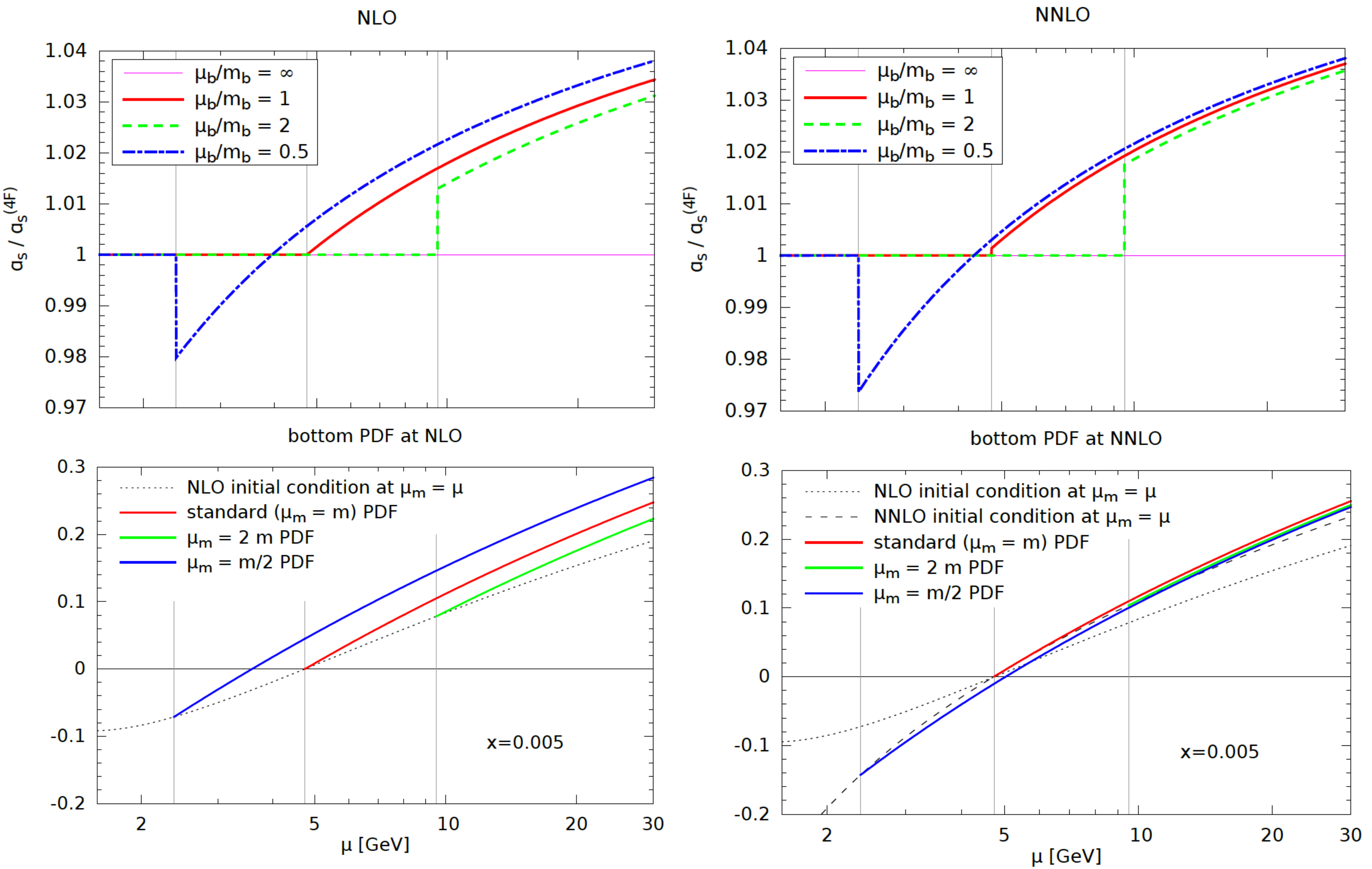


- Generally small effect which becomes large at large *x*.

Displaced Heavy-Quark Thresholds

- The implementation of the VFNS evolution both for PDFs and α_s requires **matching** factorisation schemes differing in the number of active flavours:
  - the scale at which two consecutive factorisation schemes are matched are usually referred to as **heavy-quark thresholds**.
  - Heavy-quark thresholds are usually (and for convenience) identified with the heavy quark masses by means of the so-called **matching conditions** presently know up to $O(\alpha_s^2)$ [hep-ph/9612398].
  - However, heavy-quark thresholds are actually free parameters and can be chosen **arbitrarily**.
  - If masses and thresholds are taken to be different, the matching conditions need to be "generalised" including **logarithmic terms**.
- APFEL now implements the possibility to set masses and thresholds to different values in a consistent way both in the pole mass and in the $\overline{\text{MS}}$ renormalisation schemes.

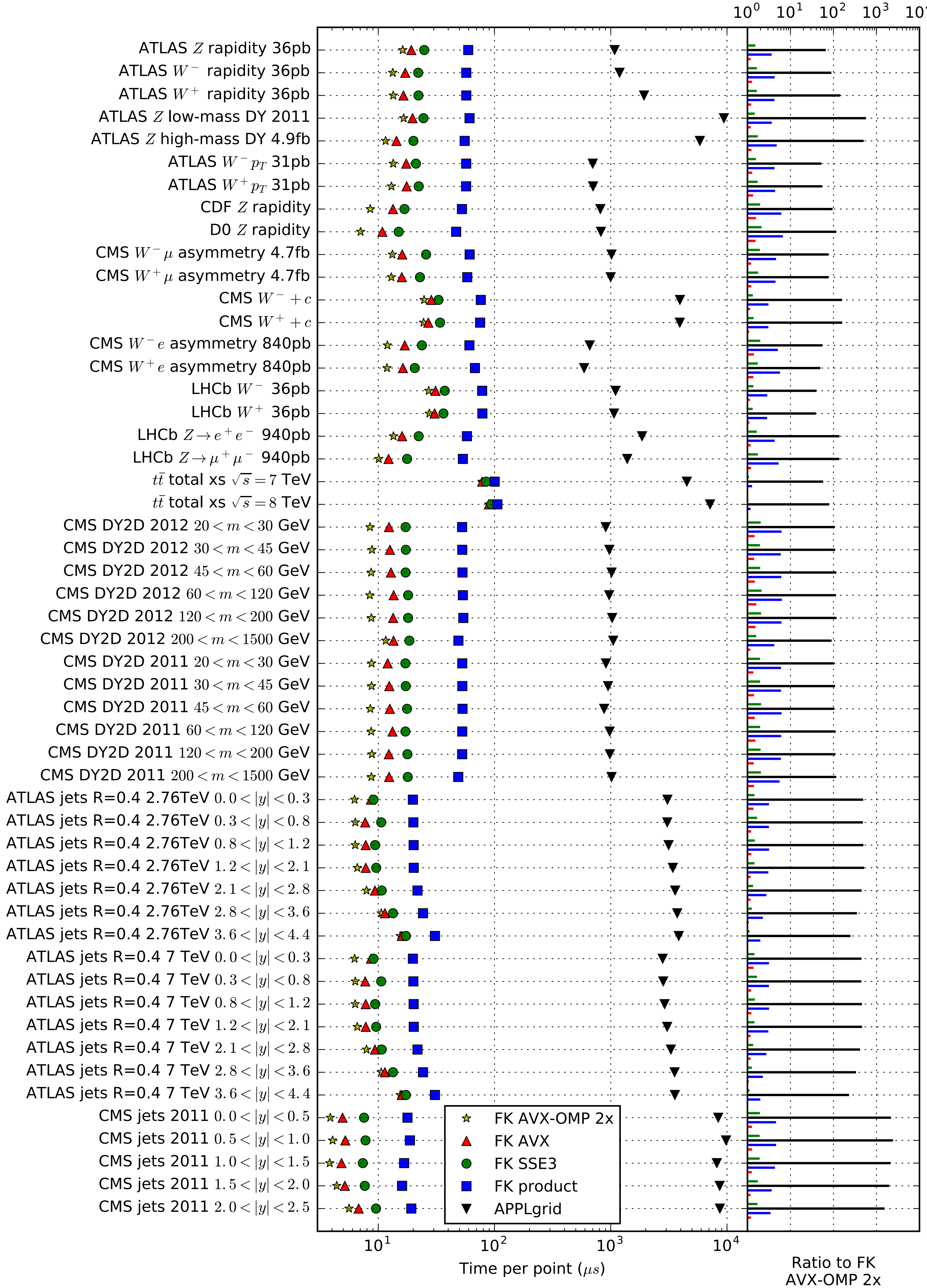
Displaced Heavy-Quark Thresholds



APFELgrid

A fast(er) interface for PDF fits

- While being extremely useful tools, **APPLgrid** and **FastNLO** might not be appropriate to be directly employed in a global PDF fit where usually **thousands of iterations** are needed:
  - need to calculate PDF and α_s **evolution in real time**.
  - **not particularly fast** convolution.
  - many tables need to be loaded with the concrete risk of **exceeding the memory limit** (pretty common on clusters).
- We developed **APFELgrid** which, starting from an APPLgrid, combines the PDF evolution from APFEL to the hard cross sections producing *derived* tables (FK tables) to be directly convoluted with the initial scale PDFs.
- APFELgrid relies on the precomputation of the evolution of α_s and PDFs:
  - less flexible than APPLgrid as the evolution parameters (perturbative order, reference value of α_s, heavy-quark thresholds, etc.) cannot be changed.



[V. Bertone, S. Carrazza, N.P. Hartland, Comput.Phys.Commun. 212 (2017) 205-209]

APFEL++

An object oriented rewriting of APFEL

- APFEL core is currently based on **FORTRAN77**.
- It thus provides "**singleton objects**", *i.e.* it is not possible to instantiate different evolutions and/or structure functions with different parameters.
- It includes a **large number of features**:
  - limited modularity ⇒ **hard to maintain and extend**,
  - static memory management ⇒ **large memory footprint** to ensure appropriate accuracy for all foreseeable applications.
- **C++** provides a natural solution to these issues:
  - possibility to instantiate any number of evolutions and structure functions,
  - **modularity** ensured by the possibility to define objects,
  - optimal memory management,
  - based on C++11 standard (Lambda funcs., auto declaration, smart pointers, etc.)

APFEL++

An object oriented rewriting of APFEL

- A good excuse to do better:
  - improve integration procedure (faster and more accurate):

```
Initialization... elapsed time: 0.209144 seconds
Tabulation... elapsed time: 0.111195 seconds
```

  - improve interpolation procedure:

```
Interpolating 1000000 times PDFs on the (x,Q) grid... elapsed time: 0.604006 seconds
```

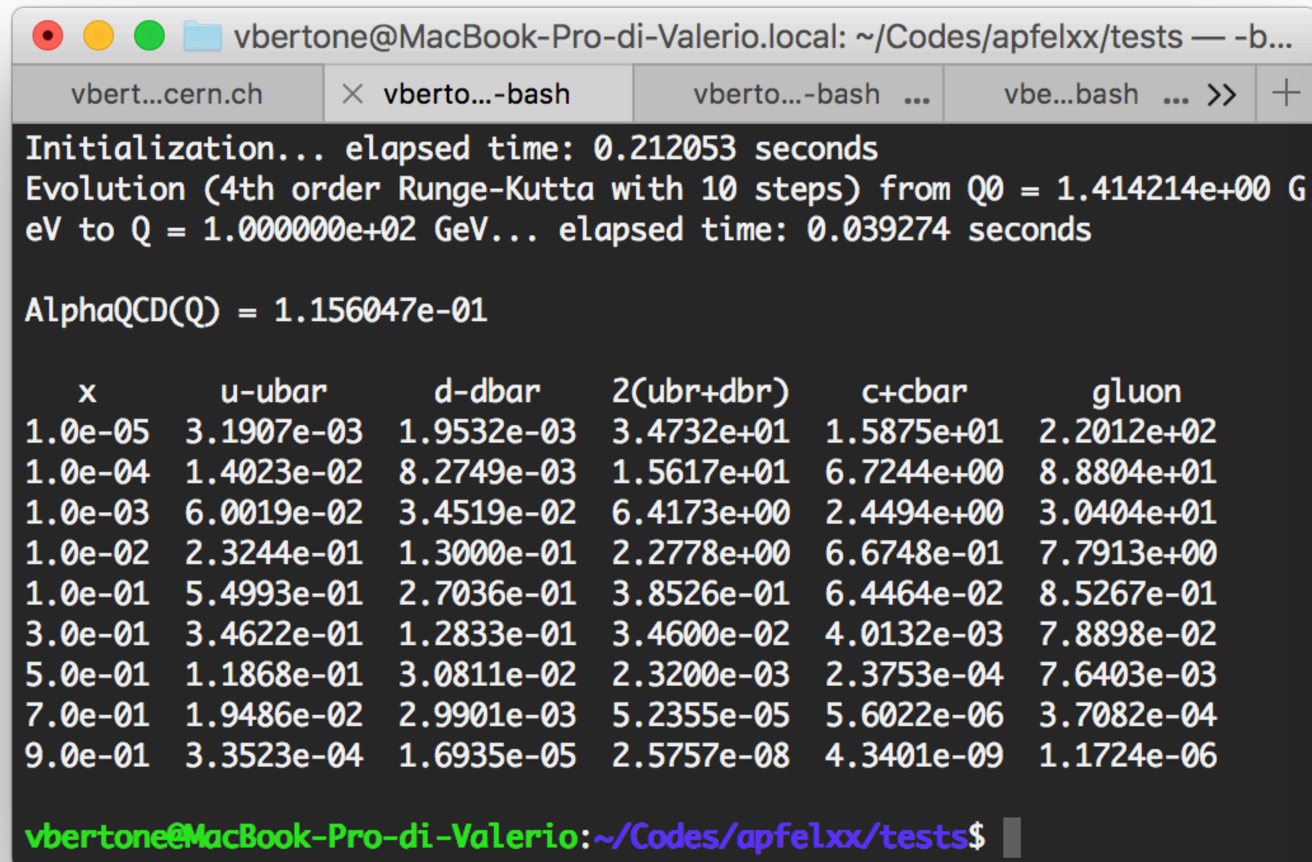
  - overload operators to make convolutions in an easy way:

```
//_________________________________________________________________________
Set<Distribution> Dglap::Derivative(int const& nf, double const& t, Set<Distribution> const& f) const
{
  return _SplittingFunctions(nf, exp(t/2)) * f;
}
```

  - use the same technology for specific hadronic observables (*e.g.* double differential DY).
  - Exploit CPU acceleration techniques (AVX, SSE).

APFEL++

An object oriented rewriting of APFEL

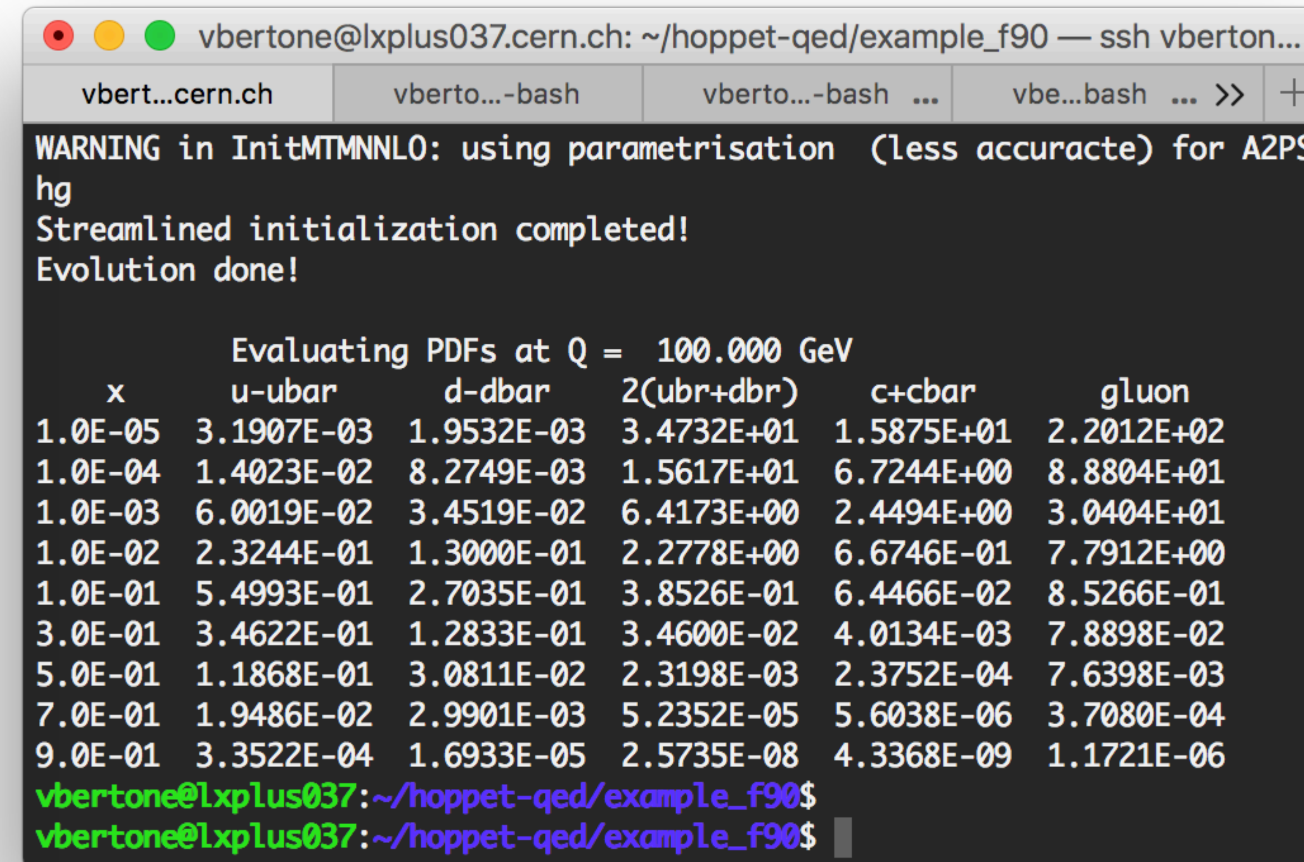


Initialization... elapsed time: 0.212053 seconds
Evolution (4th order Runge-Kutta with 10 steps) from Q0 = 1.414214e+00 GeV to Q = 1.000000e+02 GeV... elapsed time: 0.039274 seconds

AlphaQCD(Q) = 1.156047e-01

x	u-ubar	d-dbar	2(ubr+dbr)	c+cbar	gluon
1.0e-05	3.1907e-03	1.9532e-03	3.4732e+01	1.5875e+01	2.2012e+02
1.0e-04	1.4023e-02	8.2749e-03	1.5617e+01	6.7244e+00	8.8804e+01
1.0e-03	6.0019e-02	3.4519e-02	6.4173e+00	2.4494e+00	3.0404e+01
1.0e-02	2.3244e-01	1.3000e-01	2.2778e+00	6.6748e-01	7.7913e+00
1.0e-01	5.4993e-01	2.7036e-01	3.8526e-01	6.4464e-02	8.5267e-01
3.0e-01	3.4622e-01	1.2833e-01	3.4600e-02	4.0132e-03	7.8898e-02
5.0e-01	1.1868e-01	3.0811e-02	2.3200e-03	2.3753e-04	7.6403e-03
7.0e-01	1.9486e-02	2.9901e-03	5.2355e-05	5.6022e-06	3.7082e-04
9.0e-01	3.3523e-04	1.6935e-05	2.5757e-08	4.3401e-09	1.1724e-06

vbertone@MacBook-Pro-di-Valerio:~/Codes/apfelxx/tests$



WARNING in InitMTMNNLO: using parametrisation (less accuracte) for A2PShg
Streamlined initialization completed!
Evolution done!

Evaluating PDFs at Q = 100.000 GeV

x	u-ubar	d-dbar	2(ubr+dbr)	c+cbar	gluon
1.0E-05	3.1907E-03	1.9532E-03	3.4732E+01	1.5875E+01	2.2012E+02
1.0E-04	1.4023E-02	8.2749E-03	1.5617E+01	6.7244E+00	8.8804E+01
1.0E-03	6.0019E-02	3.4519E-02	6.4173E+00	2.4494E+00	3.0404E+01
1.0E-02	2.3244E-01	1.3000E-01	2.2778E+00	6.6746E-01	7.7912E+00
1.0E-01	5.4993E-01	2.7035E-01	3.8526E-01	6.4466E-02	8.5266E-01
3.0E-01	3.4622E-01	1.2833E-01	3.4600E-02	4.0134E-03	7.8898E-02
5.0E-01	1.1868E-01	3.0811E-02	2.3198E-03	2.3752E-04	7.6398E-03
7.0E-01	1.9486E-02	2.9901E-03	5.2352E-05	5.6038E-06	3.7080E-04
9.0E-01	3.3522E-04	1.6933E-05	2.5735E-08	4.3368E-09	1.1721E-06

vbertone@lxplus037:~/hoppet-qed/example_f90$
vbertone@lxplus037:~/hoppet-qed/example_f90$

APFEL

HOPPET

- Perfect agreement between APFEL and HOPPET.

Other Recent Developments

- **Intrinsic-charm** in DIS *a la* FONLL.
- **Polarised DGLAP evolution** up to NNLO.
- Independent factorisation and renormalisation **scale variations** both in the DIS structure functions and in the evolution,
- framework for the **determination of FFs**.

In the Pipeline

- Implementation of **TMD evolution** and **SIDIS cross sections**,
- Implementation of the **polarised structure functions**,
- **mass corrections** to SIA structure functions.

TMD Evolution (PDFs)

$$F_{f/P}(x, \mathbf{b}_T; \mu, \zeta) = \sum_j C_{f/j}(x, b_*; \mu_b, \zeta_F) \otimes f_{j/P}(x, \mu_b) \quad : A$$

$$\times \quad \exp\left\{ K(b_*; \mu_b) \ln \frac{\sqrt{\zeta_F}}{\mu_b} + \int_{\mu_b}^{\mu} \frac{d\mu'}{\mu'} \left[\gamma_F - \gamma_K \ln \frac{\sqrt{\zeta_F}}{\mu'} \right] \right\} \quad : B$$

$$\times \quad \exp\left\{ g_{j/P}(x, b_T) + g_K(b_T) \ln \frac{\sqrt{\zeta_F}}{\sqrt{\zeta_{F,0}}} \right\} \quad : C$$

TMD Evolution (PDFs)

$$F_{f/P}(x, \mathbf{b}_T; \mu, \zeta) = \sum_j C_{f/j}(x, b_*; \mu_b, \zeta_F) \otimes f_{j/P}(x, \mu_b) \qquad : A$$

$$\times \exp\left\{K(b_*; \mu_b) \ln \frac{\sqrt{\zeta_F}}{\mu_b} + \int_{\mu_b}^{\mu} \frac{d\mu'}{\mu'} \left[\gamma_F - \gamma_K \ln \frac{\sqrt{\zeta_F}}{\mu'}\right]\right\} \qquad : B$$

$$\times \exp\left\{g_{j/P}(x, b_T) + g_K(b_T) \ln \frac{\sqrt{\zeta_F}}{\sqrt{\zeta_{F,0}}}\right\} \qquad : C$$

- $b_\mathrm{T} \ll 1/\Lambda_\mathrm{QCD}$
- matching to the collinear region
- factorises as hard and non-perturbative
- numerically cumbersome
- precompute using the APFEL technology

- CS evolution
- perturbative

- matching between the small and large b_T
- non perturbative
- parametrised and fitted to data

TMDs in SIDIS

- In SIDIS, what enters the computation of the cross sections is:

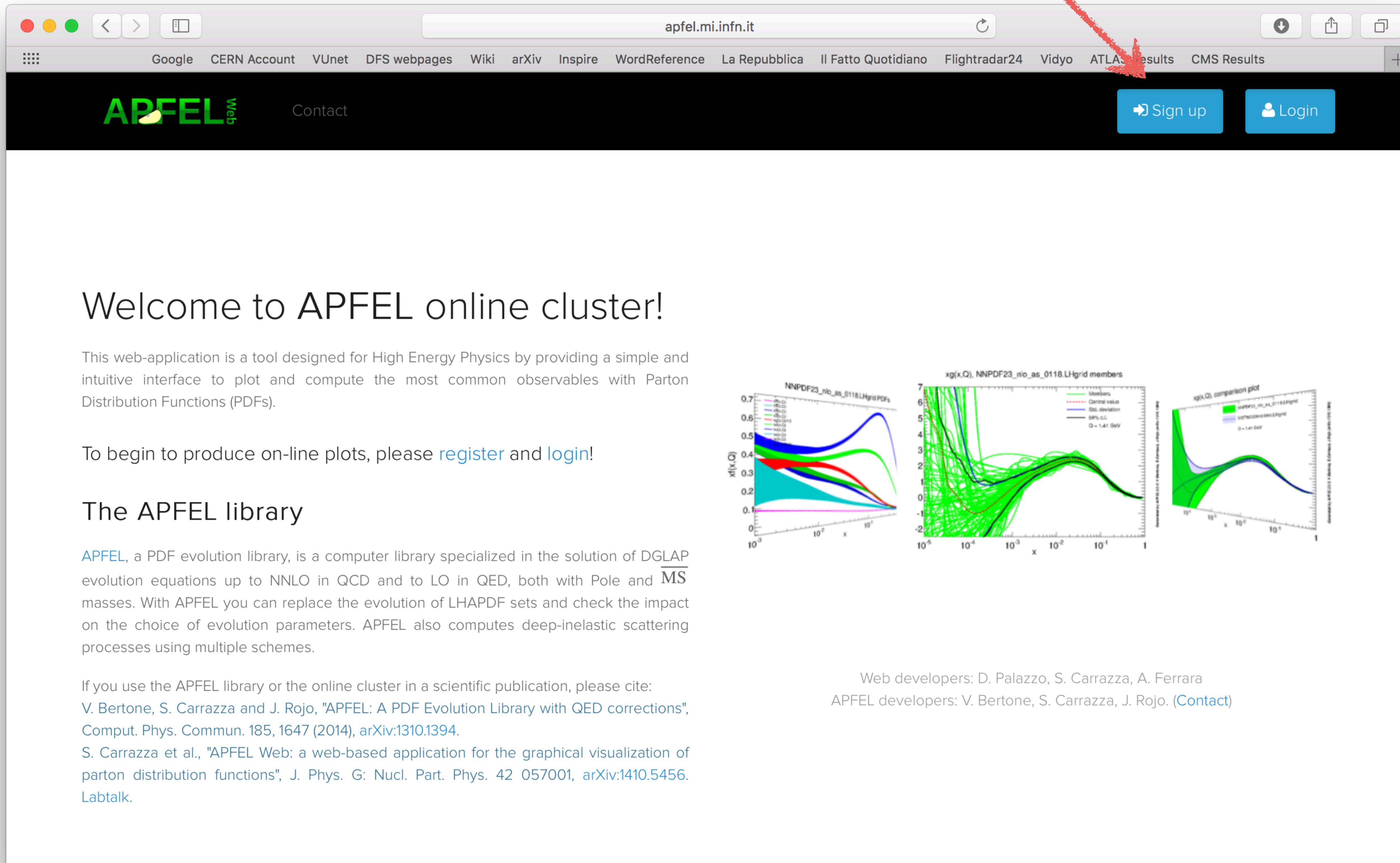
$$\mathcal{L}_{\rm SIDIS} = \int \frac{d^2\mathbf{b}_T}{(2\pi)^2} e^{-i\mathbf{q}_T\cdot\mathbf{b}_T} F_{f/P}(x,\mathbf{b}_T;\mu,\zeta_F) D_{H/f}(x,\mathbf{b}_T;\mu,\zeta_D)$$

Fourier transform PDFs FFs

- The ingredients are:
  - a set of evolved TMD-PDFs,
  - a set of evolved TMD-FFs,
  - the Fourier transform of its product.
- Complex set of tasks that have to be performed optimally
- APFEL provides the ideal environment for this computation:
  - fast and accurate interpolation techniques,
  - precomputation of the time consuming bits.

APFEL Web

Go to http://apfel.mi.infn.it and sign up



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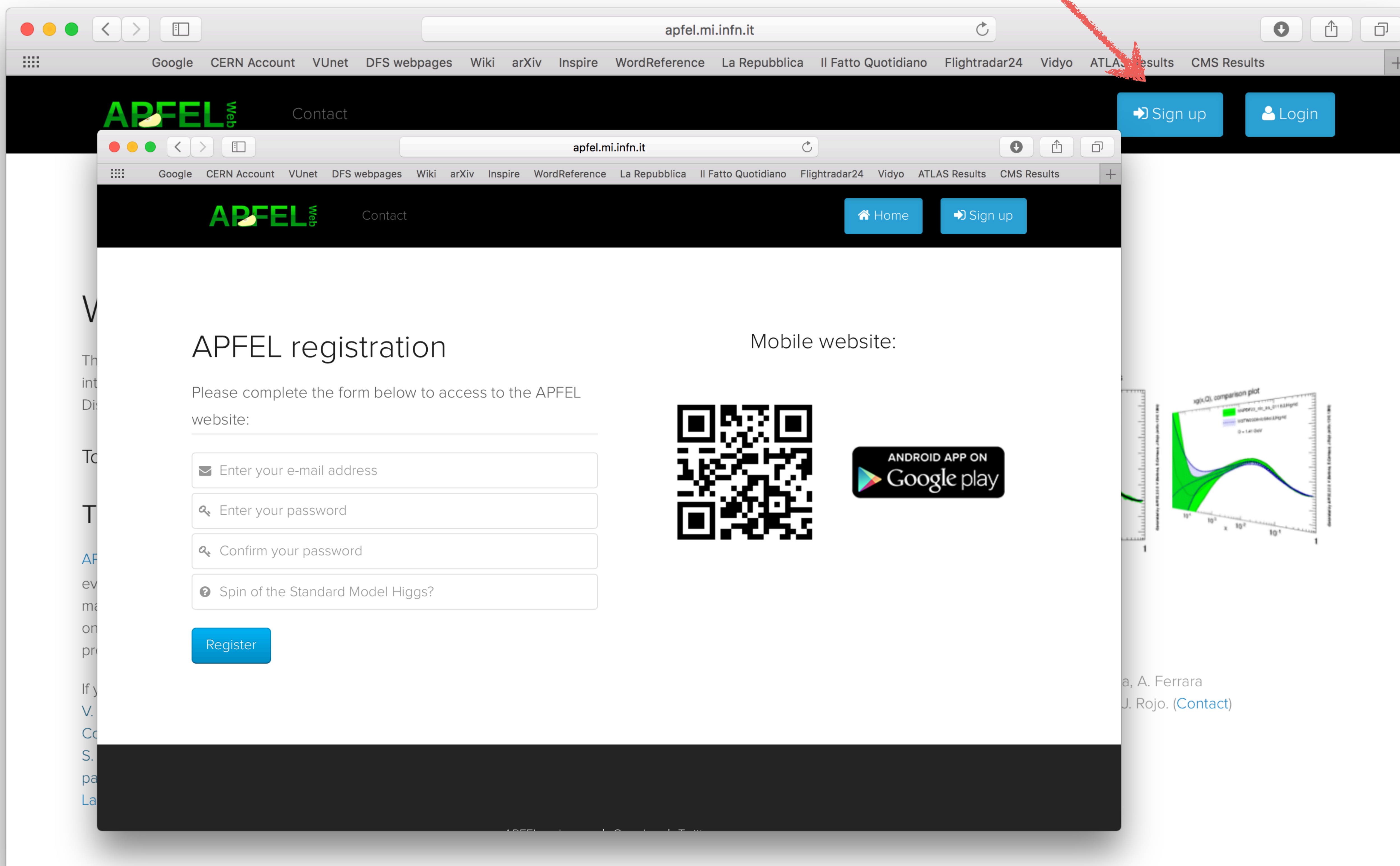
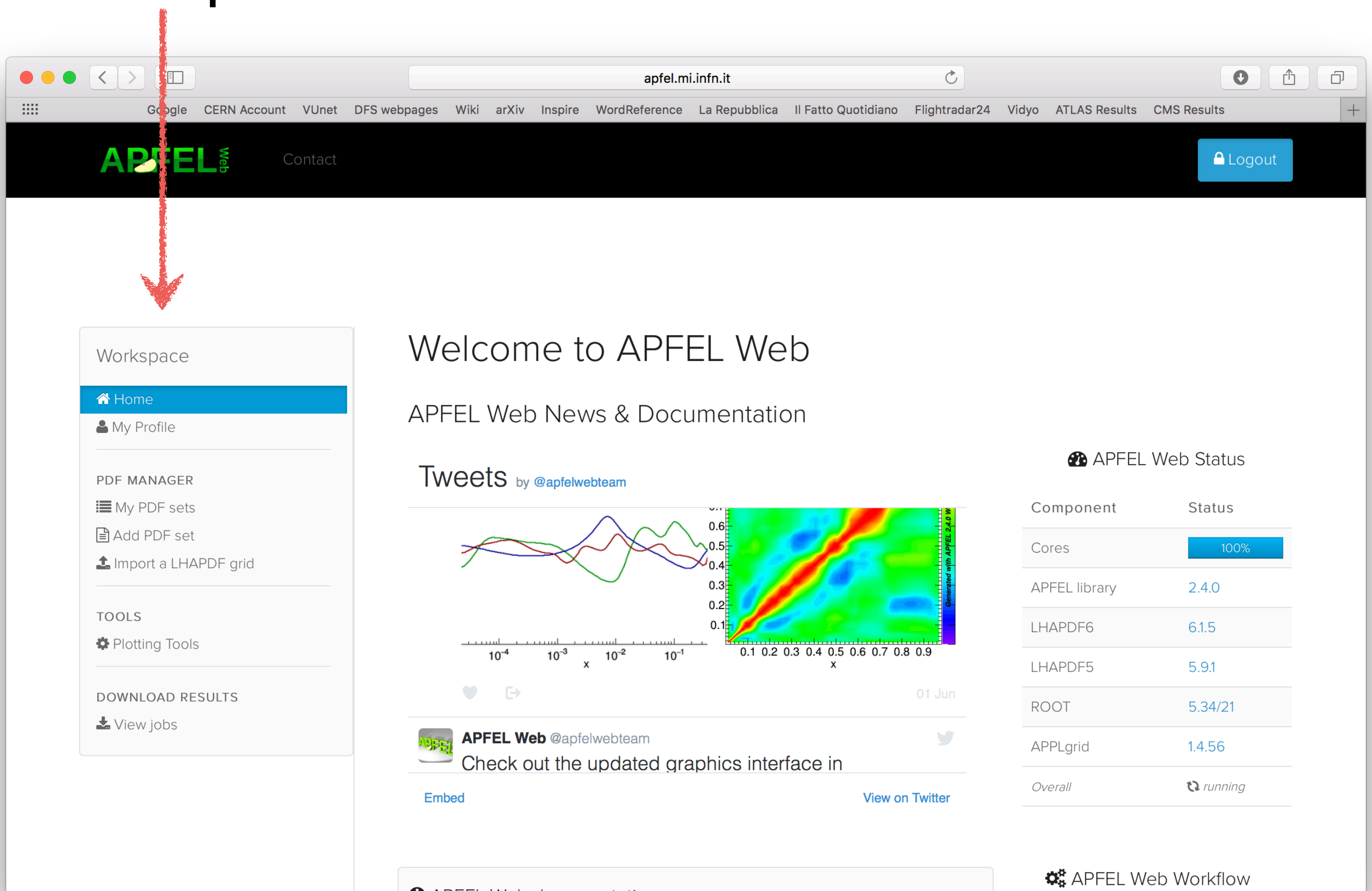


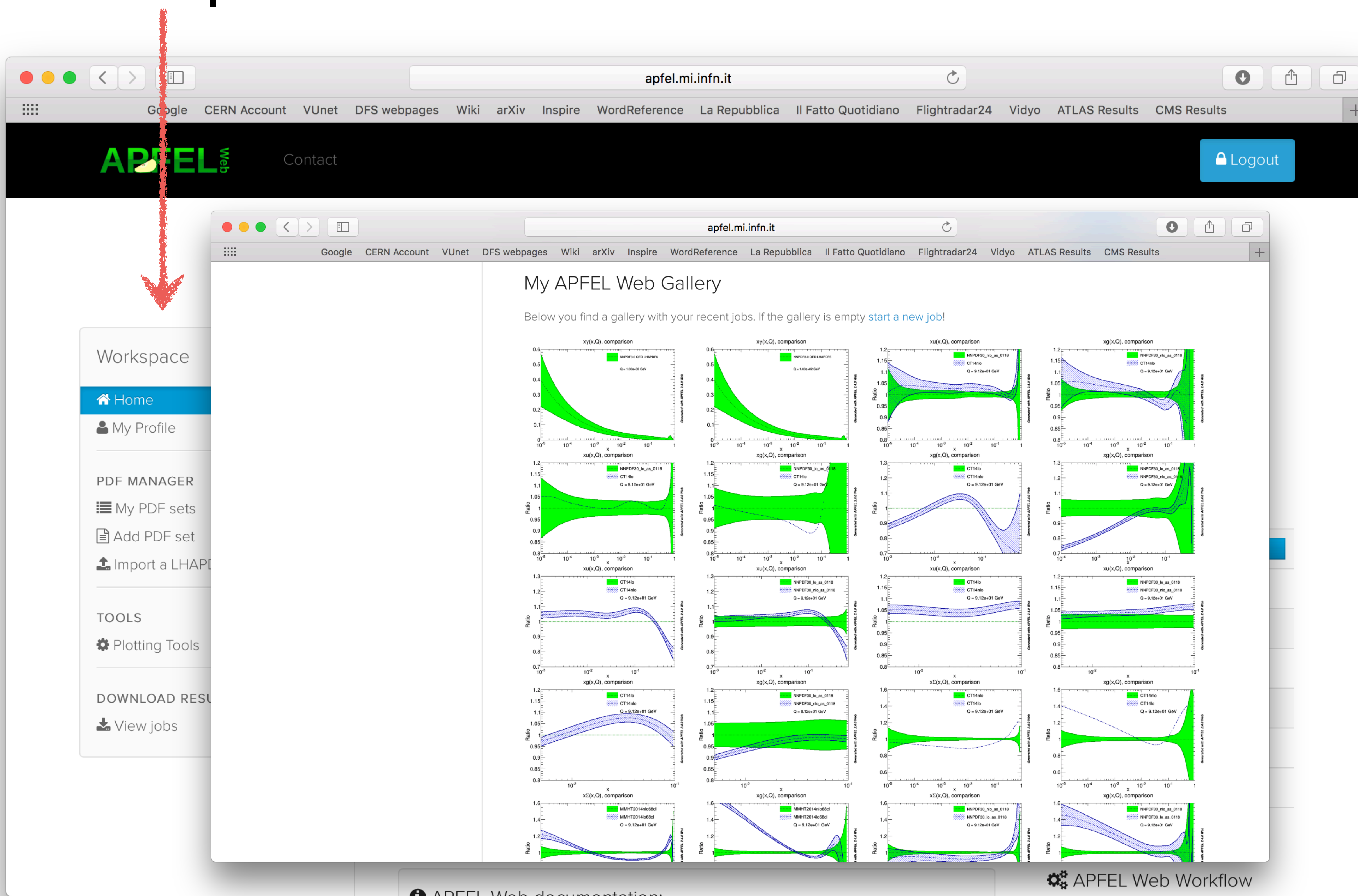
APFEL Web

Workspace



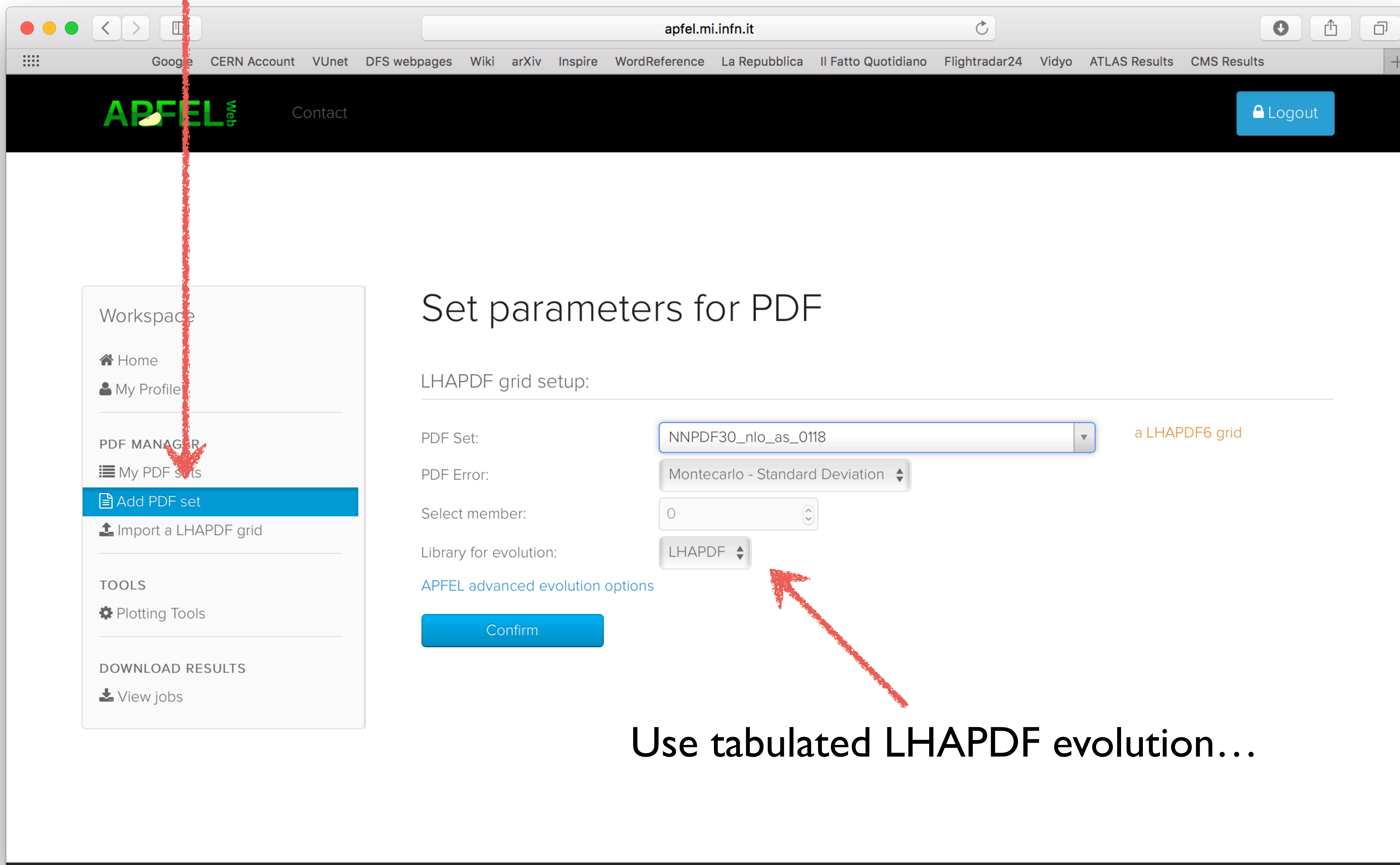
APFEL Web

Workspace



APFEL Web

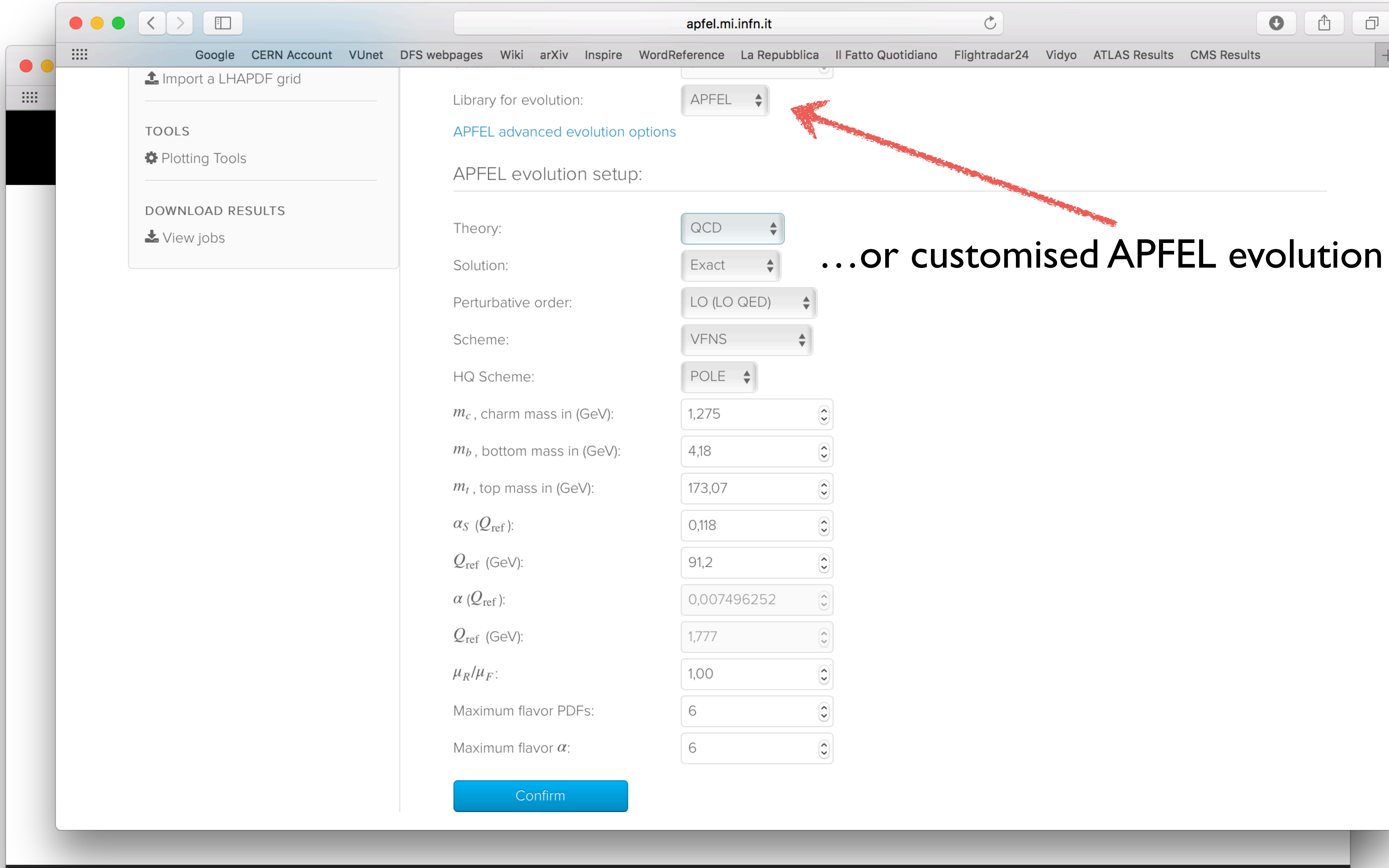
Add a new PDF set



Use tabulated LHAPDF evolution…

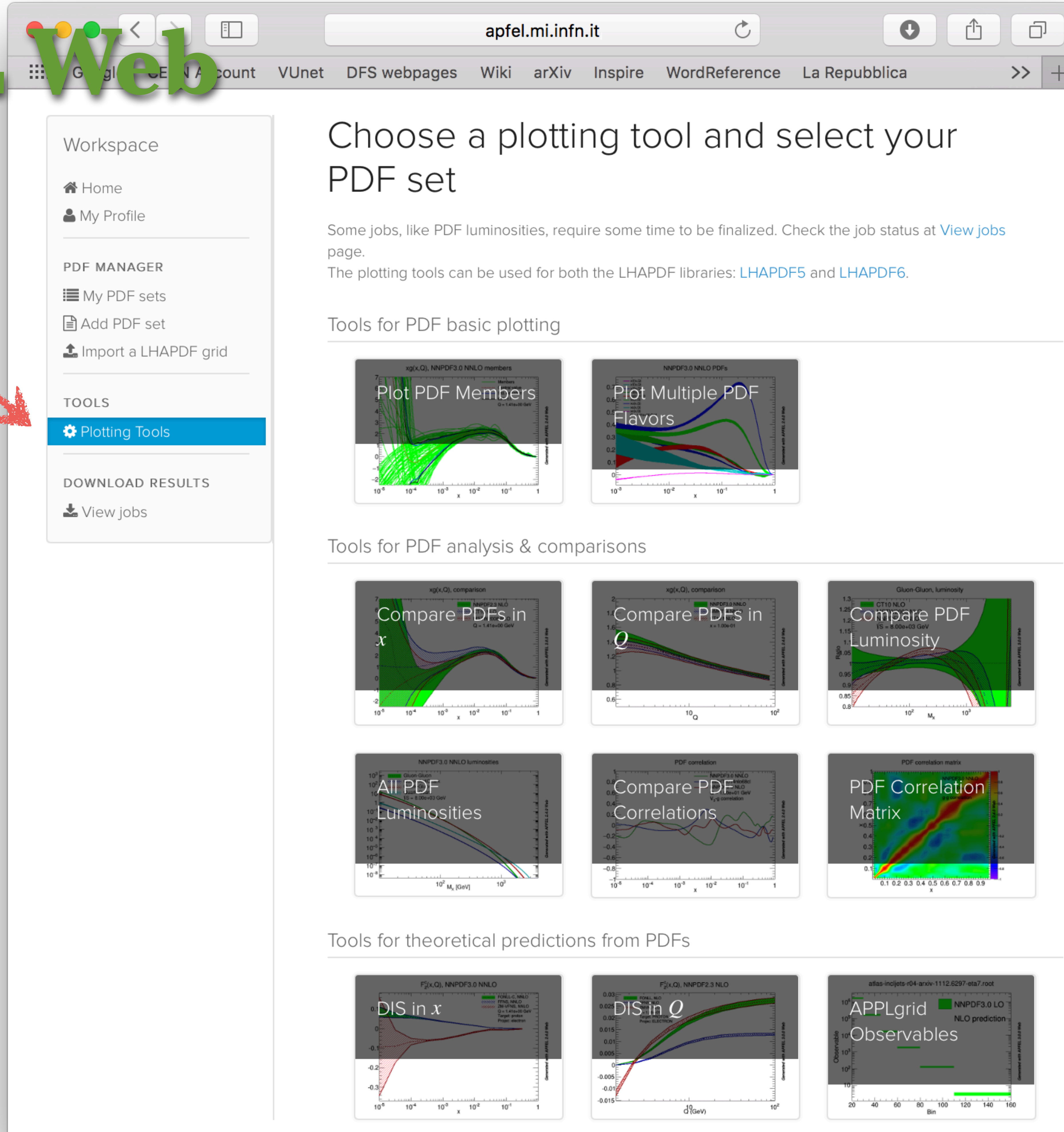
APFEL Web

Add a new PDF set



APFEL Web

Plotting tools



APFEL Web

View your jobs