

# APPLfast-NNLO

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21<sup>st</sup> March 2017

Work supported by the IPPP Associateship program

# appl(e)tise

*/'ap(ə)ltɪz/*

*noun trademark*

1. a sparkling fruit juice based drink created by blending apple juice with carbonated water (now appeltiser)

*verb*

1. to modify or add code (to an existing calculation) for the purposes of storing information (from the calculation) such that the approximate result (of the calculation) with different input parameters can be quickly determined



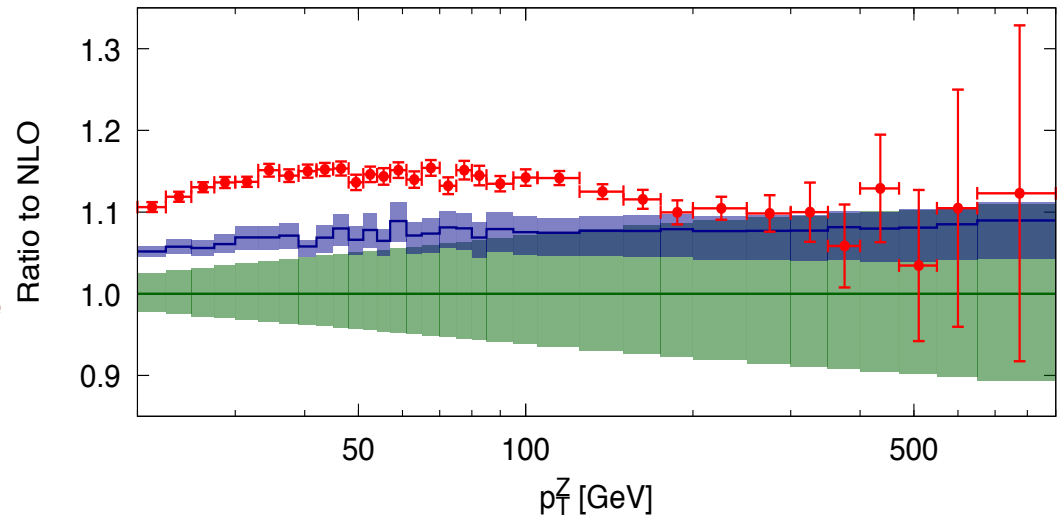
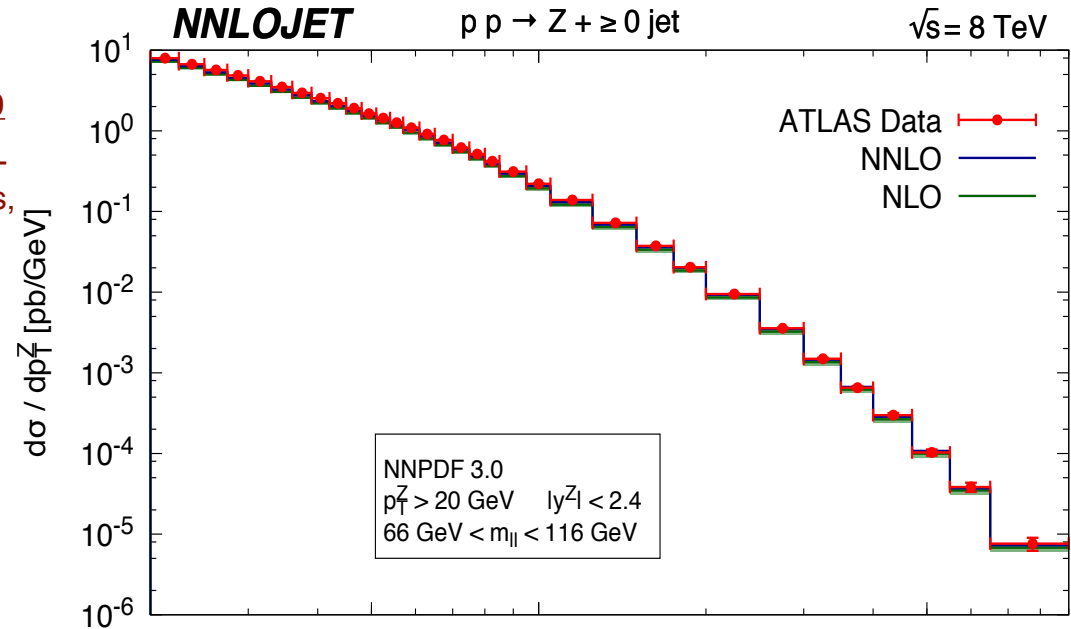
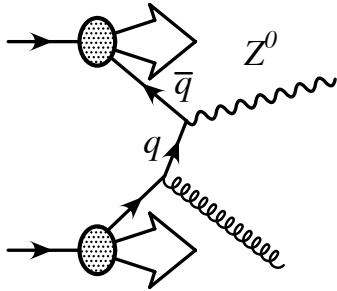
# Technical personnel

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- Project coordination :
  - Claire, Klaus, Mark
- NNLOJET modifications :
  - Alex, (Tom)
- Initial framework and validation:
  - Tom, Mark, Claire, Klaus
- pp-process developments :
  - Mark Claire, Klaus
- DIS developments :
  - Daniel, Mark
- Large scale production :
  - Klaus, Daniel, Mark, Claire
- Continuing core code development :
  - Everyone
- This program has personnel from **both fastNLO and APPLgrid**, together with developers from the **NNLOJET** developers
  - The aim is for a combined interface for the NNLO calculation with the fast grid technology
- The NNLO calculation represents an **extremely significant** and **extensive program** of work over many years
  - The extension from NLO to NNLO is non-trivial and people **should not underestimate** the effort involved
- The NNLO calculation engine is not ours - so the wishes of the NNLOJET authors are of course are primary
- The applfast developers are all **part of the wider QCD community**, so of course we are all are **very much aware** that progress in this area is eagerly awaited ...
  - Rest assured that we are working on the implementation and will make the results available as soon as we have verified output
- As soon as **verified high statistics grids** are available (for calculations in the experimental kinematic regions) that the **NNLOJET authors have endorsed**, these will be made available to the wider community

# NNLOJET (and APPLfast-NNLO)

- Semi-automated calculation of cross sections at NNLO from the IPPP, Zurich, ETH and others
  - Gehrmann-De Ridder *et al* [arXiv: 1607.01749](https://arxiv.org/abs/1607.01749)
  - In principle many processes are implemented - inclusive Z, Z+jets, W production, inclusive jets, etc
- APPLfast-NNLO
  - Developers from **NNLOJET**, **APPLgrid** and **fastNLO**
  - A single, combined interface for NNLOJET with both APPLgrid and fastNLO
- Many processes implemented in NNLOJET
  - Developing a generic interface for **all available** processes
  - Concentrating on Z + jets at NNLO for the initial development and proof-of-concept



# Grid generation

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- After establishing details of the parameterisation - number of grid nodes, interpolation order etc - using limited statistics jobs, can generate high statistics grids
1. NNLOJET warmup
    - Optimise the NNLOJET VEGAS phase space in dedicated NNLOJET job
  2. NNLOJET-applfast warmup
    - Many parallel jobs
    - Phase space only processing, run with grid filling enabled purely to establish optimised phase space - different approach for APPLgrid and fastNLO, but this is hidden in the interface
    - Reduced number of internal processes, and phase space only processing gives a significant reduction of processing
  3. Full production run
    - Run as many jobs in parallel as CPU availability allows - many thousands of CPU hours required, most notably for the double-real contribution due to the highly differential phase space
  4. Combine the output sub-grids from the production run
  5. All these stages have been implemented but still being fully validated

# Z + jets at NLO

- NNLOJET calculates using many (150) distinct internal processes, many with the same input partons
  - Automatically reduce down to 33 parton luminosities for the NLO process - combine for Real and Virtual contributions
  - Keep the internal mapping of the internal process ID to the parton luminosity

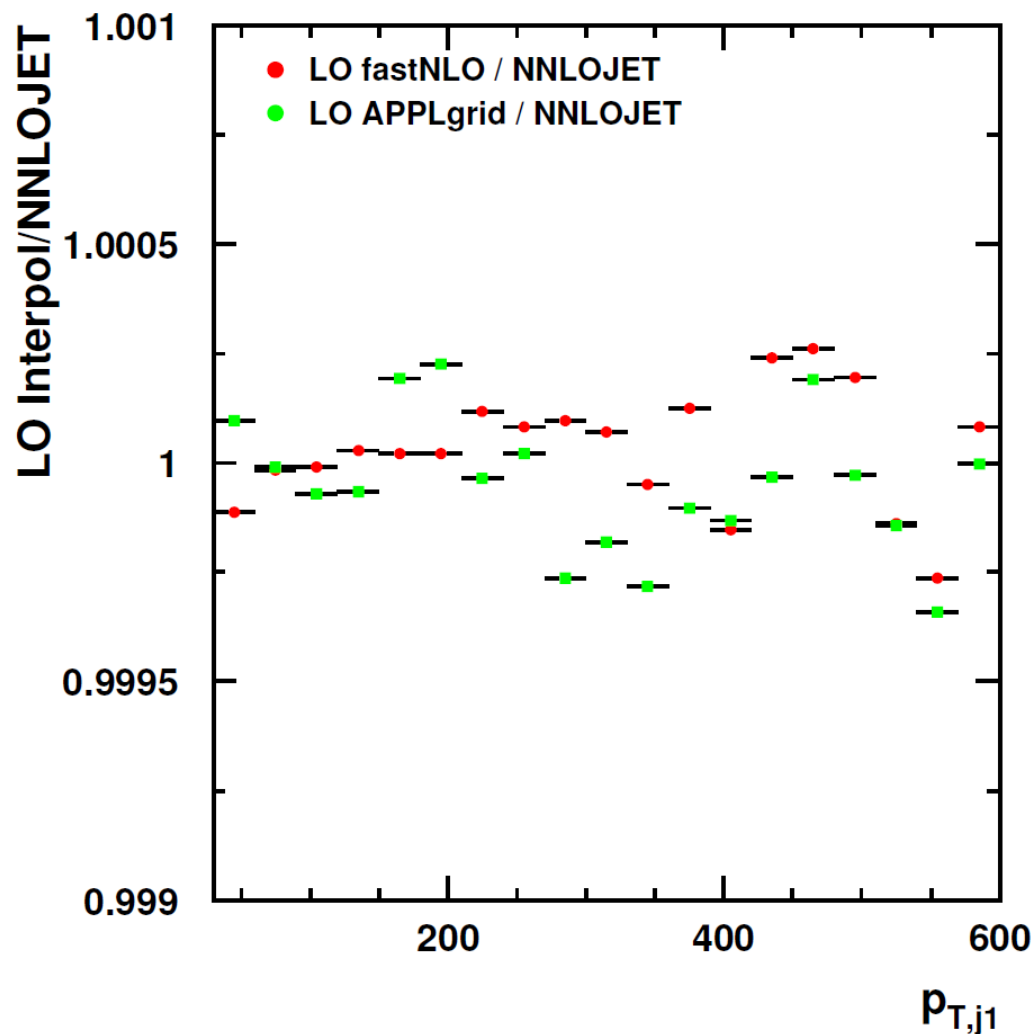
0	13 27 41 42 65 73 103 133	$(d, \bar{d}) + (s, \bar{s}) + (b, \bar{b})$
1	14 28 43 44 66 74 104 134	$(u, \bar{u}) + (c, \bar{c})$
2	15 29 45 46 67 75 105 135	$(\bar{d}, d) + (\bar{s}, s) + (\bar{b}, b)$
3	16 30 47 48 68 76 106 136	$(\bar{u}, u) + (\bar{c}, c)$
4	17 31 77 107 137	$(d, g) + (s, g) + (b, g)$
5	18 32 78 108 138	$(u, g) + (c, g)$
6	19 33 79 109 139	$(g, d) + (g, s) + (g, b)$
7	20 34 80 110 140	$(g, u) + (g, c)$
8	21 22 35 36 85 86 115 116 145 146	$(g, g)$
9	23 37 81 111 141	$(g, \bar{d}) + (g, \bar{s}) + (g, \bar{b})$
10	24 38 82 112 142	$(g, \bar{u}) + (g, \bar{c})$
11	25 39 83 113 143	$(\bar{d}, g) + (\bar{s}, g) + (\bar{b}, g)$
12	26 40 84 114 144	$(\bar{u}, g) + (\bar{c}, g)$
13	49 91 121 151	$(d, \bar{d}) + (d, \bar{s}) + (d, \bar{b}) + (s, \bar{d}) + (s, \bar{s}) + (s, \bar{b}) + (b, \bar{d}) + (b, \bar{s}) + (b, \bar{b})$
14	50 92 122 152	$(d, \bar{u}) + (d, \bar{c}) + (s, \bar{u}) + (s, \bar{c}) + (b, \bar{u}) + (b, \bar{c})$
15	51 93 123 153	$(u, \bar{d}) + (u, \bar{s}) + (u, \bar{b}) + (c, \bar{d}) + (c, \bar{s}) + (c, \bar{b})$
16	52 94 124 154	$(u, \bar{u}) + (u, \bar{c}) + (c, \bar{u}) + (c, \bar{c})$
17	53 87 117 147	$(d, d) + (d, s) + (d, b) + (s, d) + (s, s) + (s, b) + (b, d) + (b, s) + (b, b)$
18	54 88 118 148	$(d, u) + (d, c) + (s, u) + (s, c) + (b, u) + (b, c)$
19	55 89 119 149	$(u, d) + (u, s) + (u, b) + (c, d) + (c, s) + (c, b)$
20	56 90 120 150	$(u, u) + (u, c) + (c, u) + (c, c)$
21	57 99 129 159	$(\bar{d}, \bar{d}) + (\bar{d}, \bar{s}) + (\bar{d}, \bar{b}) + (\bar{s}, \bar{d}) + (\bar{s}, \bar{s}) + (\bar{s}, \bar{b}) + (\bar{b}, \bar{d}) + (\bar{b}, \bar{s}) + (\bar{b}, \bar{b})$
22	58 100 130 160	$(\bar{d}, \bar{u}) + (\bar{d}, \bar{c}) + (\bar{s}, \bar{u}) + (\bar{s}, \bar{c}) + (\bar{b}, \bar{u}) + (\bar{b}, \bar{c})$
23	59 101 131 161	$(\bar{u}, \bar{d}) + (\bar{u}, \bar{s}) + (\bar{u}, \bar{b}) + (\bar{c}, \bar{d}) + (\bar{c}, \bar{s}) + (\bar{c}, \bar{b})$
24	60 102 132 162	$(\bar{u}, \bar{u}) + (\bar{u}, \bar{c}) + (\bar{c}, \bar{u}) + (\bar{c}, \bar{c})$
25	61 95 125 155	$(\bar{d}, d) + (\bar{d}, s) + (\bar{d}, b) + (\bar{s}, d) + (\bar{s}, s) + (\bar{s}, b) + (\bar{b}, d) + (\bar{b}, s) + (\bar{b}, b)$
26	62 96 126 156	$(\bar{d}, u) + (\bar{d}, c) + (\bar{s}, u) + (\bar{s}, c) + (\bar{b}, u) + (\bar{b}, c)$
27	63 97 127 157	$(\bar{u}, d) + (\bar{u}, s) + (\bar{u}, b) + (\bar{c}, d) + (\bar{c}, s) + (\bar{c}, b)$
28	64 98 128 158	$(\bar{u}, u) + (\bar{u}, c) + (\bar{c}, u) + (\bar{c}, c)$
29	69	$(d, d) + (s, s) + (b, b)$
30	70	$(u, u) + (c, c)$
31	71	$(\bar{d}, \bar{d}) + (\bar{s}, \bar{s}) + (\bar{b}, \bar{b})$
32	72	$(\bar{u}, \bar{u}) + (\bar{c}, \bar{c})$

# Z + jets at NNLO

- Again the same 33 input parton luminosities as in the NNLO case, however, many more (794) individual internal processes

0	163 177 191 205 206 245 246 285 301 317 347 377 391 405 435 449 450 489 490 529 530 569 593 617 647 677 707 737 767 899 905 906 935 941 942	$(d, \bar{d}) + (s, \bar{s}) + (b, \bar{b})$
1	164 178 192 207 208 247 248 286 302 318 348 378 392 406 436 451 452 491 492 531 532 570 594 618 648 678 708 738 768 900 907 908 936 943 944	$(u, \bar{u}) + (c, \bar{c})$
2	165 179 193 209 210 249 250 287 303 319 349 379 393 407 437 453 454 493 494 533 534 571 595 619 649 679 709 739 769 909 917 918 945 953 954	$(\bar{d}, d) + (\bar{s}, s) + (\bar{b}, b)$
3	166 180 194 211 212 251 252 288 304 320 350 380 394 408 438 455 456 495 496 535 536 572 596 620 650 680 710 740 770 910 919 920 946 955 956	$(\bar{u}, u) + (\bar{c}, c)$
4	167 181 195 229 230 269 270 289 305 321 351 381 395 409 439 481 482 521 522 561 562 585 586 609 610 621 651 681 711 741 771 817 818 861 862 901 902 937 938	$(d, g) + (s, g) + (b, g)$
5	168 182 196 231 232 271 272 290 306 322 352 382 396 410 440 483 484 523 524 563 564 587 588 611 612 622 652 682 712 742 772 819 820 863 864 903 904 939 940	$(u, g) + (c, g)$
6	169 183 197 233 234 273 274 293 309 323 353 383 397 411 441 473 474 513 514 553 554 577 578 601 602 623 653 683 713 743 773 801 802 845 846 889 890 925 926	$(g, d) + (g, s) + (g, b)$
7	170 184 198 235 236 275 276 294 310 324 354 384 398 412 442 475 476 515 516 555 556 579 580 603 604 624 654 684 714 744 774 803 804 847 848 891 892 927 928	$(g, u) + (g, c)$
8	171 172 185 186 199 200 325 326 355 356 385 386 399 400 413 414 443 444 629 630 659 660 689 690 719 720 749 750 779 780 797 798 799 800 841 842 843 844 885 886 887 888 921 922 923 924	$(g, g)$
9	173 187 201 237 238 277 278 295 311 327 357 387 401 415 445 477 478 517 518 557 558 581 582 605 606 625 655 685 715 745 775 805 806 849 850 893 894 929 930	$(g, \bar{d}) + (g, \bar{s}) + (g, \bar{b})$
10	174 188 202 239 240 279 280 296 312 328 358 388 402 416 446 479 480 519 520 559 560 583 584 607 608 626 656 686 716 746 776 807 808 851 852 895 896 931 932	$(g, \bar{u}) + (g, \bar{c})$
11	175 189 203 241 242 281 282 297 313 329 359 389 403 417 447 485 486 525 526 565 566 589 590 613 614 627 657 687 717 747 777 833 834 877 878 913 914 949 950	$(\bar{d}, g) + (\bar{s}, g) + (\bar{b}, g)$
12	176 190 204 243 244 283 284 298 314 330 360 390 404 418 448 487 488 527 528 567 568 591 592 615 616 628 658 688 718 748 778 835 836 879 880 915 916 951 952	$(\bar{u}, g) + (\bar{c}, g)$
13	213 253 335 365 423 457 497 537 635 665 695 725 755 785 813 821 822 857 865 866	$(d, \bar{d}) + (d, \bar{s}) + (d, \bar{b}) + (s, \bar{d}) + (s, \bar{s}) + (s, \bar{b}) + (b, \bar{d}) + (b, \bar{s}) + (b, \bar{b})$
14	214 254 336 366 424 458 498 538 636 666 696 726 756 786 814 858	$(d, \bar{u}) + (d, \bar{c}) + (s, \bar{u}) + (s, \bar{c}) + (b, \bar{u}) + (b, \bar{c})$
15	215 255 337 367 425 459 499 539 637 667 697 727 757 787 815 859	$(u, \bar{d}) + (u, \bar{s}) + (u, \bar{b}) + (c, \bar{d}) + (c, \bar{s}) + (c, \bar{b})$
16	216 256 338 368 426 460 500 540 638 668 698 728 758 788 816 823 824 860 867 868	$(u, \bar{u}) + (u, \bar{c}) + (c, \bar{u}) + (c, \bar{c})$
17	217 257 331 361 419 461 501 541 631 661 691 721 751 781 809 853	$(d, d) + (d, s) + (d, b) + (s, d) + (s, s) + (s, b) + (b, d) + (b, s) + (b, b)$
18	218 258 332 362 420 462 502 542 632 662 692 722 752 782 810 854	$(d, u) + (d, c) + (s, u) + (s, c) + (b, u) + (b, c)$
19	219 259 333 363 421 463 503 543 633 663 693 723 753 783 811 855	$(u, d) + (u, s) + (u, b) + (c, d) + (c, s) + (c, b)$
20	220 260 334 364 422 464 504 544 634 664 694 724 754 784 812 856	$(u, u) + (u, c) + (c, u) + (c, c)$
21	221 261 343 373 431 465 505 545 643 673 703 733 763 793 829 873	$(\bar{d}, \bar{d}) + (\bar{d}, \bar{s}) + (\bar{d}, \bar{b}) + (\bar{s}, \bar{d}) + (\bar{s}, \bar{s}) + (\bar{s}, \bar{b}) + (\bar{b}, \bar{d}) + (\bar{b}, \bar{s}) + (\bar{b}, \bar{b})$
22	222 262 344 374 432 466 506 546 644 674 704 734 764 794 830 874	$(\bar{d}, \bar{u}) + (\bar{d}, \bar{c}) + (\bar{s}, \bar{u}) + (\bar{s}, \bar{c}) + (\bar{b}, \bar{u}) + (\bar{b}, \bar{c})$
23	223 263 345 375 433 467 507 547 645 675 705 735 765 795 831 875	$(\bar{u}, \bar{d}) + (\bar{u}, \bar{s}) + (\bar{u}, \bar{b}) + (\bar{c}, \bar{d}) + (\bar{c}, \bar{s}) + (\bar{c}, \bar{b})$
24	224 264 346 376 434 468 508 548 646 676 706 736 766 796 832 876	$(\bar{u}, \bar{u}) + (\bar{u}, \bar{c}) + (\bar{c}, \bar{u}) + (\bar{c}, \bar{c})$
25	225 265 339 369 427 469 509 549 639 669 699 729 759 789 825 837 838 869 881 882	$(\bar{d}, d) + (\bar{d}, s) + (\bar{d}, b) + (\bar{s}, d) + (\bar{s}, s) + (\bar{s}, b) + (\bar{b}, d) + (\bar{b}, s) + (\bar{b}, b)$
26	226 266 340 370 428 470 510 550 640 670 700 730 760 790 826 870	$(\bar{d}, u) + (\bar{d}, c) + (\bar{s}, u) + (\bar{s}, c) + (\bar{b}, u) + (\bar{b}, c)$
27	227 267 341 371 429 471 511 551 641 671 701 731 761 791 827 871	$(\bar{u}, d) + (\bar{u}, s) + (\bar{u}, b) + (\bar{c}, d) + (\bar{c}, s) + (\bar{c}, b)$
28	228 268 342 372 430 472 512 552 642 672 702 732 762 792 828 839 840 872 883 884	$(\bar{u}, u) + (\bar{u}, c) + (\bar{c}, u) + (\bar{c}, c)$
29	291 307 573 597 897 933	$(d, d) + (s, s) + (b, b)$
30	292 308 574 598 898 934	$(u, u) + (c, c)$
31	299 315 575 599 911 947	$(\bar{d}, \bar{d}) + (\bar{s}, \bar{s}) + (\bar{b}, \bar{b})$
32	300 316 576 600 912 948	$(\bar{u}, \bar{u}) + (\bar{c}, \bar{c})$

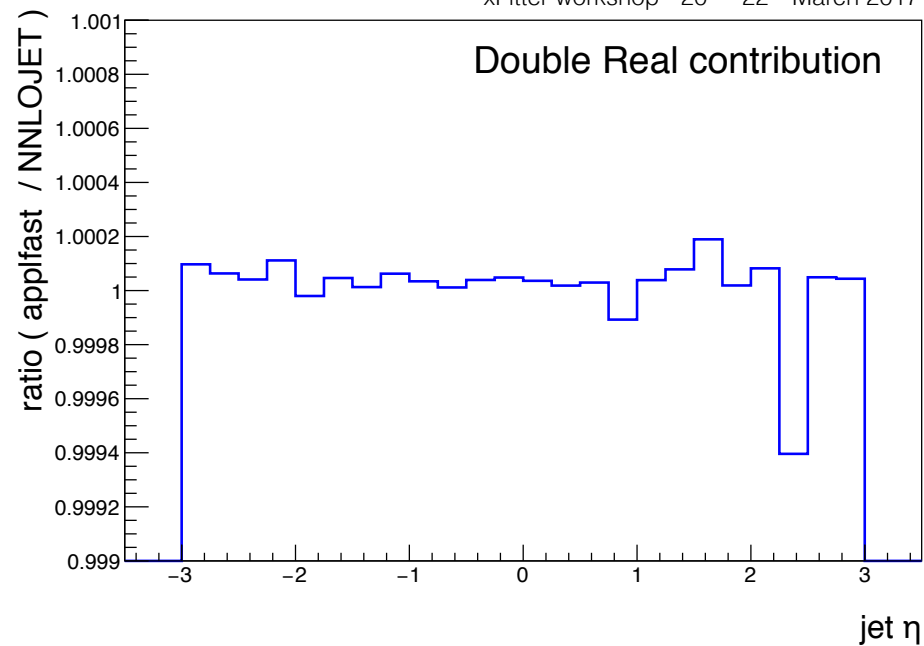
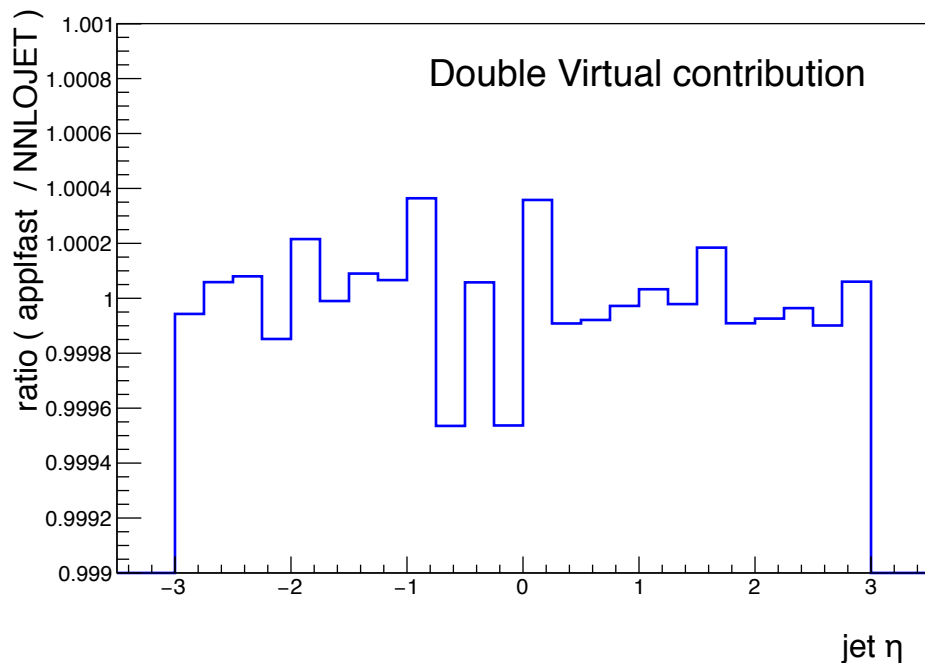
# Z+jets pre-processing



- Initial proof-of-concept validation presented at QCD@LHC in Zurich
- Sub-permille agreement reached at LO, NLO and NNLO in validation jobs
- High statistics grids can be generated ...
- Test setup:
  - Grids for  $P_{Tjet1}, \eta_{jet1}, y_Z$
  - $E_{cms} = 8 \text{ TeV}$
  - $P_{Tjet} > 30 \text{ GeV}$
  - $|y_{jet1}| < 3$
  - $|y_{l+l-}| < 5$
  - $80 < M_{ll} < 100 \text{ GeV}$
  - $\mu_r = \mu_f = M_Z$

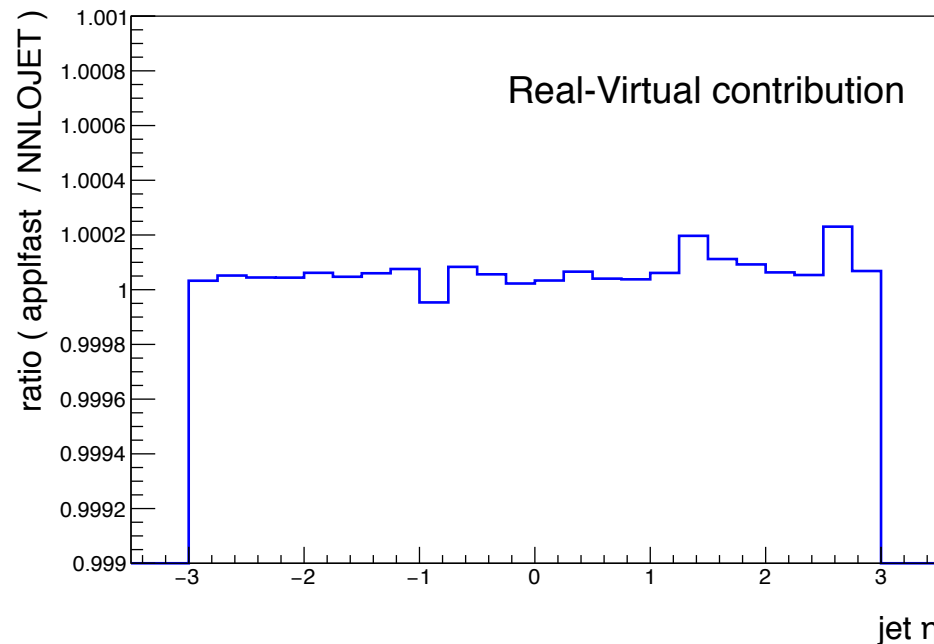
Klaus Rabbertz at PDF4LHC





## Z + jets at NNLO

- Early proof-of-concept convolutions ratios from QCD@LHC last August
  - Note large excursions for some bins due to limited statistics used for the determination of the optimised phase space



# Initial NNLOJET VEGAS warmup

- Pure NNLOJET warmup - no grid generation
  - One job per cross section contribution type - LO, R, V, RR(a,b), RV, VV etc
  - Internal NNLOJET multithreading possible

Job Type	# Jobs	Threads / Job	Events / Job	Runtime / Job	Total Runtime
LO	1	16	32 M	0.35 h	0.35 h
NLO-R	1	16	16 M	1.0 h	1.0 h
NLO-V	1	16	16 M	1.0 h	1.0 h
NNLO-RRa	1	32	5 M	17.5 h	17.5 h
NNLO-RRb	1	32	5 M	20.7 h	20.7 h
NNLO-RV	1	16	8 M	22.4 h	22.4 h
NNLO-VV	1	16	8 M	24.6 h	24.6 h
<b>Total</b>	<b>7</b>	<b>-</b>	<b>-</b>	<b>-</b>	<b>87.6 h</b>

- High statistics runs by Klaus using fastNLO grid generation on the KIT BwUniCluster, with support from the Baden-Württemberg High Performance Computing (HPC) group

# Grid phase space optimisation

<b>Job Type</b>	<b># Jobs</b>	<b>Events / Job</b>	<b>Runtime / Job</b>	<b># Events</b>	<b>Total Runtime</b>
<b>LO</b>	<b>5</b>	<b>500 M</b>	<b>12 h</b>	<b>2.5 G</b>	<b>60 h</b>
<b>NLO-R</b>	<b>5</b>	<b>300 M</b>	<b>18 h</b>	<b>1.5 G</b>	<b>90 h</b>
<b>NLO-V</b>	<b>5</b>	<b>500 M</b>	<b>13 h</b>	<b>2.5 G</b>	<b>65 h</b>
<b>NNLO-RRa</b>	<b>10</b>	<b>50 M</b>	<b>13 h</b>	<b>0.5 G</b>	<b>130 h</b>
<b>NNLO-RRb</b>	<b>10</b>	<b>50 M</b>	<b>15 h</b>	<b>0.5 G</b>	<b>150 h</b>
<b>NNLO-RV</b>	<b>5</b>	<b>300 M</b>	<b>19 h</b>	<b>1.5 G</b>	<b>90 h</b>
<b>NNLO-VV</b>	<b>5</b>	<b>500 M</b>	<b>12 h</b>	<b>2.5 G</b>	<b>60 h</b>
<b>Total</b>	<b>45</b>	<b>---</b>	<b>---</b>	<b>11.5 G</b>	<b>645 h</b>

- Run NNLOJET with only the phase space filling calculated - expensive weight calculation per sub-processed is not executed
- Jobs can be run in parallel - at least one job per contribution, LO, NLO-R, NLO-V etc
- Reasonably fast - non-trivial numbers of events are required, but nowhere near the numbers needed for the full production runs

# Processing times

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- Details of additional processing time with respect to running full NNLOJET along largely process dependent
  - Currently approximately only a factor of  $\sim 2$  slower depending on physics process and grid setup
  - Still a significant saving over re-running complete calculation ...
  
- NNLO Z+jets, no grids
  - LO:  $O(100)$  jobs  $\times$  5 min
  - NLO-V:  $O(100)$  jobs  $\times$  10 min
  - NLO-R:  $O(100)$  jobs  $\times$  15 min
  - NNLO-VV:  $O(100)$  jobs  $\times$  1 hour
  - NNLO-RV:  $O(1000)$  jobs  $\times$  10-20 hours
  - NNLO-RR:  $O(5000)$  jobs  $\times$   $\sim 20$  hours
  
- NNLO DIS jets + grids - each job typically 8 - 16 CPU hours
  - LO: 50 jobs ( 5G events)
  - NLO-V: 40 jobs ( 2G events)
  - NLO-R: 80 jobs (2G events)
  - NNLO-VV: 100 jobs (1.5G events)
  - NNLO-RV: 5000 jobs (5G events)
  - NNLO-RRa: 10000 jobs (5G events)
  - NNLO-RRb: 2000 jobs (5G events)

# Production running

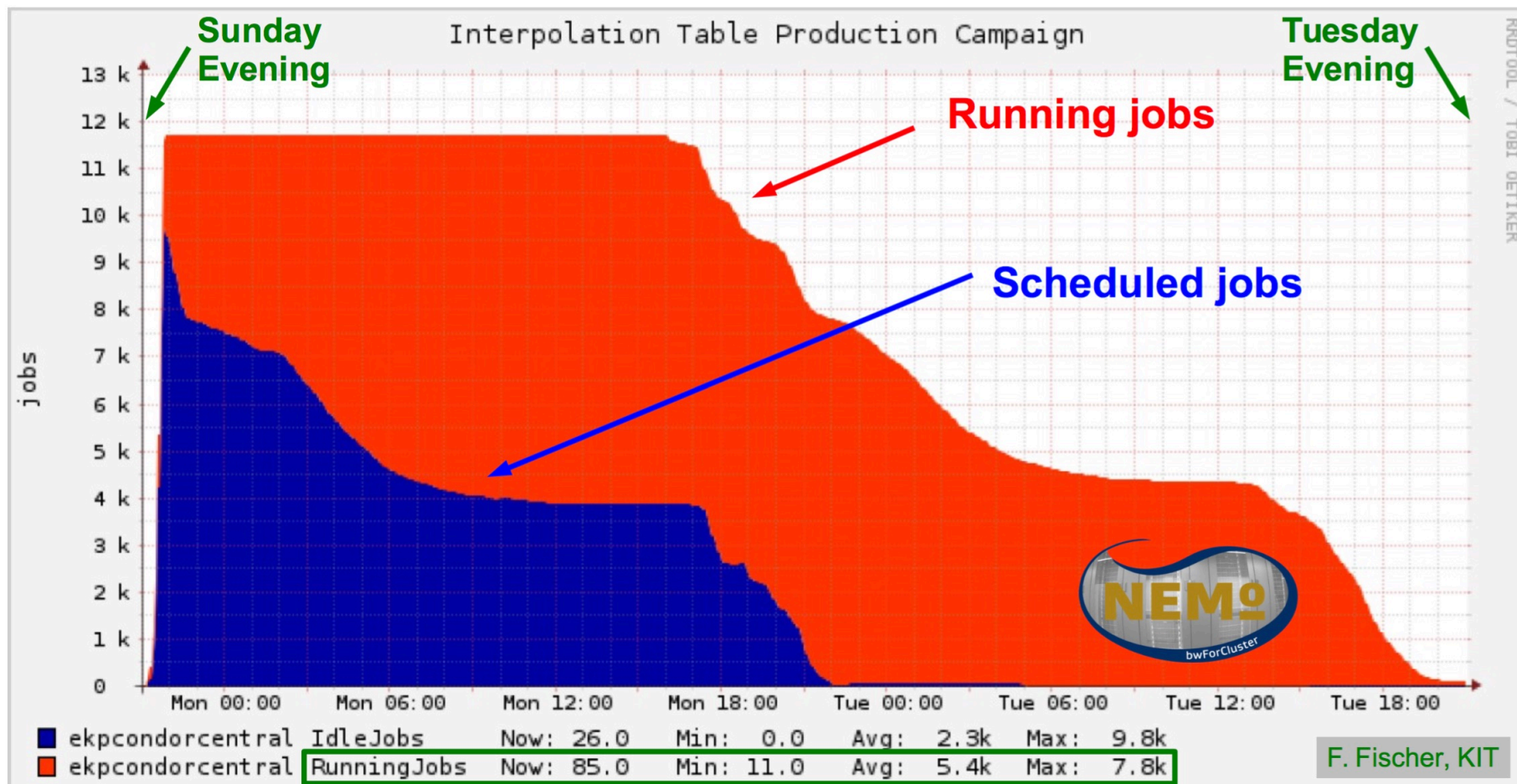
Job Type	# Jobs	Events / Job	Runtime / Job	# Events	Total Output	Total Runtime
LO	10	140 M	20.6 h	1.4 G	24 MB	206 h
NLO-R	200	6 M	19.0 h	1.2 G	1.3 GB	3800 h
NLO-V	200	5 M	21.2 h	1.0 G	1.2 GB	4240 h
NNLO-RRa	5000	60 k	22.5 h	0.3 G	26 GB	112500 h
NNLO-RRb	5000	40 k	20.3 h	0.2 G	27 GB	101500 h
NNLO-RV	1000	200 k	19.8 h	0.2 G	6.4 GB	19800 h
NNLO-VV	300	4 M	20.5 h	1.2 G	2.0 GB	6150 h
<b>Total</b>	<b>11710</b>	<b>---</b>	<b>---</b>	<b>5.5 G</b>	<b>64 GB</b>	<b>248196 h</b>

**3 times 11710 grids/tables + all NNLOJET output!  
Final 3 files for analysis are O(10 MB) each.**



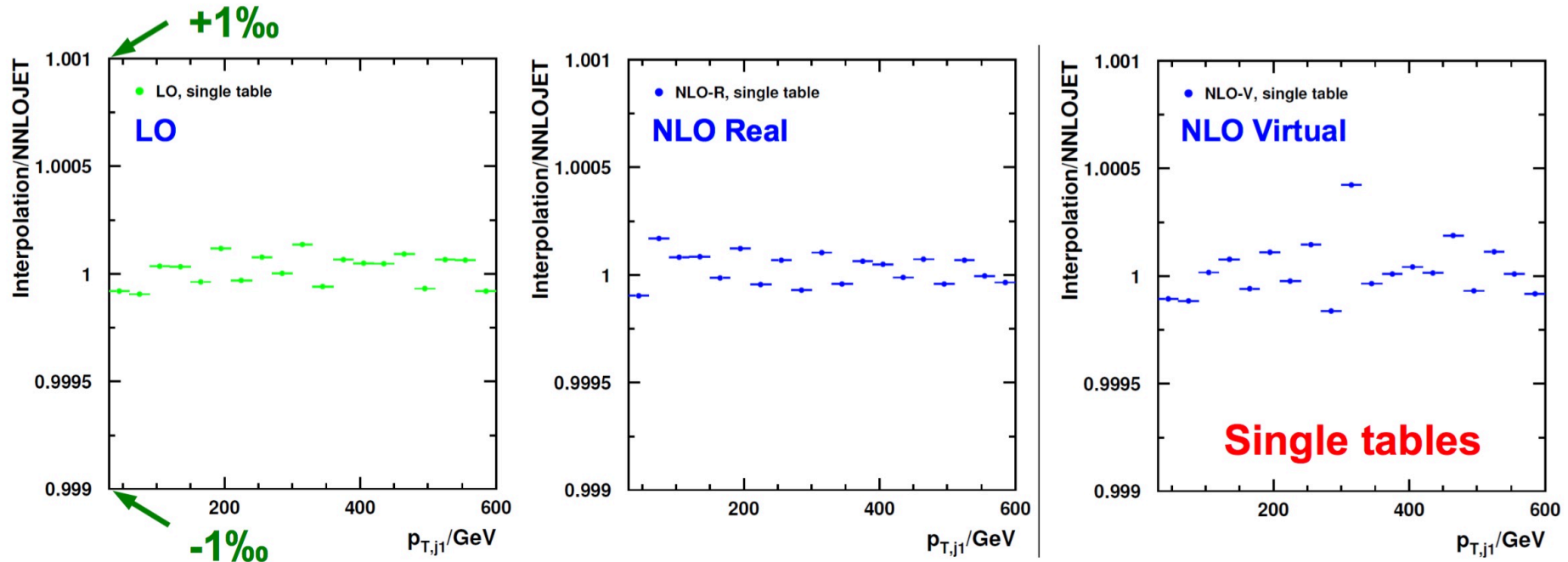
- Massive parallel processing on virtual machine farm with O(24) hours per job
- Running on BwForCluster NEMO in Friburg - support from Baden-Württemberg High Performance Computing (HPC) group

# Production farm occupancy



- Completed in approximately 2 days with maximum of around 7800 parallel jobs running at any given time

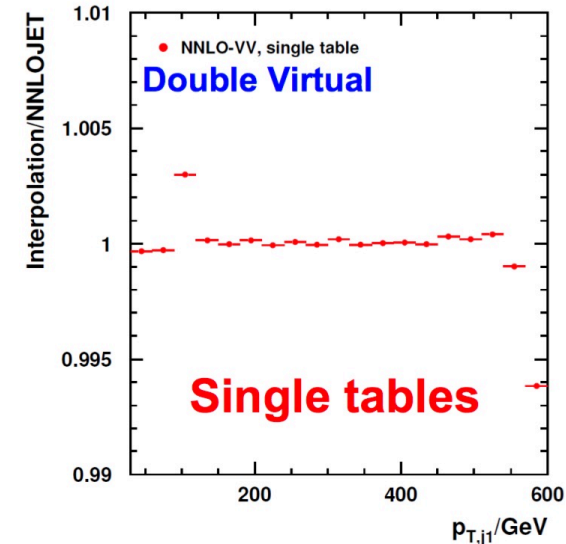
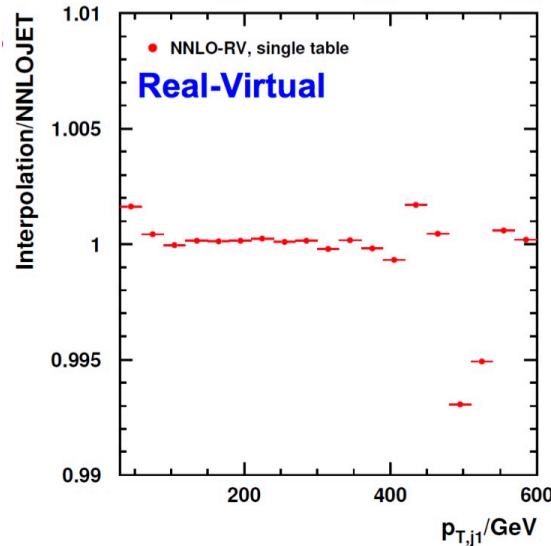
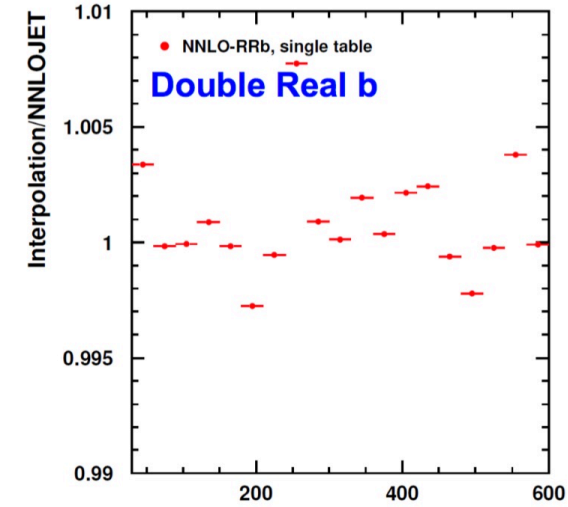
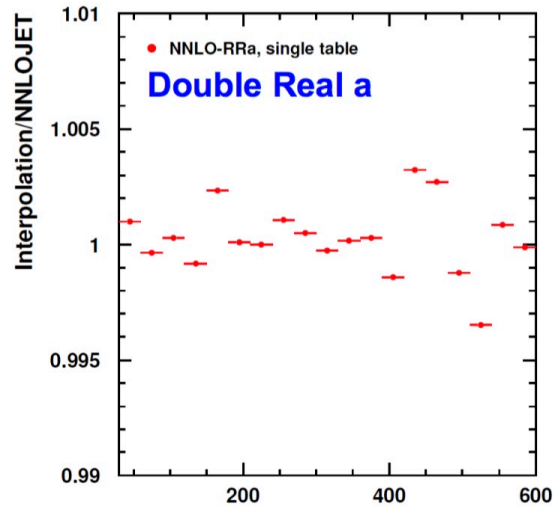
# Closure tests at Leading and Next-to-Leading order



- Consider the closure of the individual tables from each job separately
  - NB: note the 0.1% y-axis range
  - Generally agree to much better than the per mille level
- At this level, the interpolation of LHAPDF might be a limiting factor
  - Have seen previously that the pdf interpolation in LHAPDF 6 is  $O(10)$  times better than that in LHAPDF 5

# Closure tests at NNLO

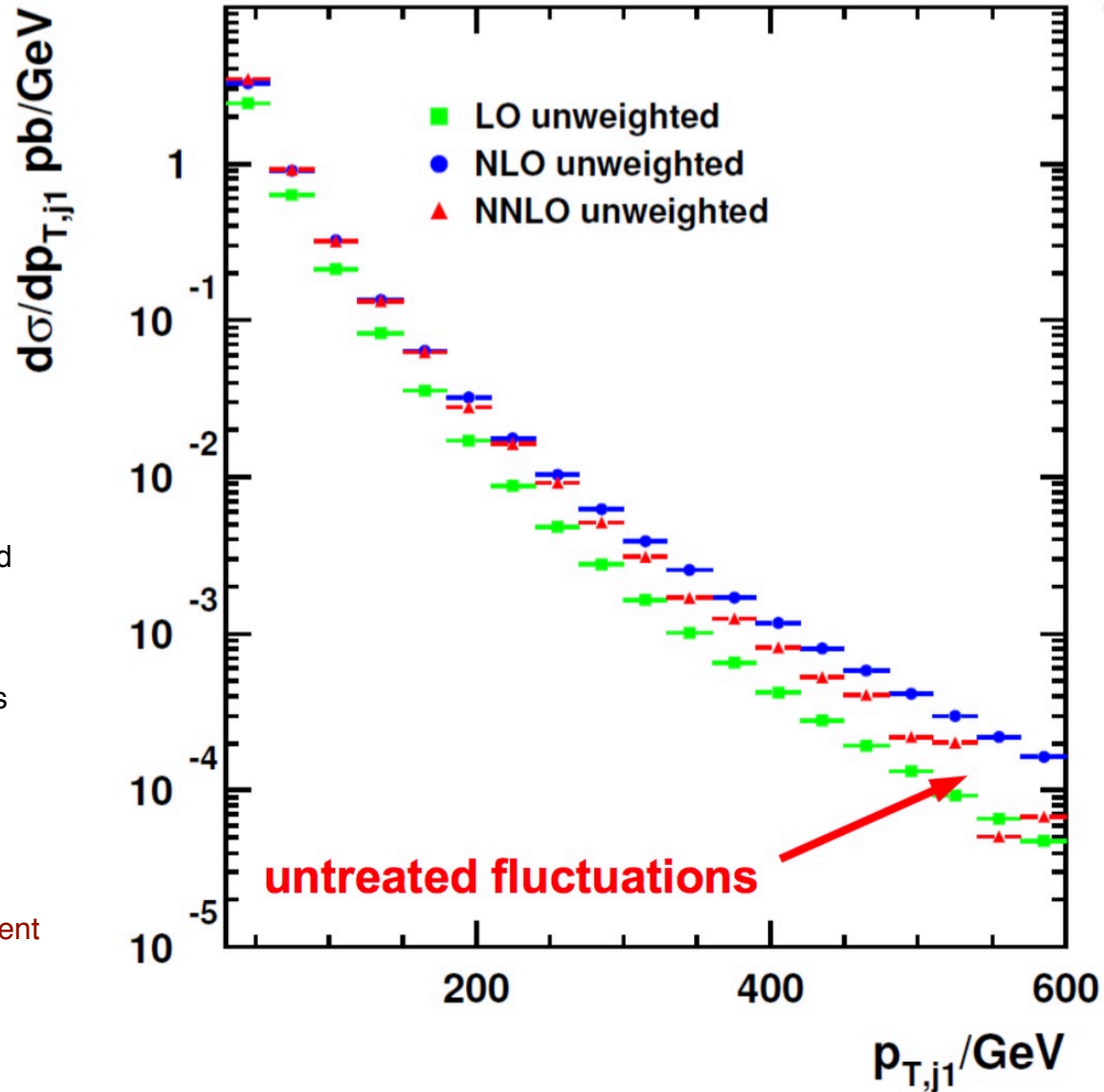
- Note increased  $\pm 1\%$  y-axis range
- Generally agreement to around the per mille level
- Double Real contribution is computed in two parts here
- Some inevitable large fluctuations
  - Incomplete cancelation between bins
  - Taken care of during the global procedure to combine the grids for the final cross section



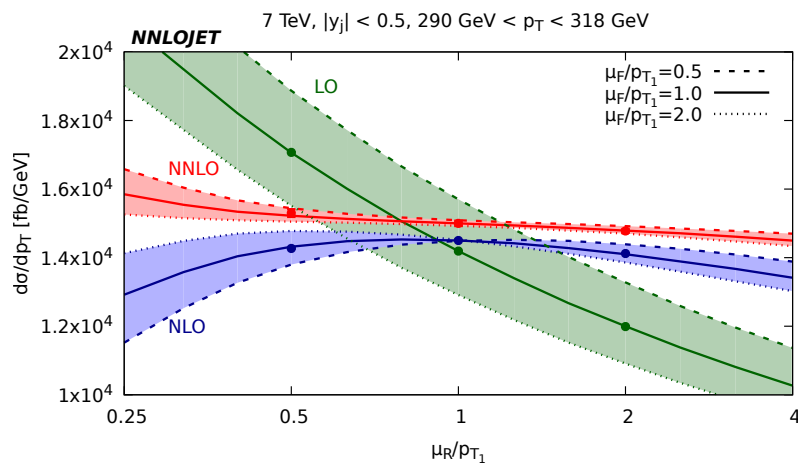


# Unweighted combination

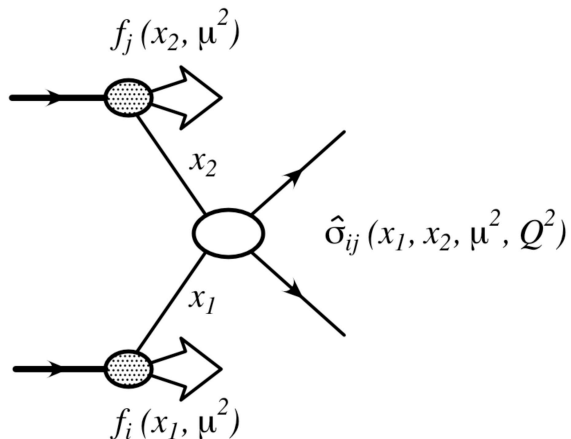
- Due to incomplete cancelation in bins in differential cross sections, a detailed weighted combination method is used
- The code for this exists and the procedure is being validated for the applfast tables
- Naïve, unweighted combination produces approximately correct cross section
  - Present of untreated fluctuations is apparent



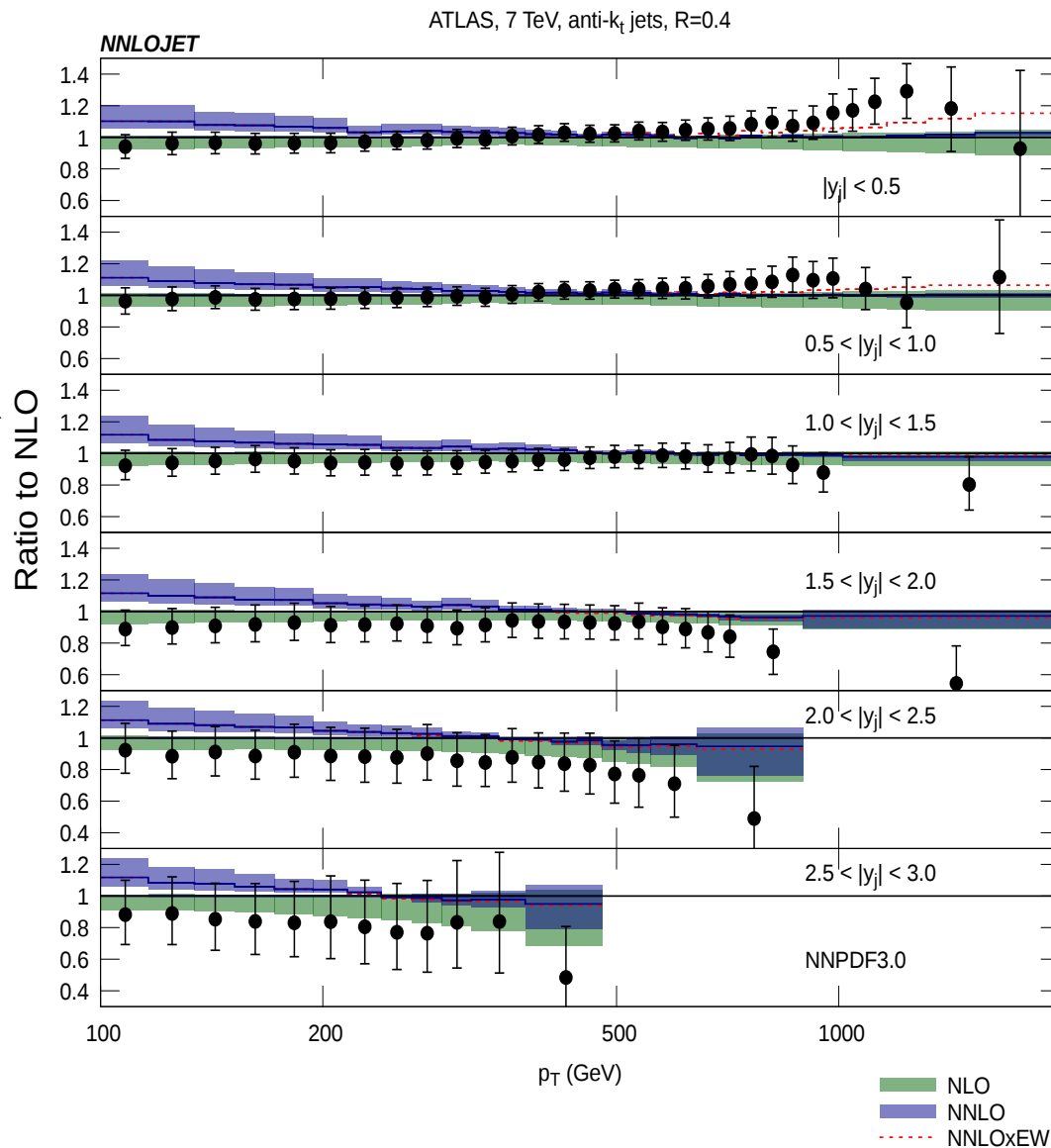
# Inclusive jet production



- NNLO QCD predictions for single jet inclusive production at the LHC
  - J. Currie, E.W.N. Glover, J. Pires [arXiv: 1611.01460](https://arxiv.org/abs/1611.01460)



- For initial limited statistics grid validation studies, use scale choice,  $\mu_R = \mu_F = p_{T \text{ lead}}$



# Jet production technicalities

- Leading order - 25 internal processes

0	1 6 13	$(g, g)$
1	2 9	$(d, g) + (u, g) + (s, g) + (c, g) + (b, g)$
2	3 10 18 22	$(d, \bar{d}) + (u, \bar{u}) + (s, \bar{s}) + (c, \bar{c}) + (b, \bar{b})$
3	4 11	$(g, d) + (g, u) + (g, s) + (g, c) + (g, b)$
4	5 12 19 24	$(\bar{d}, d) + (\bar{u}, u) + (\bar{s}, s) + (\bar{c}, c) + (\bar{b}, b)$
5	7 14	$(g, \bar{d}) + (g, \bar{u}) + (g, \bar{s}) + (g, \bar{c}) + (g, \bar{b})$
6	8 15	$(\bar{d}, g) + (\bar{u}, g) + (\bar{s}, g) + (\bar{c}, g) + (\bar{b}, g)$
7	16	$(d, \bar{d}) + (d, \bar{u}) + (d, \bar{s}) + (d, \bar{c}) + (d, \bar{b}) + (u, \bar{d}) + (u, \bar{u}) + (u, \bar{s}) + (u, \bar{c}) + (u, \bar{b}) + (s, \bar{d}) + (s, \bar{u}) + (s, \bar{s}) + (s, \bar{c}) + (s, \bar{b}) + (c, \bar{d}) + (c, \bar{u}) + (c, \bar{s}) + (c, \bar{c}) + (c, \bar{b}) + (b, \bar{d}) + (b, \bar{u}) + (b, \bar{s}) + (b, \bar{c}) + (b, \bar{b})$
8	17	$(d, d) + (d, u) + (d, s) + (d, c) + (d, b) + (u, d) + (u, u) + (u, s) + (u, c) + (u, b) + (s, d) + (s, u) + (s, s) + (s, c) + (s, b) + (c, d) + (c, u) + (c, s) + (c, c) + (c, b) + (b, d) + (b, u) + (b, s) + (b, c) + (b, b)$
9	20	$(\bar{d}, \bar{d}) + (\bar{d}, \bar{u}) + (\bar{d}, \bar{s}) + (\bar{d}, \bar{c}) + (\bar{d}, \bar{b}) + (\bar{u}, \bar{d}) + (\bar{u}, \bar{u}) + (\bar{u}, \bar{s}) + (\bar{u}, \bar{c}) + (\bar{u}, \bar{b}) + (\bar{s}, \bar{d}) + (\bar{s}, \bar{u}) + (\bar{s}, \bar{s}) + (\bar{s}, \bar{c}) + (\bar{s}, \bar{b}) + (\bar{c}, \bar{d}) + (\bar{c}, \bar{u}) + (\bar{c}, \bar{s}) + (\bar{c}, \bar{c}) + (\bar{c}, \bar{b}) + (\bar{b}, \bar{d}) + (\bar{b}, \bar{u}) + (\bar{b}, \bar{s}) + (\bar{b}, \bar{c}) + (\bar{b}, \bar{b})$
10	21	$(\bar{d}, d) + (\bar{d}, u) + (\bar{d}, s) + (\bar{d}, c) + (\bar{d}, b) + (\bar{u}, d) + (\bar{u}, u) + (\bar{u}, s) + (\bar{u}, c) + (\bar{u}, b) + (\bar{s}, d) + (\bar{s}, u) + (\bar{s}, s) + (\bar{s}, c) + (\bar{s}, b) + (\bar{c}, d) + (\bar{c}, u) + (\bar{c}, s) + (\bar{c}, c) + (\bar{c}, b) + (\bar{b}, d) + (\bar{b}, u) + (\bar{b}, s) + (\bar{b}, c) + (\bar{b}, b)$
11	23	$(d, d) + (u, u) + (s, s) + (c, c) + (b, b)$
12	25	$(\bar{d}, \bar{d}) + (\bar{u}, \bar{u}) + (\bar{s}, \bar{s}) + (\bar{c}, \bar{c}) + (\bar{b}, \bar{b})$

- And at NLO - 150 internal processes

0	26 31 38 45 84 89 98 109 120 127 134	$(g, g)$
1	27 34 41 51 61 70 78 87 92 94 105 116 123 130 145 155 165 173 181 189	$(d, g) + (u, g) + (s, g) + (c, g) + (b, g)$
2	28 35 42 48 58 68 76 95 106 117 124 131 139 149 159 167 175 183	$(d, \bar{d}) + (u, \bar{u}) + (s, \bar{s}) + (c, \bar{c}) + (b, \bar{b})$
3	29 36 43 53 63 72 80 85 90 96 107 118 125 132 143 153 163 171 179 187	$(g, d) + (g, u) + (g, s) + (g, c) + (g, b)$
4	30 37 44 52 62 71 79 97 108 119 126 133 140 150 160 169 177 185	$(\bar{d}, d) + (\bar{u}, u) + (\bar{s}, s) + (\bar{c}, c) + (\bar{b}, b)$
5	32 39 46 57 67 75 83 86 91 99 110 121 128 135 144 154 164 172 180 188	$(g, \bar{d}) + (g, \bar{u}) + (g, \bar{s}) + (g, \bar{c}) + (g, \bar{b})$
6	33 40 47 56 66 74 82 88 93 100 111 122 129 136 146 156 166 174 182 190	$(\bar{d}, g) + (\bar{u}, g) + (\bar{s}, g) + (\bar{c}, g) + (\bar{b}, g)$
7	49 59 101 112 138 148 158	$(d, d) + (d, u) + (d, s) + (d, c) + (d, b) + (u, d) + (u, u) + (u, s) + (u, c) + (u, b) + (s, d) + (s, u) + (s, s) + (s, c) + (s, b) + (c, d) + (c, u) + (c, s) + (c, c) + (c, b) + (b, d) + (b, u) + (b, s) + (b, c) + (b, b)$
8	50 60 102 113 137 147 157	$(d, \bar{d}) + (d, \bar{u}) + (d, \bar{s}) + (d, \bar{c}) + (d, \bar{b}) + (u, \bar{d}) + (u, \bar{u}) + (u, \bar{s}) + (u, \bar{c}) + (u, \bar{b}) + (s, \bar{d}) + (s, \bar{u}) + (s, \bar{s}) + (s, \bar{c}) + (s, \bar{b}) + (c, \bar{d}) + (c, \bar{u}) + (c, \bar{s}) + (c, \bar{c}) + (c, \bar{b}) + (b, \bar{d}) + (b, \bar{u}) + (b, \bar{s}) + (b, \bar{c}) + (b, \bar{b})$
9	54 64 103 114 142 152 162	$(\bar{d}, d) + (\bar{d}, u) + (\bar{d}, s) + (\bar{d}, c) + (\bar{d}, b) + (\bar{u}, d) + (\bar{u}, u) + (\bar{u}, s) + (\bar{u}, c) + (\bar{u}, b) + (\bar{s}, d) + (\bar{s}, u) + (\bar{s}, s) + (\bar{s}, c) + (\bar{s}, b) + (\bar{c}, d) + (\bar{c}, u) + (\bar{c}, s) + (\bar{c}, c) + (\bar{c}, b) + (\bar{b}, d) + (\bar{b}, u) + (\bar{b}, s) + (\bar{b}, c) + (\bar{b}, b)$
10	55 65 104 115 141 151 161	$(\bar{d}, \bar{d}) + (\bar{d}, \bar{u}) + (\bar{d}, \bar{s}) + (\bar{d}, \bar{c}) + (\bar{d}, \bar{b}) + (\bar{u}, \bar{d}) + (\bar{u}, \bar{u}) + (\bar{u}, \bar{s}) + (\bar{u}, \bar{c}) + (\bar{u}, \bar{b}) + (\bar{s}, \bar{d}) + (\bar{s}, \bar{u}) + (\bar{s}, \bar{s}) + (\bar{s}, \bar{c}) + (\bar{s}, \bar{b}) + (\bar{c}, \bar{d}) + (\bar{c}, \bar{u}) + (\bar{c}, \bar{s}) + (\bar{c}, \bar{c}) + (\bar{c}, \bar{b}) + (\bar{b}, \bar{d}) + (\bar{b}, \bar{u}) + (\bar{b}, \bar{s}) + (\bar{b}, \bar{c}) + (\bar{b}, \bar{b})$
11	69 77 168 176 184	$(d, d) + (u, u) + (s, s) + (c, c) + (b, b)$
12	73 81 170 178 186	$(\bar{d}, \bar{d}) + (\bar{u}, \bar{u}) + (\bar{s}, \bar{s}) + (\bar{c}, \bar{c}) + (\bar{b}, \bar{b})$

# Jet production

- Double virtual - 93 internal processes

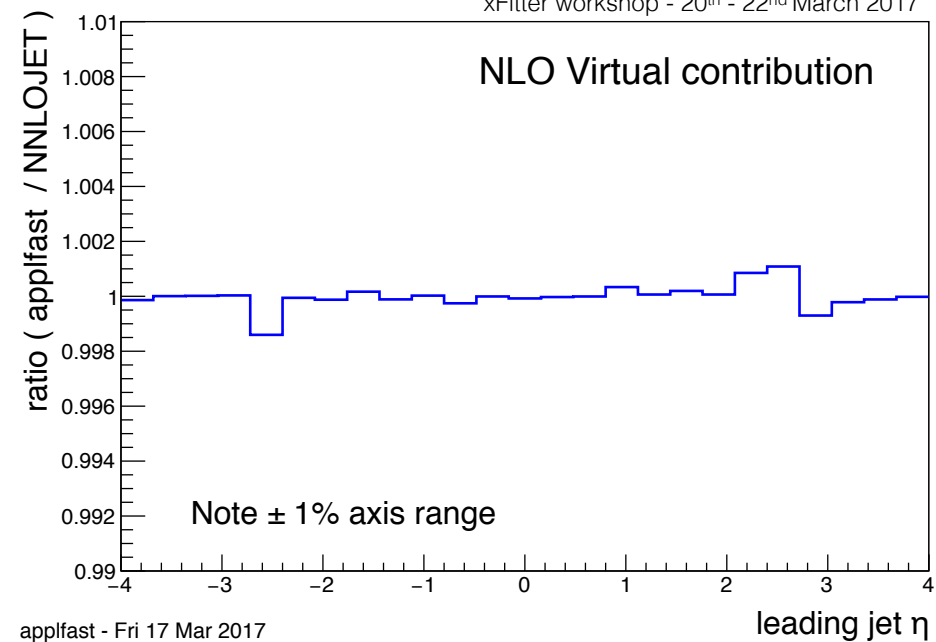
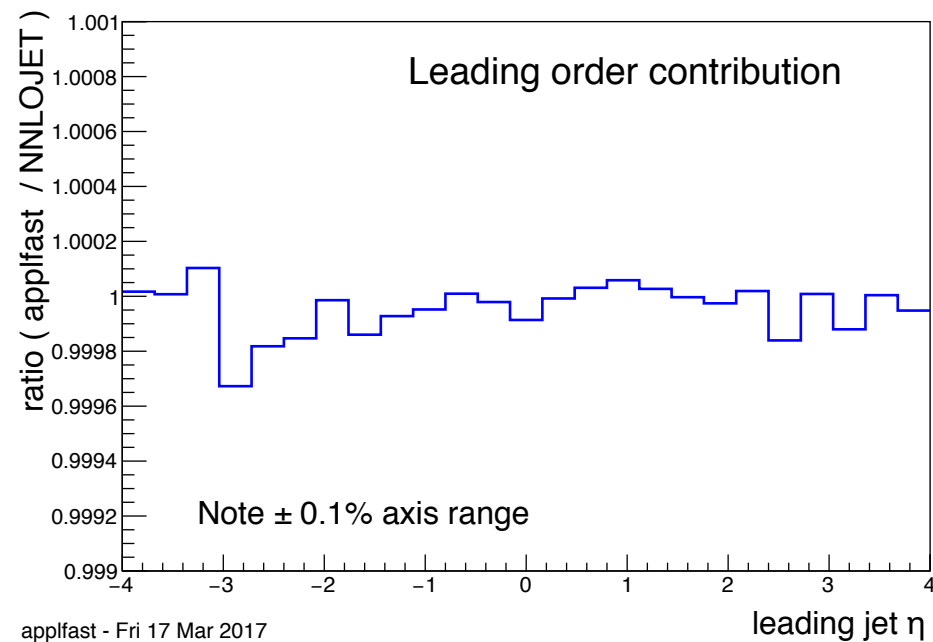
0	270 279 288 301 312 323 336 347 358	$(g, g)$
1	271 280 289 299 310 321 337 348 359	$(g, d) + (g, u) + (g, s) + (g, c) + (g, b)$
2	272 281 290 302 313 324 338 349 360	$(g, \bar{d}) + (g, \bar{u}) + (g, \bar{s}) + (g, \bar{c}) + (g, \bar{b})$
3	273 282 291 304 315 326 331 342 353	$(d, d) + (d, u) + (d, s) + (d, c) + (d, b) + (u, d) + (u, u) + (u, s) + (u, c) + (u, b) + (s, d) + (s, u) + (s, s) + (s, c) + (s, b) + (c, d) + (c, u) + (c, s) + (c, c) + (c, b) + (b, d) + (b, u) + (b, s) + (b, c) + (b, b)$
4	274 283 292 305 316 327 330 341 352	$(d, \bar{d}) + (d, \bar{u}) + (d, \bar{s}) + (d, \bar{c}) + (d, \bar{b}) + (u, \bar{d}) + (u, \bar{u}) + (u, \bar{s}) + (u, \bar{c}) + (u, \bar{b}) + (s, \bar{d}) + (s, \bar{u}) + (s, \bar{s}) + (s, \bar{c}) + (s, \bar{b}) + (c, \bar{d}) + (c, \bar{u}) + (c, \bar{s}) + (c, \bar{c}) + (c, \bar{b}) + (b, \bar{d}) + (b, \bar{u}) + (b, \bar{s}) + (b, \bar{c}) + (b, \bar{b})$
5	275 284 293 297 308 319 339 350 361	$(d, g) + (u, g) + (s, g) + (c, g) + (b, g)$
6	276 285 294 306 317 328 335 346 357	$(\bar{d}, d) + (\bar{d}, u) + (\bar{d}, s) + (\bar{d}, c) + (\bar{d}, b) + (\bar{u}, d) + (\bar{u}, u) + (\bar{u}, s) + (\bar{u}, c) + (\bar{u}, b) + (\bar{s}, d) + (\bar{s}, u) + (\bar{s}, s) + (\bar{s}, c) + (\bar{s}, b) + (\bar{c}, d) + (\bar{c}, u) + (\bar{c}, s) + (\bar{c}, c) + (\bar{c}, b) + (\bar{b}, d) + (\bar{b}, u) + (\bar{b}, s) + (\bar{b}, c) + (\bar{b}, b)$
7	277 286 295 307 318 329 334 345 356	$(\bar{d}, \bar{d}) + (\bar{d}, \bar{u}) + (\bar{d}, \bar{s}) + (\bar{d}, \bar{c}) + (\bar{d}, \bar{b}) + (\bar{u}, \bar{d}) + (\bar{u}, \bar{u}) + (\bar{u}, \bar{s}) + (\bar{u}, \bar{c}) + (\bar{u}, \bar{b}) + (\bar{s}, \bar{d}) + (\bar{s}, \bar{u}) + (\bar{s}, \bar{s}) + (\bar{s}, \bar{c}) + (\bar{s}, \bar{b}) + (\bar{c}, \bar{d}) + (\bar{c}, \bar{u}) + (\bar{c}, \bar{s}) + (\bar{c}, \bar{c}) + (\bar{c}, \bar{b}) + (\bar{b}, \bar{d}) + (\bar{b}, \bar{u}) + (\bar{b}, \bar{s}) + (\bar{b}, \bar{c}) + (\bar{b}, \bar{b})$
8	278 287 296 303 314 325 340 351 362	$(\bar{d}, g) + (\bar{u}, g) + (\bar{s}, g) + (\bar{c}, g) + (\bar{b}, g)$
9	298 309 320 332 343 354	$(d, \bar{d}) + (u, \bar{u}) + (s, \bar{s}) + (c, \bar{c}) + (b, \bar{b})$
10	300 311 322 333 344 355	$(\bar{d}, d) + (\bar{u}, u) + (\bar{s}, s) + (\bar{c}, c) + (\bar{b}, b)$

- Real-virtual - 54 internal processes

0	216 221 230 241 258 269	$(g, g)$
1	217 222 228 239 253 264	$(g, d) + (g, u) + (g, s) + (g, c) + (g, b)$
2	218 223 231 242 257 268	$(g, \bar{d}) + (g, \bar{u}) + (g, \bar{s}) + (g, \bar{c}) + (g, \bar{b})$
3	219 224 226 237 251 262	$(d, g) + (u, g) + (s, g) + (c, g) + (b, g)$
4	220 225 232 243 256 267	$(\bar{d}, g) + (\bar{u}, g) + (\bar{s}, g) + (\bar{c}, g) + (\bar{b}, g)$
5	227 238 248 259	$(d, \bar{d}) + (u, \bar{u}) + (s, \bar{s}) + (c, \bar{c}) + (b, \bar{b})$
6	229 240 252 263	$(\bar{d}, d) + (\bar{u}, u) + (\bar{s}, s) + (\bar{c}, c) + (\bar{b}, b)$
7	233 244 249 260	$(d, d) + (d, u) + (d, s) + (d, c) + (d, b) + (u, d) + (u, u) + (u, s) + (u, c) + (u, b) + (s, d) + (s, u) + (s, s) + (s, c) + (s, b) + (c, d) + (c, u) + (c, s) + (c, c) + (c, b) + (b, d) + (b, u) + (b, s) + (b, c) + (b, b)$
8	234 245 250 261	$(d, \bar{d}) + (d, \bar{u}) + (d, \bar{s}) + (d, \bar{c}) + (d, \bar{b}) + (u, \bar{d}) + (u, \bar{u}) + (u, \bar{s}) + (u, \bar{c}) + (u, \bar{b}) + (s, \bar{d}) + (s, \bar{u}) + (s, \bar{s}) + (s, \bar{c}) + (s, \bar{b}) + (c, \bar{d}) + (c, \bar{u}) + (c, \bar{s}) + (c, \bar{c}) + (c, \bar{b}) + (b, \bar{d}) + (b, \bar{u}) + (b, \bar{s}) + (b, \bar{c}) + (b, \bar{b})$
9	235 246 254 265	$(\bar{d}, d) + (\bar{d}, u) + (\bar{d}, s) + (\bar{d}, c) + (\bar{d}, b) + (\bar{u}, d) + (\bar{u}, u) + (\bar{u}, s) + (\bar{u}, c) + (\bar{u}, b) + (\bar{s}, d) + (\bar{s}, u) + (\bar{s}, s) + (\bar{s}, c) + (\bar{s}, b) + (\bar{c}, d) + (\bar{c}, u) + (\bar{c}, s) + (\bar{c}, c) + (\bar{c}, b) + (\bar{b}, d) + (\bar{b}, u) + (\bar{b}, s) + (\bar{b}, c) + (\bar{b}, b)$
10	236 247 255 266	$(\bar{d}, \bar{d}) + (\bar{d}, \bar{u}) + (\bar{d}, \bar{s}) + (\bar{d}, \bar{c}) + (\bar{d}, \bar{b}) + (\bar{u}, \bar{d}) + (\bar{u}, \bar{u}) + (\bar{u}, \bar{s}) + (\bar{u}, \bar{c}) + (\bar{u}, \bar{b}) + (\bar{s}, \bar{d}) + (\bar{s}, \bar{u}) + (\bar{s}, \bar{s}) + (\bar{s}, \bar{c}) + (\bar{s}, \bar{b}) + (\bar{c}, \bar{d}) + (\bar{c}, \bar{u}) + (\bar{c}, \bar{s}) + (\bar{c}, \bar{c}) + (\bar{c}, \bar{b}) + (\bar{b}, \bar{d}) + (\bar{b}, \bar{u}) + (\bar{b}, \bar{s}) + (\bar{b}, \bar{c}) + (\bar{b}, \bar{b})$

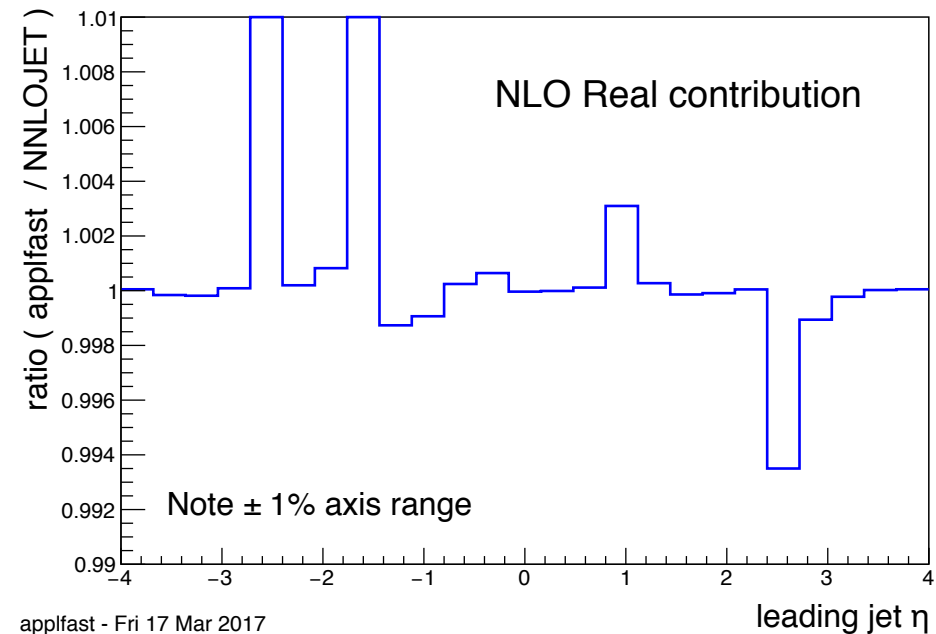
- Double real - 25 internal processes

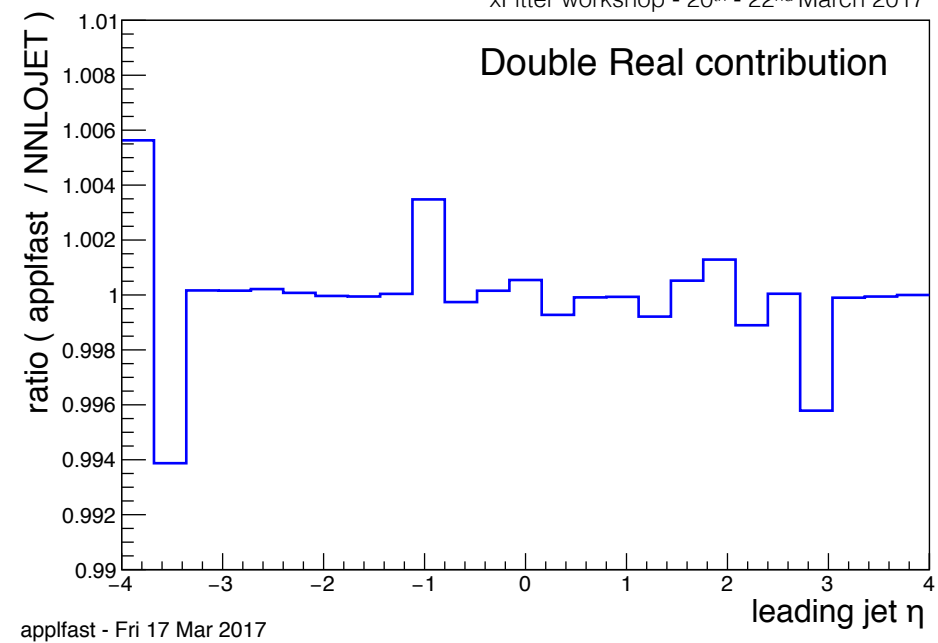
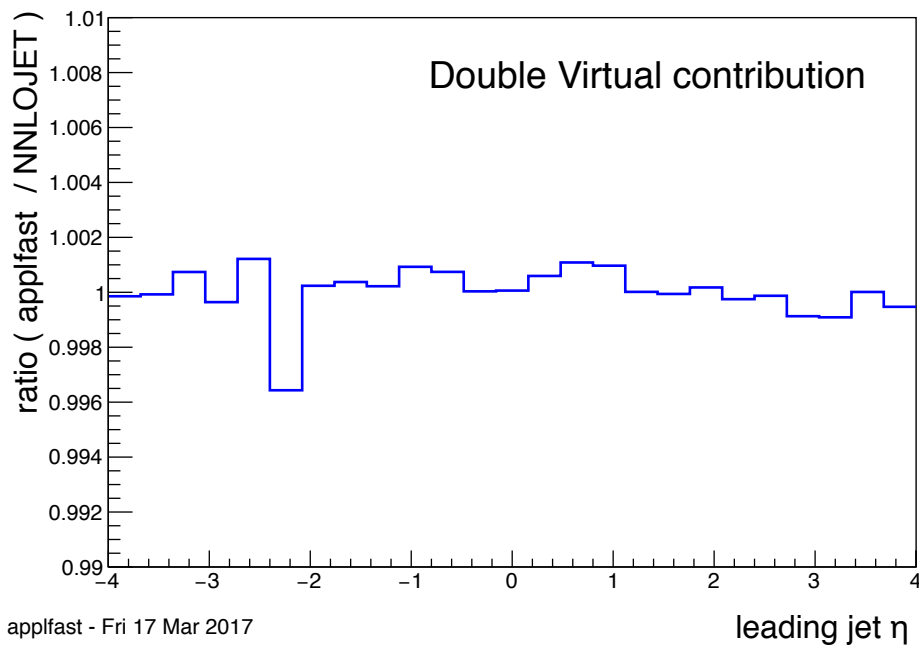
0	191 196 209	$(g, g)$
1	192 202	$(d, g) + (u, g) + (s, g) + (c, g) + (b, g)$
2	193 199 210	$(d, \bar{d}) + (u, \bar{u}) + (s, \bar{s}) + (c, \bar{c}) + (b, \bar{b})$
3	194 204	$(g, d) + (g, u) + (g, s) + (g, c) + (g, b)$
4	195 203 213	$(d, d) + (\bar{u}, u) + (\bar{s}, s) + (\bar{c}, c) + (\bar{b}, b)$
5	197 208	$(g, \bar{d}) + (g, \bar{u}) + (g, \bar{s}) + (g, \bar{c}) + (g, \bar{b})$
6	198 207	$(d, g) + (\bar{u}, g) + (\bar{s}, g) + (\bar{c}, g) + (\bar{b}, g)$
7	200 211	$(d, d) + (d, u) + (d, s) + (d, c) + (d, b) + (u, d) + (u, u) + (u, s) + (u, c) + (u, b) + (s, d) + (s, u) + (s, s) + (s, c) + (s, b) + (c, d) + (c, u) + (c, s) + (c, c) + (c, b) + (b, d) + (b, u) + (b, s) + (b, c) + (b, b)$
8	201 212	$(d, \bar{d}) + (d, \bar{u}) + (d, \bar{s}) + (d, \bar{c}) + (d, \bar{b}) + (u, \bar{d}) + (u, \bar{u}) + (u, \bar{s}) + (u, \bar{c}) + (u, \bar{b}) + (s, \bar{d}) + (s, \bar{u}) + (s, \bar{s}) + (s, \bar{c}) + (s, \bar{b}) + (c, \bar{d}) + (c, \bar{u}) + (c, \bar{s}) + (c, \bar{c}) + (c, \bar{b}) + (b, \bar{d}) + (b, \bar{u}) + (b, \bar{s}) + (b, \bar{c}) + (b, \bar{b})$
9	205 214	$(\bar{d}, d) + (\bar{d}, u) + (\bar{d}, s) + (\bar{d}, c) + (\bar{d}, b) + (\bar{u}, d) + (\bar{u}, u) + (\bar{u}, s) + (\bar{u}, c) + (\bar{u}, b) + (\bar{s}, d) + (\bar{s}, u) + (\bar{s}, s) + (\bar{s}, c) + (\bar{s}, b) + (\bar{c}, d) + (\bar{c}, u) + (\bar{c}, s) + (\bar{c}, c) + (\bar{c}, b) + (\bar{b}, d) + (\bar{b}, u) + (\bar{b}, s) + (\bar{b}, c) + (\bar{b}, b)$
10	206 215	$(\bar{d}, \bar{d}) + (\bar{d}, \bar{u}) + (\bar{d}, \bar{s}) + (\bar{d}, \bar{c}) + (\bar{d}, \bar{b}) + (\bar{u}, \bar{d}) + (\bar{u}, \bar{u}) + (\bar{u}, \bar{s}) + (\bar{u}, \bar{c}) + (\bar{u}, \bar{b}) + (\bar{s}, \bar{d}) + (\bar{s}, \bar{u}) + (\bar{s}, \bar{s}) + (\bar{s}, \bar{c}) + (\bar{s}, \bar{b}) + (\bar{c}, \bar{d}) + (\bar{c}, \bar{u}) + (\bar{c}, \bar{s}) + (\bar{c}, \bar{c}) + (\bar{c}, \bar{b}) + (\bar{b}, \bar{d}) + (\bar{b}, \bar{u}) + (\bar{b}, \bar{s}) + (\bar{b}, \bar{c}) + (\bar{b}, \bar{b})$



## Inclusive jets at NLO

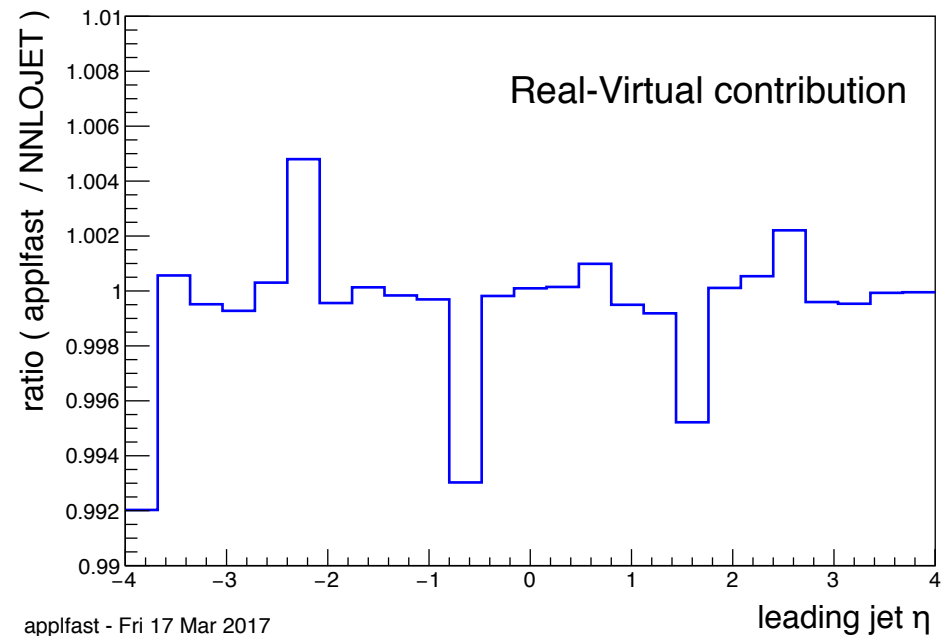
- Tests of grid closure of the NLO components
  - Only low statistics runs so far - completely consistent with NLO jet production from numerous sources
  - Even with short warmup, agreement generally better than 0.1%





## Inclusive jets at NNLO

- Similarly NNLO contributions also generally better than 0.1 %
  - Should improve with longer warmup
- Intend to start larger scale production in the near future



# Outlook

- The NNLOJET project is reaching fruition ...
  - Provides a common interface for multifarious physics processes - Inclusive Z production Z+jet, W inclusive, Inclusive Higgs production, DIS jets, multijets in  $e^+e^-$  ...
- The APPLfast-NNLO proof of concept development is starting to mature
  - Proof of concept validated with common interface for both APPLgrid and fastNLO at LO, NLO and NNLO order
- Large scale production launched for Z+jets in pp collisions, DIS jets
  - More processes currently under development: inclusive Z production, inclusive jets ...
  - Working on completion of correct combination of large scale production results
- Looking forward to completion of the validation for many new NNLO grids to provide a veritable smörgåsbord of physics processes of the highest order.
- We gratefully acknowledge support from the IPPP Associateship program and from Baden-Württemberg HPC through the BwUniCluster and BwForCluster.



*non frustra vivimus !*