TPC Analysis Jamboree – Orsay (May 2009)

Space charge: PART I

Analytical solution of the Laplace equ. for a coaxial cavity

Space charge: PART II

Space charge scenarios for the ALICE TPC

Space charge: PART III

Space point distortions due to space charges and more ...

Stefan Rossegger

Space charge: PART I

Analytical solution of the Laplace equ. for a coaxial cavity

Stefan Rossegger

What is needed ...

... to calculate the field distortions due to space charges?

➡ A solution of the **3D-Laplace equation** for a given charge accumulation within the TPC geometry

- **Numerical solution? ...**

  ... high accuracy takes time

- **Analytical approach** by means of Green's functions

  ... that's a nice idea ... ;-)

Theoretical base ...

- If a fast converging Green's function is found, one just multiplies it with the charge distribution and integrates over its volume ...

$$\Phi(r,\phi,z) = \frac{1}{\epsilon_0} \int r'dr' \int d\phi' \int dz' \;\; \rho_c(r',\phi',z') \cdot G(r,\phi,z,r',\phi',z')$$

$$E(r,\phi,z) = -\nabla\Phi(r,\phi,z)$$

... which results in the potential and the electric field components

Which Green's function?

A solution is given in [1]

[1] W. R. **Smythe**, **Static and dynamic electricity**, New York-Toronto-London: McGraw-Hill 1950.

$$G(r,\phi,z;r',\phi',z') =$$
$$= \frac{1}{\pi L}\sum_{m=0}^{\infty}\sum_{n=1}^{\infty}(2-\delta_{m0})\cos[m(\phi-\phi')]\sin(\beta_n z)\sin(\beta_n z')\,\frac{R_{mn\,1}(r_<)R_{mn\,2}(r_>)}{I_m(\beta_n a)K_m(\beta_n b)-I_m(\beta_n b)K_m(\beta_n a)}.$$

$$R_{mn\,1}(r) = K_m(\beta_n a)I_m(\beta_n r) - I_m(\beta_n a)K_m(\beta_n r).$$

Good news: non-oscillating solution in r-direction, we get **Er**

Bad news: **Ez, Eϕ** show slow convergence, or are even divergent (in r'-plane)!

CLEARY NOT SUFFICIENT for a 3D SPACE-CHARGE-DISTR. !!!

... more Green's functions ?

After a few months of fighting with merciless Bessel functions, two more solution were derived **[CERN-OPEN-2009-003]**

(with different regions of slow convergence ...)

(1)

$$G(r,\phi,z;r',\phi',z') = \frac{1}{2\pi}\sum_{m=0}^{\infty}\sum_{n=1}^{\infty}(2-\delta_{m0})\cos[m(\phi-\phi')]\frac{R_{mn}(r)R_{mn}(r')}{\bar{N}^2_{mn}}\frac{\sinh(\beta_{mn}z_<)\sinh(\beta_{mn}(L-z_>))}{\beta_{mn}\sinh(\beta_{mn}L)},$$

$$R_{mn}(r) = Y_m(\beta_{mn}a)J_m(\beta_{mn}r) - J_m(\beta_{mn}a)Y_m(\beta_{mn}r).$$

(2)

$$G(r,\phi,z;r',\phi',z') = \frac{1}{\pi L}\sum_{m=0}^{\infty}\sum_{n=1}^{\infty}(2-\delta_{m0})\cos[m(\phi-\phi')]\sin(\beta_n z)\sin(\beta_n z')\frac{R_{mn\,1}(r_<)R_{mn\,2}(r_>)}{I_m(\beta_n a)K_m(\beta_n b)-I_m(\beta_n b)K_m(\beta_n a)}.$$

$$R_{mn\,1}(r) = K_m(\beta_n a)I_m(\beta_n r) - I_m(\beta_n a)K_m(\beta_n r).$$

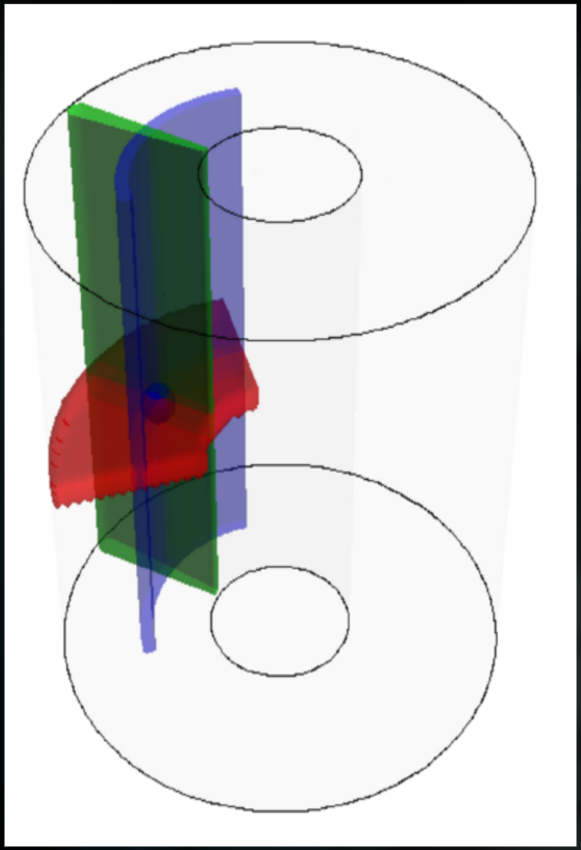
(3)

$$G(r,\phi,z;r',\phi',z') = \frac{1}{L}\sum_{k=1}^{\infty}\sum_{n=1}^{\infty}\frac{\cosh[\mu_{nk}(\pi-|\phi-\phi'|)]}{\mu_{nk}\sinh(\pi\mu_{nk})}\sin(\beta_n z)\sin(\beta_n z')\frac{R_{nk}(r)R_{nk}(r')}{N^2_{nk}}.$$

$$R_n(r) = L_{i\mu}(\beta_n a)K_{i\mu}(\beta_n r) - K_{i\mu}(\beta_n a)L_{i\mu}(\beta_n r).$$

Convergence properties ...

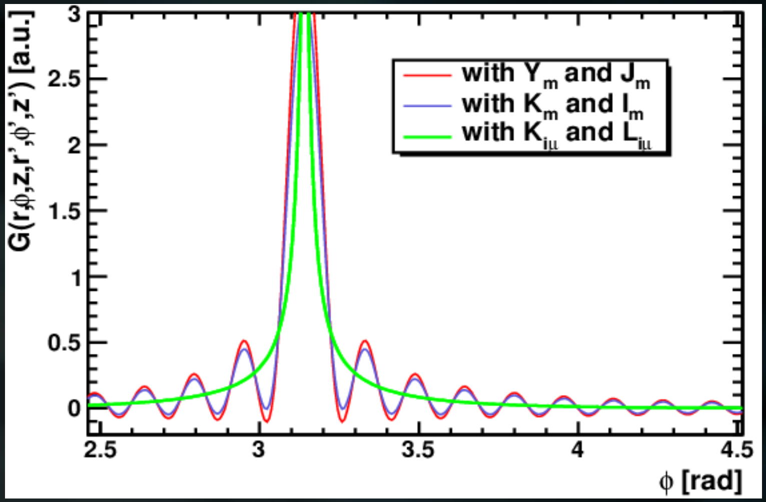
Regions of slow convergence



(1) allows fast converging **Ez**

(2) allows fast converging **Er**

(3) allows fast converging **Eϕ**

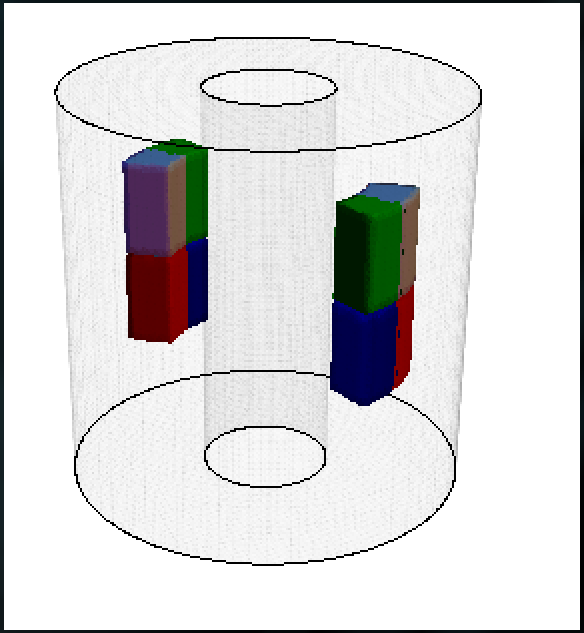


FINALLY, handling of any 3D case (not only radial symmetric ones) including correct Dirichlet boundaries in r and z-direction !!

Evaluation ...

Pre-calculation of the fields for a const. charge within a "Piece-Of-Cake" volume (P.O.C charge) at every possible position within the volume

THIS RESULTS IN A LOOKUP TABLE



Approximation of the desired charge accumulation through SIMPLE SUMMATION of the (weighted) P.O.C charges ... VERY FAST!

How fast? How accurate?

Tested for some realistic cases of: $\rho(r,z) = (A_1 + A_2\, z) / r^2$

Comparison of discretization with the analytic solutions:

- A slicing of "P.O.C" boxes of about **10 cm** size
- Summation limit for the Look-Up table generation was less then 30 terms

➡ **Accuracy** around **10^-7** for all E field components

➡ Calculation time **less then a 1 sec** !

CONCLUSION

- With the presented ANALYTICAL SOLUTIONS you can calculate EVERY FIELD component for ANY DESIRED SPACE CHARGE DISTRIBUTION.

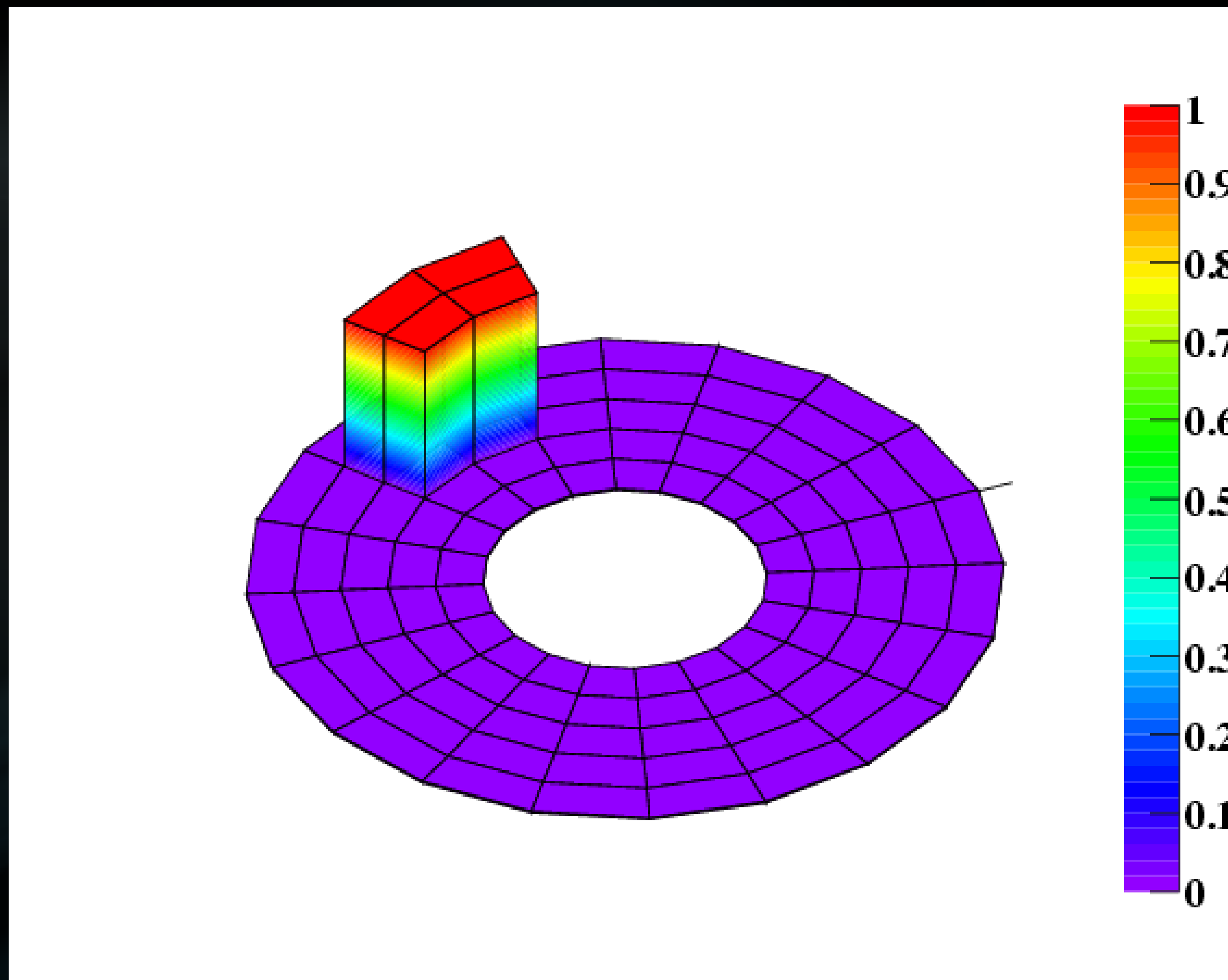
- BUT, special mathematical packages, like GSL and Alg831, are needed and the summation needs time

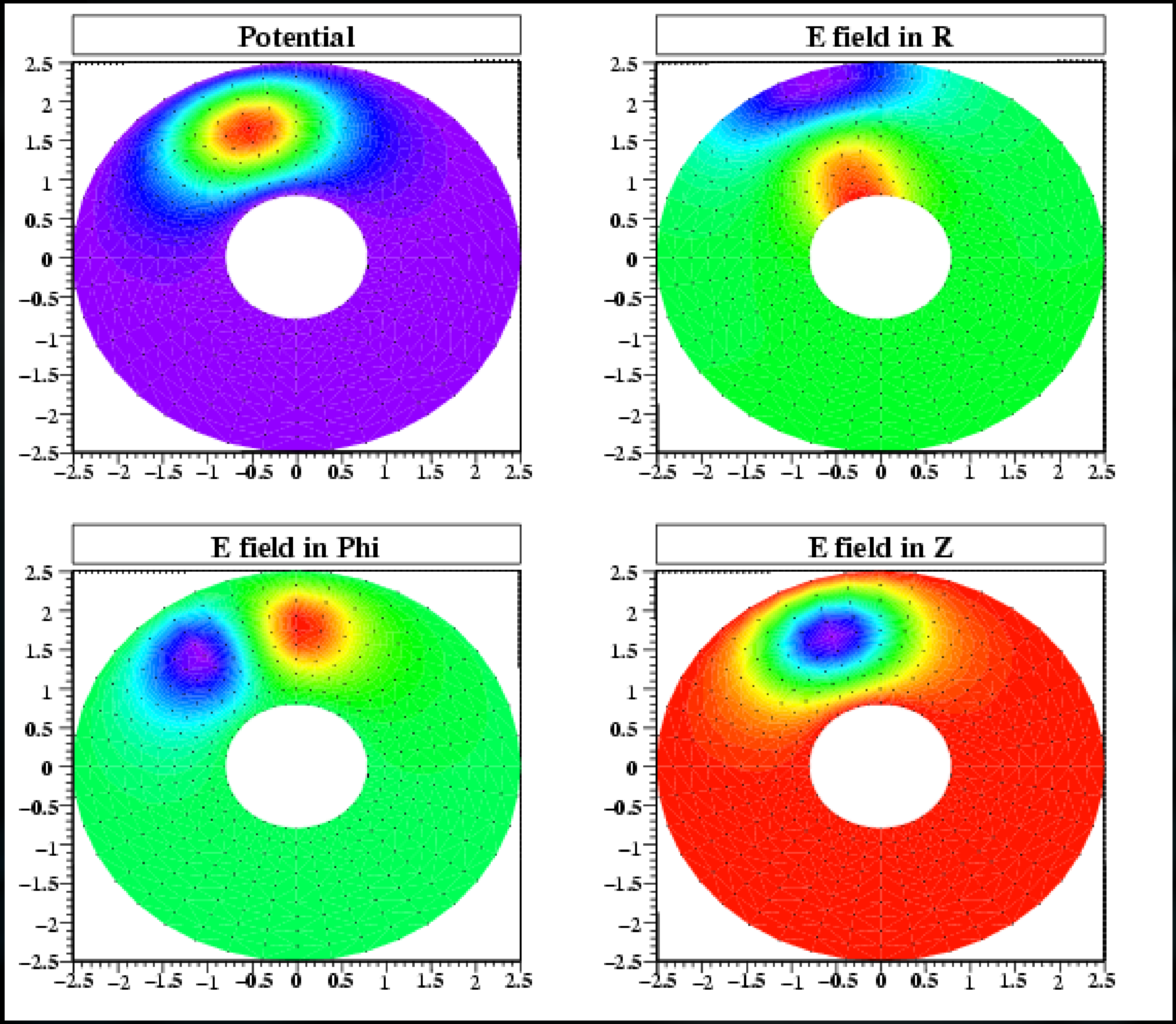
- With a LOOK UP TABLE (of const. 'P.O.C' charges) you can calculate a highly accurate approximation for every possible configuration within “NO TIME”

Example 1:

SIMPLE Charge distribution

$\rho(r,\phi,z) = 1$ ... within a box

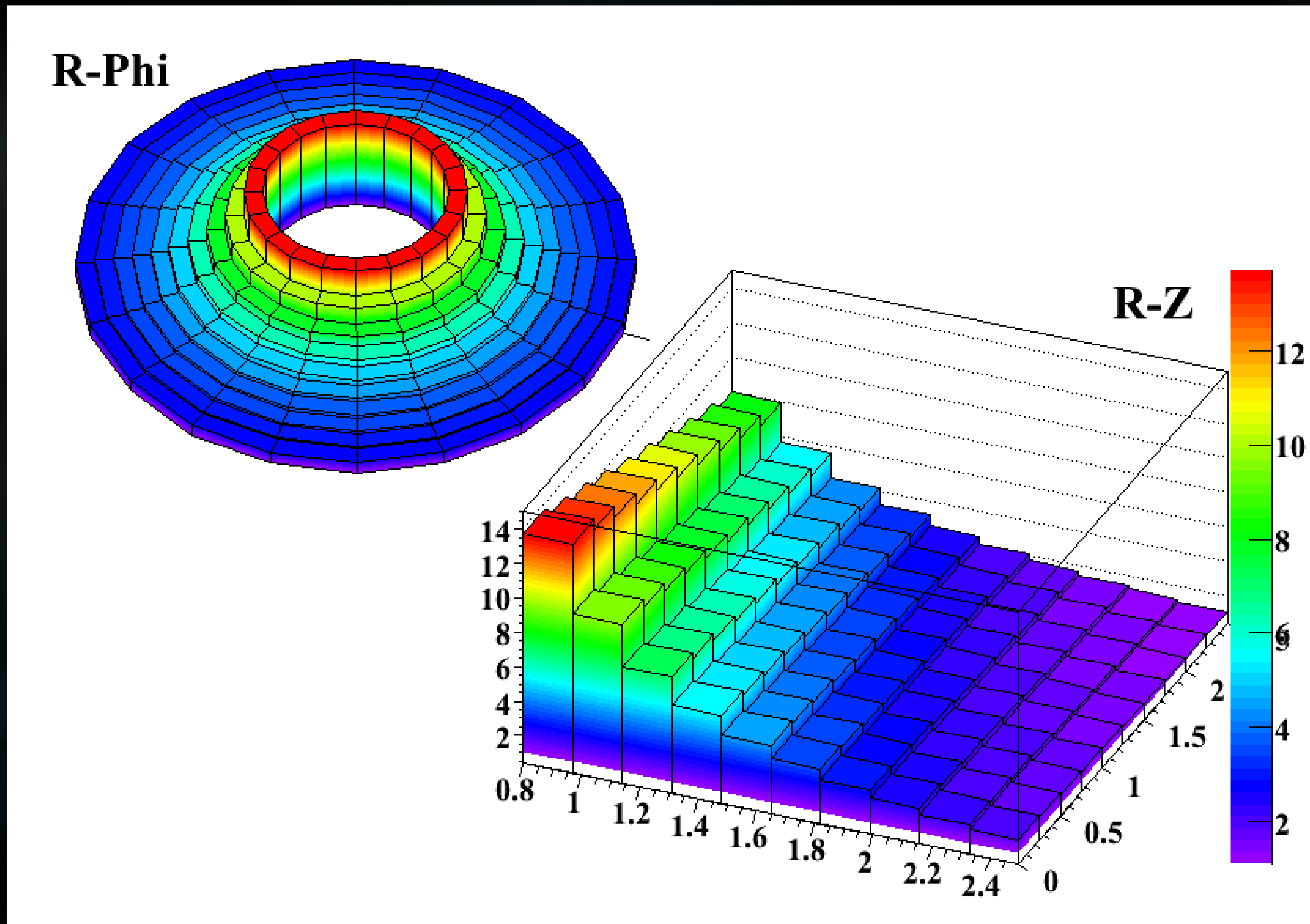


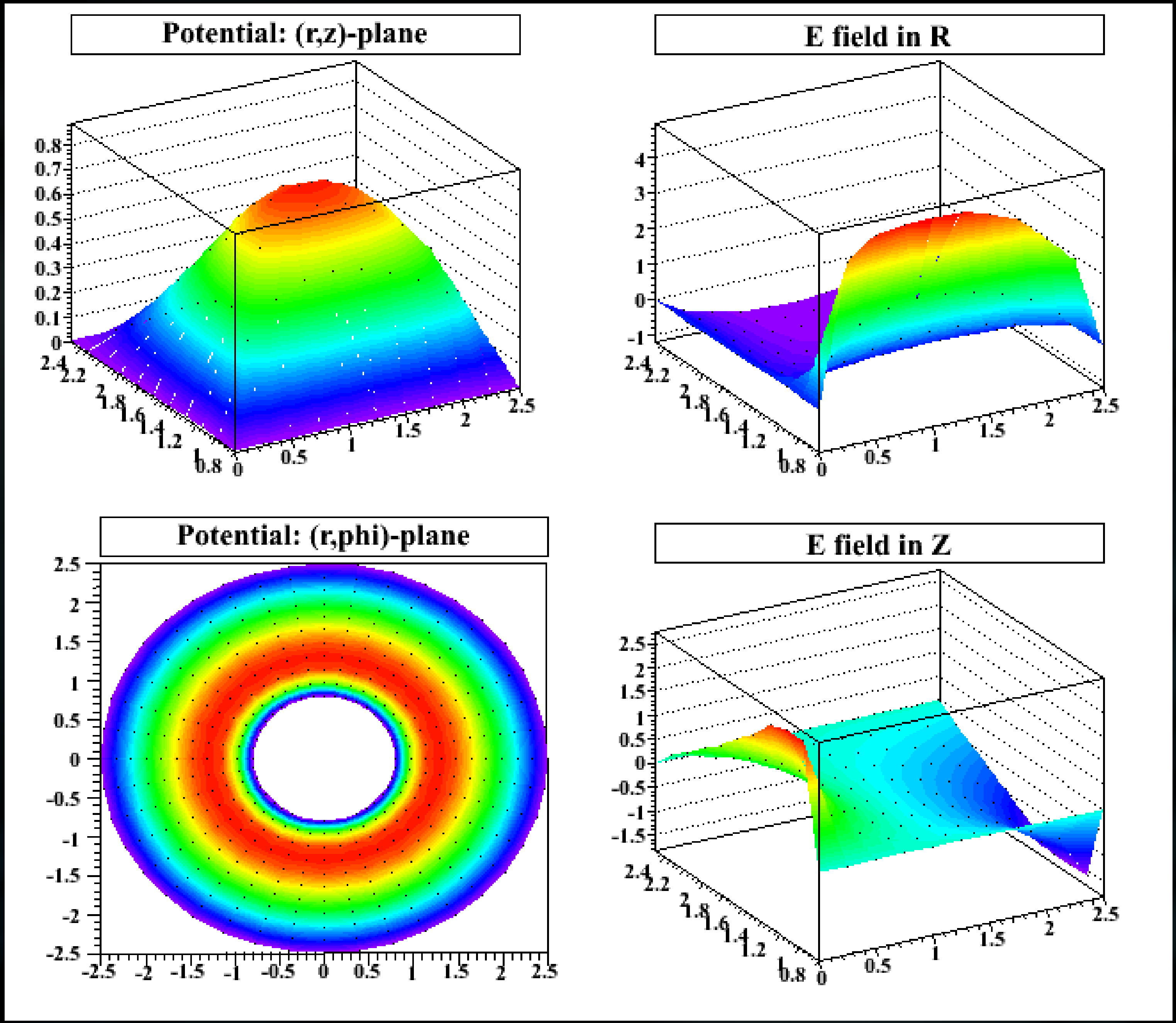

Potential
E field in R
E field in Phi
E field in Z
2.5
2
1.5
1
0.5
0
−0.5
−1
−1.5
−2
−2.5

Example 2:

REALISTIC   Charge distribution

$\rho(r,z) = 1/r^2 * k*z$ ... (used in STAR)



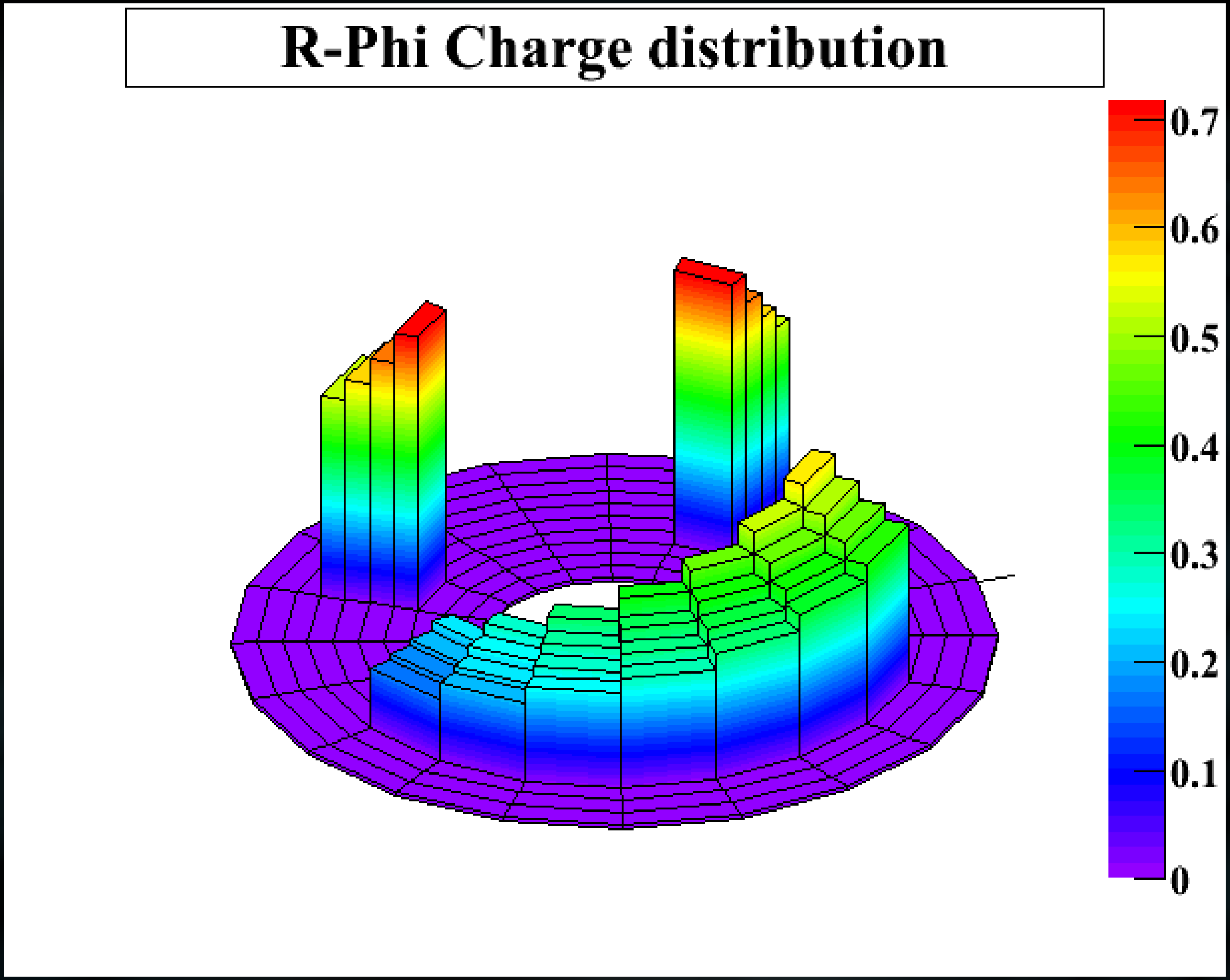


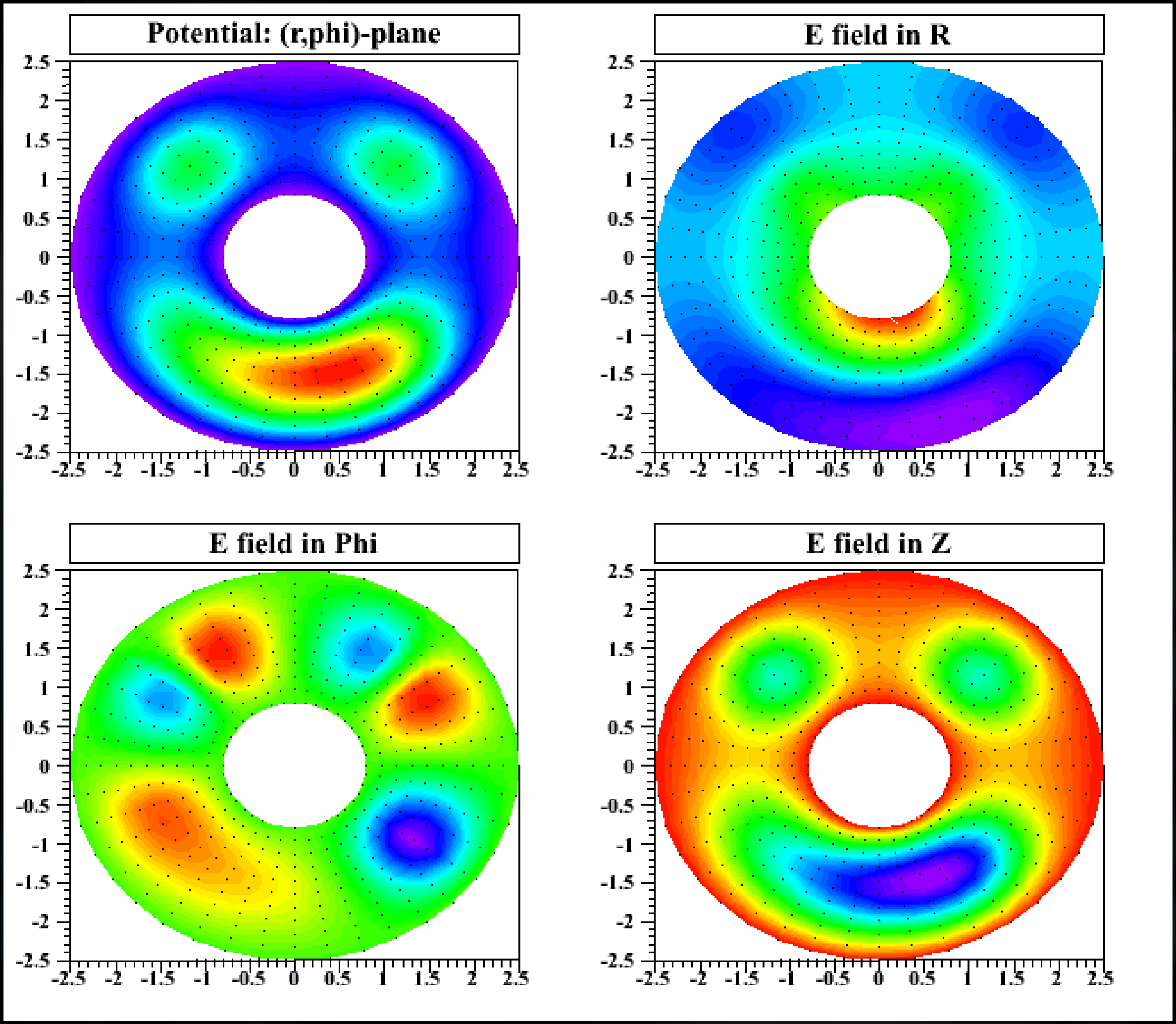
Potential: (r,z)-plane
E field in R
Potential: (r,phi)-plane
E field in Z

Example 3:

FANCY Charge distribution

$\rho(r,\phi,z)$ = ?? who can guess ??





Potential: (r,phi)-plane
E field in R
E field in Phi
E field in Z
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5

BACKUP slide:

(4)[th] solution: Green's function with fast convergence close to the Point Charge, in case you don't want to integrate over const. “Piece-Of-Cake” boxes ...

$$
\begin{aligned}
&G(r,\phi,z,r',\phi',z') = \\
&\quad = G_C(r,\phi,r',\phi',z-z') - G_C(r,\phi,r',\phi',z+z') - G_C(r,\phi,r',\phi',2L-z-z') + \\
&\qquad + \frac{1}{2\pi}\sum_{m=0}^{\infty}\sum_{n=1}^{\infty}(2-\delta_{m0})\cos[m(\phi-\phi')]\frac{R_{mn}(r)R_{mn}(r')}{N_{mn}^2}\frac{g_s(\beta_{mn})}{\beta_{mn}} \qquad (5.56)
\end{aligned}
$$

$$
\begin{aligned}
g_s &= g_0 - \frac{1}{2}\left[e^{-\beta(z_> - z_<)} - e^{-\beta(z_> + z_<)} - e^{-\beta(2L - z_> - z_<)}\right] = \\
&= \frac{e^{-\beta(2L-z_>+z_<)} + e^{-\beta(2L+z_>-z_<)} - e^{-\beta(2L+z_>+z_<)} - e^{-\beta(4L-z_>-z_<)}}{2(1-e^{-2\beta L})} \qquad (5.54)
\end{aligned}
$$

$$
\begin{aligned}
&G_C(r,\phi,r',\phi',\alpha) = \\
&\quad = G_F(r,\phi,r',\phi',\alpha) + H_C(r,\phi,r',\phi',\alpha) \\
&\quad = \frac{1}{4\pi}\frac{1}{\sqrt{r^2 - 2rr'\cos(\phi-\phi') + r'^2 + \alpha^2}} - \ldots \\
&\quad - \frac{1}{2\pi^2}\sum_{m=0}^{\infty}\epsilon_m\cos[m(\phi-\phi')]\int_0^{\infty} d\lambda\cos(\lambda\alpha)\left[I_m(\lambda r)c_m(\lambda) + K_m(\lambda r)d_m(\lambda)\right]
\end{aligned}
$$

with

$$
\begin{aligned}
\epsilon_m &= (2-\delta_{m0}) \\
c_m(\lambda) &= K_m(\lambda b)R_m(a,r')/N_m \\
d_m(\lambda) &= I_m(\lambda a)R_m(r',b)/N_m \\
R_m(s,t) &= I_m(\lambda s)K_m(\lambda t) - I_m(\lambda t)K_m(\lambda s) \\
N_m &= I_m(\lambda a)K_m(\lambda b) - I_m(\lambda b)K_m(\lambda a)
\end{aligned}
$$

(to be published)